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Modelling boundary and nonlinear effects in porous media flow

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ABSTRACT

We consider the problem of employing a Brinkman–Forchheimer system to model flow in a porous medium when Newton cooling conditions are appropriate at the boundary of the body. Specifically it is shown that the solution depends continuously on the Forchheimer coefficient and on the coefficient in the Newton cooling law at the boundary. Since we are dealing with non-slow flow rates and a porosity which is close to one we employ the Brinkman–Forchheimer equations and this leads to a second order differential inequality in the analysis as opposed to the first order one often found.

1. Introduction

In the field of stability, continuous dependence on changes in the model itself is believed to be as important a topic as the traditional idea of stability. Within the field of continuum mechanics the topic of continuous dependence on the model, or structural stability as the subject is alternatively known, was shown to be highly relevant by Knops and Payne [1], with further improvements by Knops and Payne [2] and by Payne [3–5]. Since the appearance of these pioneering articles, structural stability has been investigated for several models of flow in porous media, see e.g. [6–19]. For general modelling of flows in porous media including viscoelastic flows in such, we refer to the papers of Hayat and his co-workers, see [20–27].

In the field of porous media the theory of Darcy has, rightly, occupied a prominent position, cf. Straughan [17]. However, when the flow rate in the porous medium is no longer small, or when the porosity is close to unity, Darcy's theory is believed inadequate and various alternatives have been proposed. Among these some of the most important are the equations of Brinkman, Forchheimer, or Brinkman–Forchheimer. Such equations are analyzed in detail in many places, e.g. [6–12,14,16–19,28–31].

The aim of the present paper is to continue in the framework of structural stability and we analyze further the equations for flow in a Brinkman–Forchheimer material, but we allow for a non-isothermal situation. Previous analyses have mostly concentrated on the isothermal case.

It is worth observing that the equations of Brinkman, Forchheimer, and Brinkman–Forchheimer have recently been employed in many real life situations. For example, van der Sman [32] has shown how they apply to airflow through stacked fruit, such as oranges or tomatoes, during transit. Moreira *et al.* [33], Petrasch *et al.* [34], Zhao *et al.* [35], Phanikumar and Mahajan [36], Roy and Sundar [37], apply these equations to analyze fluid and heat transfer in high porosity ceramics or high porosity metallic foams. Saravanan and Brindha [38] provide a very interesting nonlinear stability analysis by means of an energy method.

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Since we are dealing with relatively high flow rates and a porosity close to one, we include fluid inertia in the momentum equation. In the Darcy theory, and even some Brinkman theories, inertia is often neglected, the argument being that the acceleration is so low that one may remove this term. However, Vadasz [39] has shown that the inclusion of inertia in a study of thermal convection in a porous medium leads to a striking result whereby convection can commence via an oscillating instability. Due to the importance of the inertia term we retain it here. We, in particular, study continuous dependence on the Newton cooling coefficient and on the coefficient of the nonlinear Forchheimer term. It should be noted that within Darcy theory continuous dependence on the Newton cooling coefficient was previously investigated by Payne and Straughan [15]. However, there the fluid inertia was neglected. Inclusion of fluid inertia necessitates a different technique to that of [15] and this we devise here. We find the analysis leads to a second order differential inequality (as opposed to a first order one as previously found) and we need to develop an appropriate method to deal with such an inequality.

2. The Brinkman–Forchheimer equations for temperature dependent flow in porous media

Let now Ω be a bounded domain in $\mathbb{R}^3$ with boundary Γ sufficiently smooth to allow applications of the divergence theorem. Then, for generality sufficient for the class of continuous dependence to be investigated here, the Brinkman–Forchheimer equations for non-isothermal flow in a porous medium may be taken to be

$$\begin{aligned} &\frac{1}{v_a}\frac{\partial v_i}{\partial t} = \Delta v_i - v_i - b|\mathbf{v}|v_i - p_{,i} + g_i T, \\ &v_{i,i} = 0, \\ &\frac{\partial T}{\partial t} + v_i T_{,i} = \Delta T. \end{aligned} \tag{1}$$

In these equations v_i, T, p denote the velocity, temperature, and pressure, Δ is the Laplacian operator, and standard indicial notation is employed whereby, for example, $p_{,i} \equiv \partial p/\partial x_i$. The terms v_a and b are the Vadasz number (inertial term) and the Forchheimer coefficient and g_i is a gravity field which, without loss of generality, we assume is such that $|\mathbf{g}| \le 1$. In particular, we assume $0 < v_a < \infty$, and we note $v_a = \infty$ corresponds to the zero inertia case. Instead of including the term $(1/v_a)\partial v_i/\partial t$ as inertia one may argue that the pressure gradient depends on a relaxation of the velocity field and then one arrives at Eq. $(1)_1$ but with a coefficient corresponding to the viscoelastic delay, rather than the Vadasz number. The corresponding model in a Darcy porous medium is discussed by Hussain *et al.* [22], see their Eq. (8).

Eqs. (1) hold on the domain $\Omega \times (0, \mathcal{T})$ for some $\mathcal{T} < \infty$, and v_i, T are subject to the boundary conditions,

$$v_i = 0, \qquad \frac{\partial T}{\partial n} = -k(T - T_a), \quad \text{on } \Gamma \times (0, \mathcal{T}), \tag{2}$$

where $\partial/\partial n$ is the outward normal derivative on the boundary, k is the Newton cooling coefficient, and $T_a(\mathbf{x}, t)$ is the ambient outside temperature. In addition, v_i, T, are subject to the initial conditions

$$v_i(\mathbf{x}, 0) = v_i^0(\mathbf{x}), \qquad T(\mathbf{x}, 0) = T_0(\mathbf{x}), \tag{3}$$

$\mathbf{x} \in \Omega$, for v_i^0, T_0 prescribed functions. We denote the boundary-initial value problem comprised of (1)–(3) by $\mathcal{P}$.

Since our goal here is to establish that v_i, T depend continuously on changes in the coefficients b and k we now let (v_i^*, T^*, p^*) and (v_i, T, p) be two solutions to $\mathcal{P}$ for the same initial data v_i^0, T_0, and for the same values of v_a, g_i and T_a, but for different values of b^*, k^* and b, k, respectively. Then, set

$$u_i = v_i^* - v_i, \qquad \theta = T^* - T, \qquad \pi = p^* - p, \qquad h = b^* - b, \qquad \omega = k^* - k. \tag{4}$$

One finds that the boundary-initial value problem for (u_i, θ, π) is composed of the equations

$$\begin{aligned} &\frac{1}{v_a}\frac{\partial u_i}{\partial t} = \Delta u_i - u_i - (b^*|\mathbf{v}^*|v_i^* - b|\mathbf{v}|v_i) - \pi_{,i} + g_i\theta, \\ &u_{i,i} = 0 \\ &\frac{\partial \theta}{\partial t} + v_i^*\frac{\partial \theta}{\partial x_i} + u_i\frac{\partial T}{\partial x_i} = \Delta\theta, \end{aligned} \tag{5}$$

on $\Omega \times (0, \mathcal{T})$. The boundary conditions are

$$u_i = 0, \qquad \frac{\partial \theta}{\partial n} = -k^*\theta - \omega(T - T_a), \quad \text{on } \Gamma \times (0, \mathcal{T}), \tag{6}$$

while the initial conditions are

$$u_i(\mathbf{x}, 0) = 0, \qquad \theta(\mathbf{x}, 0) = 0, \quad \mathbf{x} \in \Omega. \tag{7}$$

To proceed we let $\|\cdot\|$ and $(\cdot, \cdot)$ denote the norm and inner product on $L^2(\Omega)$ while $\|\cdot\|_3$ denotes the norm on $L^3(\Omega)$.

Before deriving continuous dependence on b and k we find it necessary to derive some *a priori* bounds.

3. A priori bounds for the temperature and velocity

We need *a priori* bounds for the L^∞ norm of T and for the L^3 norm of v_i. To achieve the first of these we follow Payne and Straughan [15], (2.15)–(2.20). The idea is to construct a differential inequality for the $L^p(\Omega)$ norm of T and let $p \to \infty$. The procedure is exactly as in [15] and one finds

$$\sup_\Omega |T| \le T_m, \tag{8}$$

where

$$T_m = \max\left\{\sup_\Omega |T_0|\,, \sup_{\Gamma\times[0,\mathcal{T}]} |T_a|\right\}. \tag{9}$$

To find a bound for the L^3 norm of v_i we multiply Eq. (1) $_1$ by v_i and integrate over Ω to find, using the divergence theorem and the boundary conditions,

$$\frac{d}{dt}\frac{1}{2v_a}\|\mathbf{v}\|^2 = -\|\nabla\mathbf{v}\|^2 - \|\mathbf{v}\|^2 - b\|\mathbf{v}\|_3^3 + (g_iT, v_i). \tag{10}$$

Likewise multiplying Eq. (1) $_3$ by T and integrating over Ω leads, with the aid of the boundary conditions, to

$$\frac{d}{dt}\frac{1}{2}\|T\|^2 = -\|\nabla T\|^2 - k\oint_\Gamma T(T-T_a)dS. \tag{11}$$

We next employ the arithmetic–geometric mean inequality on the g_i term in (10) and also use Poincaré's inequality to obtain

$$\frac{d}{dt}\frac{1}{2v_a}\|\mathbf{v}\|^2 + b\|\mathbf{v}\|_3^3 \le \frac{1}{4(\lambda_1+1)}\|T\|^2, \tag{12}$$

where λ_1 is the first eigenvalue in the membrane problem for Ω. Now employ the bound (9) in (12) and integrate to find

$$\begin{aligned}\frac{1}{2v_a}\|\mathbf{v}(t)\|^2 + b\int_0^t\|\mathbf{v}\|_3^3\,ds &\le \frac{\|\mathbf{v}_0\|^2}{2v_a} + \frac{1}{4(\lambda_1+1)}\int_0^t\|T\|^2\,ds\\ &\le \frac{1}{2v_a}\|\mathbf{v}_0\|^2 + \frac{VT_m^2t}{4(\lambda_1+1)},\end{aligned} \tag{13}$$

where V = volume of Ω. Hence, if we define the data term $\mathcal{D}_2(t)$ by,

$$\mathcal{D}_2(t) = \frac{1}{2v_ab}\|\mathbf{v}_0\|^2 + \frac{VT_m^2t}{4b(\lambda_1+1)}, \tag{14}$$

then from (13)

$$\int_0^t\|\mathbf{v}\|_3^3\,ds \le \mathcal{D}_2(t). \tag{15}$$

A similar bound for $\mathbf{v}^*$ holds with b^* replacing b in (14). Henceforth we employ $\mathcal{D}_2$ to mean the larger of (14) when b or b^* is used.

4. Continuous dependence on the Forchheimer and Newton cooling coefficients

In establishing continuous dependence we commence by multiplying $(5)_1$ by u_i and integrating over Ω, and by multiplying (5) $_3$ by θ and integrating over Ω. Upon use of the boundary conditions (6) one may show

$$\frac{d}{dt}\frac{1}{2}\|\theta\|^2 + (u_i, T_{,i}\theta) = -\|\nabla\theta\|^2 - k^*\oint_\Gamma\theta^2dS - \omega\oint_\Gamma(T-T_a)\theta dS, \tag{16}$$

and

$$\frac{1}{2v_a}\frac{d}{dt}\|\mathbf{u}\|^2 = -\|\nabla\mathbf{u}\|^2 - \|\mathbf{u}\|^2 + (g_i\theta, u_i) - \int_\Omega u_i(b^*|\mathbf{v}^*|v_i^* - b|\mathbf{v}|v_i)dx. \tag{17}$$

We now write

$$b^*|\mathbf{v}^*|v_i^* - b|\mathbf{v}|v_i = \frac{1}{2}h(|\mathbf{v}^*|v_i^* + |\mathbf{v}|v_i) + m(|\mathbf{v}^*|v_i^* - |\mathbf{v}|v_i), \tag{18}$$

where $m = (b+b^*)/2$. Further, one may rewrite the last term in (17) as shown below, cf. Payne and Straughan [16],

$$\begin{aligned}-\int_\Omega u_i(b^*|\mathbf{v}^*|v_i^* - b|\mathbf{v}|v_i)dx &= -\frac{h}{2}\int_\Omega(|\mathbf{v}^*|v_i^*u_i + |\mathbf{v}|v_iu_i)dx\\ &\quad -\frac{m}{2}\int_\Omega\left[(|\mathbf{v}^*|+|\mathbf{v}|)u_iu_i + (|\mathbf{v}^*|-|\mathbf{v}|)^2(|\mathbf{v}^*|+|\mathbf{v}|)\right]dx.\end{aligned} \tag{19}$$

Then, employing (19) in (17) we discard the last term in (19) (which is negative) and may arrive at

$$\frac{1}{2v_a}\frac{d}{dt}\|\mathbf{u}\|^2 \le -\|\nabla\mathbf{u}\|^2 - \|\mathbf{u}\|^2 + (g_i\theta, u_i) - \frac{h}{2}\int_\Omega (|\mathbf{v}^*|v_i^* u_i + |\mathbf{v}|v_i u_i)dx - \frac{m}{2}\int_\Omega (|\mathbf{v}^*| + |\mathbf{v}|)u_i u_i dx. \tag{20}$$

The next step is to take the $(u_i, T_{,i}\theta)$ term to the right hand side of (16) and integrate by parts to rewrite it as $(u_i, T\theta_{,i})$. We then employ the bound (8) and the arithmetic–geometric mean inequality in (16) to derive

$$\frac{d}{dt}\frac{1}{2}\|\theta\|^2 \le \frac{T_m^2}{4}\|\mathbf{u}\|^2 + \mathcal{D}_1\omega^2, \tag{21}$$

where $\mathcal{D}_1(t)$ is the data term

$$\mathcal{D}_1(t) = \frac{1}{4k^*}\oint_\Gamma |T_m + T_a|^2 \, dS. \tag{22}$$

Now, employ the arithmetic–geometric mean inequality on the g_i term in (20), followed by the Poincaré inequality on $\|\nabla\mathbf{u}\|^2$ and use of Young's inequality on the h terms. Then, after integration in t, one obtains

$$\frac{1}{2v_a}\|\mathbf{u}(t)\|^2 \le \left(\frac{\lambda_1 + 1}{4}\right)\int_0^t \|\theta\|^2 \, ds + \frac{h^2}{8m}\int_0^t (\|\mathbf{v}\|_3^3 + \|\mathbf{v}^*\|_3^3) ds. \tag{23}$$

Thus, employing (15) we see that

$$\frac{1}{2v_a}\|\mathbf{u}(t)\|^2 \le \left(\frac{\lambda_1 + 1}{4}\right)\int_0^t \|\theta\|^2 \, ds + \mathcal{K}_1(t)h^2, \tag{24}$$

where

$$\mathcal{K}_1(t) = \frac{\mathcal{D}_1(t)}{8m}. \tag{25}$$

Now form (21) $+ (T_m^2 v_a/2)$(24) to find

$$\frac{d}{dt}\|\theta\|^2 \le \mu_1^2 F + c_1 h^2 + \mathcal{D}_1\omega^2, \tag{26}$$

where

$$\mu_1^2 = \frac{T_m^2 v_a(\lambda_1 + 1)}{4}, \qquad c_1 = \frac{T_m^2 v_a}{2}\mathcal{K}_1, \tag{27}$$

and where we have defined $F(t)$ by

$$F(t) = \int_0^t \|\theta\|^2 \, ds. \tag{28}$$

If we define the data term $\mathcal{D}(t)$ by

$$\mathcal{D}(t) = c_1(t)h^2 + \mathcal{D}_1(t)\omega^2, \tag{29}$$

then (26) is the second order differential inequality

$$F'' - \mu_1^2 F \le \mathcal{D}. \tag{30}$$

This inequality may be integrated to obtain

$$\int_0^t \|\theta\|^2 \, ds \le \mathcal{D}H(t), \tag{31}$$

where

$$H(t) = \frac{1}{2\mu_1^2}\left[\exp\left(\frac{\mu_1 t}{2}\right) - \exp\left(-\frac{\mu_1 t}{2}\right)\right]^2. \tag{32}$$

In addition, now employing inequalities (24) and (31) one may derive

$$\|\mathbf{u}(t)\|^2 \le (\lambda_1 + 1)\frac{v_a}{2}H(t)\mathcal{D}(t) + 2v_a\mathcal{K}_1(t)h^2. \tag{33}$$

Inequalities (31) and (33) demonstrate continuous dependence on the coefficients b and k in the measures $\|\mathbf{u}(t)\|^2$ and $\int_0^t \|\theta\|^2 \, ds$.

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