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Review of Optimal Sensor Location Models for Travel Time Estimation

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The problem of optimally locating fixed sensors on a traffic network infrastructure has been object of growing interest in the past few years. Sensor location decisions models differ from each other according to the type of sensors that are to be located and the objective that one would like to optimize. This paper surveys the existing contributions in the literature related to the problem of locating fixed sensors on the network to estimate travel times. The review consists of two parts: the first part reviews the methodological approaches for the optimal location of counting sensors on a freeway for travel time estimation; the second part focuses on the results related to the optimal location of Automatic Vehicle Identification (AVI) readers on the links of a network to get travel time information.

Contents

1. Introduction

Transportation networks are the blood veins of the urban form. They provide conduits and channels for people to physically connect, for workers to keep the engines of the economy functioning, for delivery and purchase of food, fuel, and services for a sustained quality of life, for distribution of resources, and in general multiplying the extent and livability of a populated region. All this depends on the performance of the transportation system. Therefore, efficiently operating and maintaining it becomes crucial for mobility and urban sustainability.

One of the key considerations in the performance of a transportation network is the congestion level measured in terms of delays and travel times. Real-time estimation of travel times is a useful tool for traffic management and for providing information to travelers. Travel time estimation can be computed by collecting data on vehicles (flows, speeds, queues, densities etc.) on the network. This data can be collected either manually (by questionnaires, surveys, census, or even standing at street corners and counting vehicles), or by installing sensors (hardware and software) that collect such or similar data. Traditionally, collection of observations for travel time estimation has been carried out by transportation agencies through an extensive use of counting sensors (i.e., loop inductance detectors) uniformly located on freeways and highways usually at distances of 0.5-1.0 miles. The origin of such a practice is based on past analysis that has shown uniform spacing among detectors is useful for Automatic Incident Detection algorithms (ADI). The role of ADI algorithms, however, is now less important for traffic management [18, 19], due, on the one hand, to the poor performances of these algorithms in terms of detection rates, false alarm rates and mean time to detect, and, on the other hand, to the widespread use of mobile phones that have become increasingly important in the detection of incidents.

The massive use of cellular phones with GPS is now the most common source of real time data to estimate travel times. However, unlike fixed sensor data which is anonymous and is freely usable by transportation agencies vehicle/individual GPS trajectory data comes with privacy concerns. On the other hand, public vehicles with GPS like buses can be used as probe vehicles[1] that also give travel times on their routes. However, for situations where this kind of information is not readily available, the location of sensors is still an important and relevant approach to gather real time data for estimating and predicting travel times.

Studies on locating sensors have grown rapidly due to development of new methods and technologies. Following this vein, few papers, some by us and other co-authors, have studied various models that optimally locate sensors to:

(a) estimate vehicle flows on links and routes [1-9]
(b) estimate OD trips [10-12]
(c) observe the exact flows on routes [13, 14]
(d) collect data on traffic network for some general measures [15-17]

In this paper, we survey the main contributions in the literature related to the problem of optimally locating fixed sensors on a transportation network to estimate travel times. The types of sensors we will consider are the following:

(a) *Passive fixed sensors.* These do not require any active information being provided from a passing vehicle. Most common passive detectors (we use the term sensors and detectors interchangeably), and most widely used, are the single or double induction loop detectors mounted in the pavement at fixed locations. When a vehicle passes over it, the metal mass of the vehicles causes a change in the magnetic flux in the loop producing an impulse in the current through the loop; these impulses thereby count cars. Often in traffic signal operations such loops are used as counters that indicate the vehicle flow rate approaching the signalized intersection. *Single loop detectors* can be used as counting sensors and speed sensors, although as speed sensors there are errors introduced because of an underlying assumption that equal standard sized vehicles pass over the sensors. *Double*

[1] Vehicles that can be tracked via GPS locations can be used as probes to monitor the traffic; such vehicles could be specifically used by traffic agencies or could be vehicles such as buses and trucks that provide similar data.

loop detectors have two loops embedded a few feet apart in the pavement.. Double loops measure speeds more accurately since a vehicle size need not be assumed. There are other technologies used for sensing vehicles producing (i) acoustic detectors, (ii) magnetic detectors, (iii) microwave detectors, (iv) ultrasonic sensors, (v) infrared sensors, and (vi) video or image sensors; all these detect vehicles passing over a fixed point where the sensor is pointed. Some image sensors can measure vehicle queues and track vehicles in its field of view using image-processing techniques. An appropriately placed radar detector can also detect and track vehicles in the sensor's field of view. Roadside sensors are also available that can measure emissions from vehicles using real-time chemical analyses.

In this paper, we will use the term (fixed) *counting sensors* to refer to these class of detectors.

(b) *Active Fixed Detectors*. Active detectors require vehicles to actively provide its identification, needed for specific purposes such as toll collection. License plate readers, Bluetooth sniffers, and RFID or bar coded tags on vehicles are other examples. In this paper, we will use the term *Automatic Vehicle Identification* (AVI) sensors or readers.

This review focuses on fixed sensors, that is, sensor location is fixed with respect to the network infrastructure. There are also mobile sensors (piloted helicopter or drones) which can be used to provide information on traffic conditions. These contributions are not reviewed in this paper.

There are several contributions which use empirical data or simulation models to evaluate some functional performance of a predefined set of potential sensor locations for specific networks. To cite a few examples: (i) the effect of detectors spacing on travel time estimation quality was studied by Fujito et al. [26] based on real data from the field in freeways corridors in Cincinnati and Atlanta; (ii) the quality of different travel time estimation methods was studied by Li et al. [28] by using empirical data from motorways in Melbourne (Australia) and by Liu et al. [27] by using data from simulation models of I-70 corridor in Maryland; (iii) a microscopic traffic simulation model of a freeway segment located in Prince William County and Fairfax County in Norther Virginia was used by Kim et al. [29] to evaluate the location of sensors for a genetic algorithm to optimally locate them; and (iv) Thomas [47] developed a simulation model to simulate a three-mile section of Southern Avenue arterial located in Tempe and Mesa (AZ) and to compare the performance of four different sensor placement patterns. These type of contributions are not reviewed in this paper. The paper instead focuses on contributions which propose analytical models to optimally locate sensors for travel time estimation which are not limited to a specific network under study or a specific set of data.

In this context, the existing contributions can be classified into two main classes: analytical models to optimally locate counting sensors on a freeway, and analytical models to optimally locate AVI readers on a network. These two classes are reviewed in Section 2 and Section 3, respectively.

2. Locating Counting Sensors to Estimate Travel Times on a Freeway

As already mentioned, freeways have been usually equipped with counting sensors (mainly inductive loop detectors) to support traffic management and incident detection. Current focus of traffic managers is on how to use these existing sensors, and, how new ones should be located, to improve travel time estimates. Although studies on travel time estimation models are numerous, limited research has been done on the location of sensors on freeways (or on highways with uninterrupted flows) to improve the quality of travel time estimation. However, the interest in this topic is growing and different approaches to address the problem have been proposed in the last few years. Many of these approaches do not propose a general methodology to solve the location problem but are approaches based either on real data or simulated data that are specific for the network under study [23],[27]-[29]. There are some contributions in the literature, however, that propose a general methodology to locate sensors on freeways and highways that are not specific for a particular network and that can be applied in a general context. All of these approaches propose different ways to reconstruct (approximate) vehicle's trajectory from sensor speed data and hence to estimate, from such an approximated trajectory, the vehicle travel time on the freeway. These approaches are reviewed in the next section.

We now introduce the general problem by means of an example.

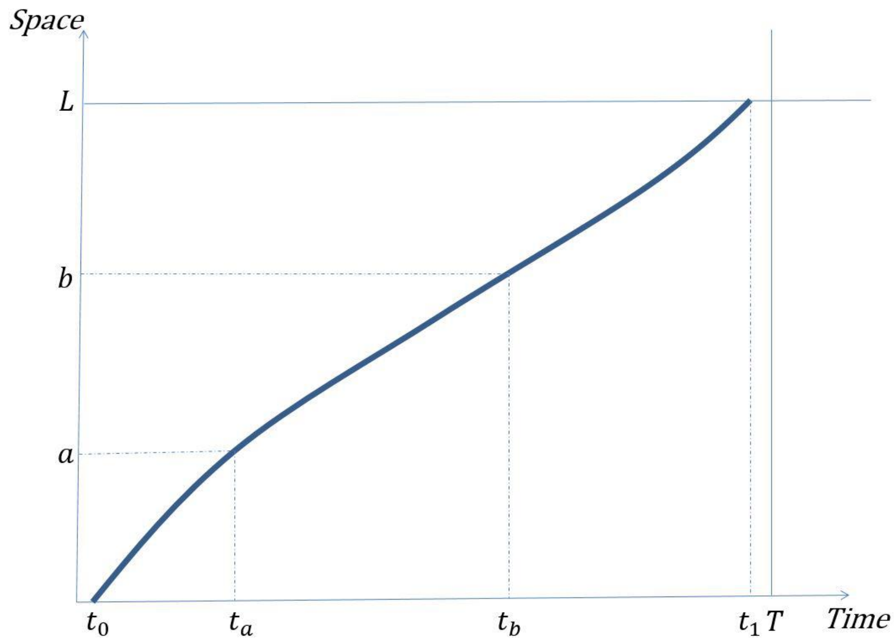

Figure 1: Space-Time diagram representing the trajectory of a vehicle.

Consider a typical freeway corridor of length L where, at the beginning of the study period $[0, T]$, flow enters the upstream point of the freeway. The motion of a vehicle traveling on the freeway can be represented by a line, referred to as *trajectory* or *profile* of the vehicle, in a space-time diagram as the one shown in Figure 1. This diagram describes the relationship between the location of a vehicle on the freeway and the time, as the vehicle travels on the freeway. The line represents the trajectory of a vehicle traversing a freeway of total length L during the given time period. The horizontal axis represents time, the vertical axis represents distance from a reference point on the freeway. Without loss of generality, we can assume the origin represents the starting point of the freeway where data collection begins. Given the trajectory of a given vehicle m, we can compute the ground-truth travel time τ^m of the vehicle as the difference between the time the vehicle exists the freeway and the time it enters. For the vehicle's trajectory in Figure 1, the ground-truth travel time can be computed as the difference between t_1 and t_0. The same approach can be used also to compute the time required by the vehicle to traverse any portion of the freeway. For example, the time required for the vehicle to go from point a to point b on the freeway is given by the difference $t_b - t_a$. If all the trajectories of all the vehicles traversing the freeway during the study period were known, we could evaluate the ground-truth travel time of each vehicle and hence compute the average travel time required to traverse the freeway during the study period. This is, of course, not possible. However, the location of sensors on the freeway could give sample information about the trajectory of the detected vehicles. This information can be used to approximate vehicles' freeway travel times and hence to compute the average travel time required to traverse the entire freeway. In particular, the location of a counting sensor at a given point of the freeway allows the knowledge of the speed of the vehicles passing on it in a given time. This data can be used to approximate the trajectory of a hypothetical vehicle on the freeway and therefore to predict (estimate) its travel time on the freeway.

Let us consider the single trajectory (that is not known) depicted in Figure 2. Let us assume that a counting sensor is located at point x_1 and that the corresponding detected speed of the vehicle when it passes over x_1 is v_1. We could approximate the motion of the vehicle on the freeway segment, by assuming constant speed of the vehicle on the entire segment and hence, travel time of the vehicle can be computed as the ratio between the length of the segment and the detected speed[2]. Hence, the estimate of vehicle's travel time $\hat{\tau}$ on the segment of length L would be:

$$\hat{\tau} = \frac{L}{v_1}$$

and the trajectory of the vehicle would be approximated as the dotted line shown in Figure 2. Such a line approximates the trajectory of the vehicle entering the study area at time $t = t_0$ and assumes constant speed of the vehicle, that is equal to the speed detected at location x_1. This is, of course, a rough approximation of the trajectory and of the resulting travel time estimation of the vehicle. The error ε in the estimation can be computed, for example, as the difference

[2] This is known in the literature as the *instantaneous model* for travel time estimation and it will be better explained in section 2.3

between the ground truth travel time τ and the estimated one $\hat{\tau}$, as depicted in Figure 2. Such an approximation, and hence, the resulting travel time estimation of the vehicle could be different if the sensor is located at a different location and a different speed is considered. Moreover, if, for example, we assume three sensors are located on the freeway respectively at point x_1, x_2 and x_3 (see Figure 3) and assume the detected speeds are respectively v_1, v_2 and v_3, a different and improved estimate would result. Indeed, in this case we could associate with each sensor i an *influence area* s_i that is a portion of the freeway that we consider to be traversed at constant speed equal to v_i. Assume, for the purpose of this example, the influence area of the sensor located at point x_1 is s_1 with length ℓ_1, the influence area of the sensor at x_2 is s_2 with length ℓ_2 and of the sensor at x_3 is s_3 with length ℓ_3, as depicted in Figure 3. The travel time of the vehicle $\hat{\tau}$ can then be estimated as $\hat{\tau} = \hat{\tau}_1 + \hat{\tau}_2 + \hat{\tau}_3$ where $\hat{\tau}_i = \frac{\ell_i}{v_i}$. The piecewise linear dotted line in Figure 3 represents the approximation of the vehicle trajectory. Again, considering a different location of the three sensors, a different travel time estimate would result.

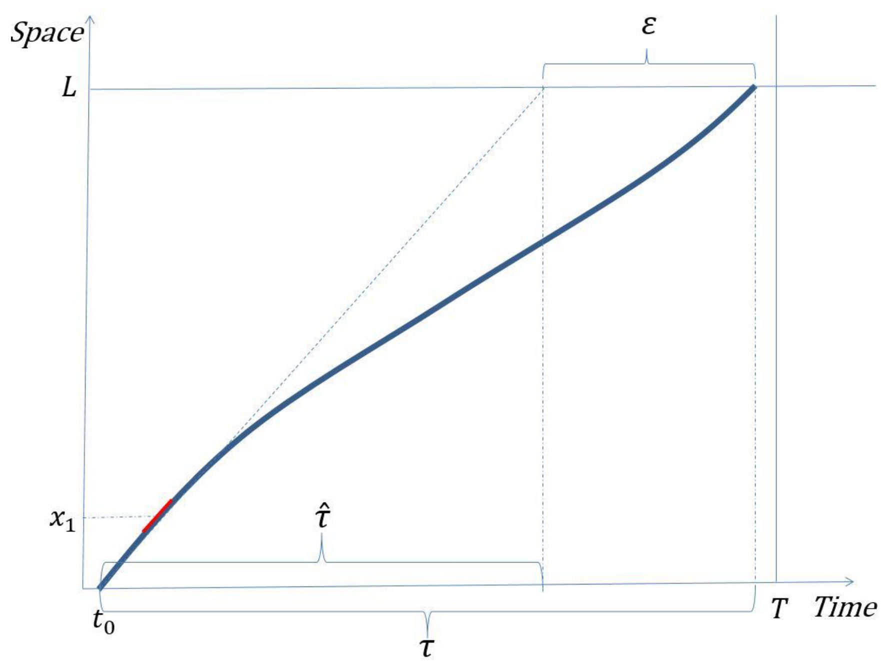

Figure 2: Space-Time diagram representing the trajectory of a vehicle and its approximation by means of the instantaneous model after the location of one sensor on x_1.

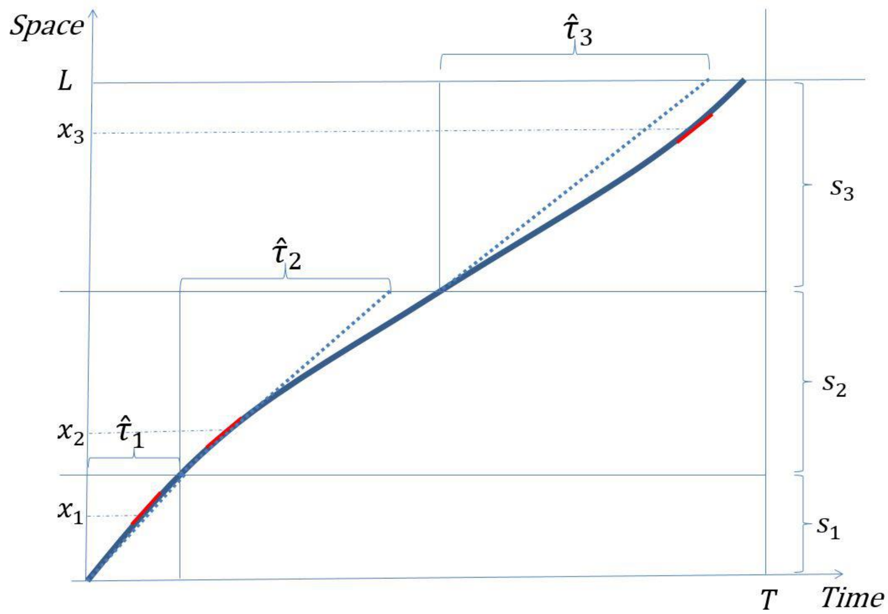

Figure 3: Space-Time diagram representing the trajectory of a vehicle and its approximation by means of the instantaneous model after the location of three sensors on x_1, x_2, and x_3

The location problem in this context is then the following: *how to locate a given number of sensors on the freeway (or how to locate sensors whose total installation cost does not exceed a given budget) so that the error in the estimation of the freeway travel time is lowest possible*. Since we do not know trajectories of all the vehicles on the freeway, we cannot compute the true estimation error associated with each trajectory; however, we could compute the error associated with a given set of locations with respect to known historical vehicle trajectories. Indeed, we will assume that historical trajectories of a given set of M vehicles are known and the estimation errors will be computed with respect to this prior information. There are different methodologies that use this combined information (i.e., detected speeds from located sensors and prior vehicles' trajectories), and several factors that influence the results, including:

- *Total number of located sensors and their location.*

  We can assume, indeed, the more sensors that are located the better are the resulting final travel time estimates.

- *Length of the influence area associated with each located sensor.*

In our illustrative example, we arbitrarily[3] defined the influence areas s_1, s_2 and s_3 for the three sensors. Had we chosen different areas the resulting travel time estimates would have been different.

- *Estimation Method.*

  We assumed, according to the instantaneous model, constant speed in each influence area, that is equal to the detected speed of the vehicle. However, different models could be adopted (these will be briefly reviewed in section 2.5) resulting in different estimates.

- *Prior vehicles' trajectories.*

  It is of course important to point out that available vehicle trajectories and vehicle speed variability, together with the other factors mentioned above, influence the final estimation accuracy.

Above factors are briefly discussed in the next subsection, where some basic notation and definitions are also given. Reported methodological approaches are then reviewed in section 2.5.

2.1 Basic Notation and Definitions

All the existing approaches are based on a set of common assumptions. In particular, it is usually assumed that the freeway is divided into n cells of equal small length δ_L, such that $L = n\delta_L$. Without loss of generality, the cells are considered to be numbered along the traffic direction from 1 to n. Each cell j is identified by an upstream boundary and a downstream boundary, the distance between them being δ_L. The length δ_L of the cell is small enough to assume that the speed of a vehicle does not vary much inside the cell; hence, the middle point of each cell represents a possible location for a counting sensor. With each located sensor an *influence area* is associated, referred to as *section* (there are different ways to associate sensors and sections as it will be explained later in section 2.2). A section on the freeway corresponds to a portion of the freeway that is represented by a collection of contiguous cells. Consider Figure 4 as an example. The upper part of the figure shows a simple freeway corridor divided into $n = 25$ cells, the

[3] We actually defined the influence areas of the three sensors according to the ZOI method that is explained in section 2.2.

lower part shows the freeway partitioned into five sections where, in particular, section $i = 1$ contains cells from 1 to 4, section $i = 2$ contains cells 5 and 6, section $i = 3$ contains cells from 7 to 12, section $i = 4$ contains cells from 13 to 16, and the last section contains cells from 17 to 25.

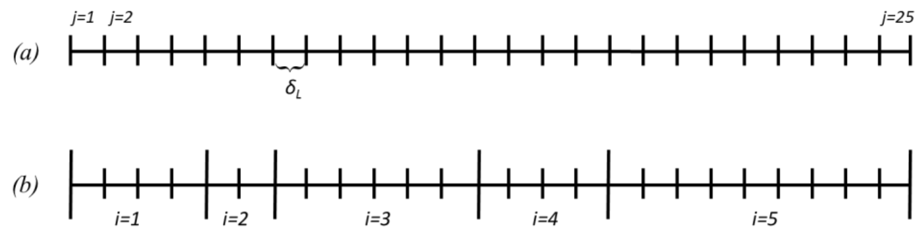

Figure 4: (a) Example of a freeway corridor divided into $n = 25$ equal cells of length δ_L. (b) Example of how the freeway can be partitioned into sections that correspond to subsets of contiguous cells.

Moreover, it is usually assumed that the trajectory of M vehicles is known from historical data for a given time period and for the given study segment and, hence, that the ground truth travel time τ^m of each of the M vehicles is known. The ground-truth travel time τ_j^m of vehicle m on cell j is also assumed to be known, since it can be computed as the difference between the time the vehicle enters the upstream boundary of the cell and the time it passes the downstream boundary of the cell. In addition, the time domain can also be divided into p time intervals of duration δ_T, such that $p\delta_T = T$. In this way the space-time diagram is discretized into a grid of np boxes. Each box (j, t) in the grid represents a data collection unit at a given location which is active for a given time period δ_T. This discretization is useful to mimic the way loop detectors collect count and speed data. Indeed, when a sensor is located, it reports average speeds for periods that are generally 20 or 30 or 60 seconds durations. With each box (j, t) in the grid, where $j = 1, 2, \dots, n$ and $t = 1, 2, \dots, p$, a speed v_{jt}^m can be associated with vehicle m that passes through cell j at time interval t. The speed data can be aggregated in different ways and at different level of aggregations according to the methodology that is used to approximate vehicle's trajectories. What is usually done is to associate with each box (j, t) the speed v_{jt} that is the average speed of the vehicles that pass cell j at time interval t, that is:

$$v_{jt} = \frac{\sum_{m=1}^{M} v_{jt}^m}{M}$$

and also, with each cell j the speed v_j, that is the average speed of the vehicles that pass cell j during the entire study period, is associated[4]:

$$v_j = \frac{\sum_{t=1}^{p} \sum_{m=1}^{M} v_{jt}^{m}}{pM}$$

These speed data can be used to reconstruct the trajectory of the M vehicles which start their trip at the beginning of the freeway segment at a given time once an influence area is associated with each sensor and an estimation method is adopted. When a sensor i is located on cell j, a section s_i, containing cell j, is associated with the sensor and the detected speed v_j (and v_{jt} at time interval t) is associated with the corresponding section. The section travel time estimate can be computed according to an estimation model (for example the instantaneous model). Different associations between sensors and sections result in different ways of associating speed to sections, and hence, result in different travel time estimations for the section. There are different methods to associate sensors with sections; these are explained next. Moreover, different methods to estimate travel times are discussed in section 2.3. Finally, in section 2.4 different ways to measure estimation error are showed. Different location models are reviewed in section 2.5.

2.2 Sensor-Section Associations

We can distinguish four different ways of associating sensors and sections: *Zone of Influence* (ZOI) method, *Neighborhood Sensor* method, *Midpoint* method and *Optimal Placement* method. The first two methods correspond to a *Locate & Divide* approach, where sensors are located first and sections are then defined. The two last methods correspond to a *Divide & Locate* approach where the boundary of the sections are defined first and sensors are successively located inside each section. These four methods are illustrated next for the simple freeway corridor shown in Figure 4.

- *Zone of Influence* (ZOI) method: The section associated with sensor h contains the portion of the freeway that corresponds to half of the distance between sensor $h-1$ and sensor h and half of the distance between sensor h and sensor $h+1$. Let us

[4] If the are some 'blank' boxes, that is boxes that are not covered by any of the M trajectories, the estimated speed v_{jt} can be computed using the average speeds associated with the surrounding boxes [20].

assume $k = 5$ sensors are located on the freeway and that cells 2, 7, 9, 16, and 21 are selected as location of the 5 sensors. The resulting five sections are shown in Figure 5 where with each section i a sensor h is associated, located on one of the cells of the section. Note that the first section (section $i = 1$ in the figure) and the last section (section $i = 5$ in the figure) are defined slightly differently. The first section is defined as the portion of the freeway from the beginning of the freeway up to half of the distance between sensor 1 and sensor 2, while the last section ends at the end of the freeway. The speeds v_{jt}^{m} associated with section i correspond with the speeds monitored by the corresponding sensor that is located on cell j of the section.

- *Neighborhood Sensor* method: A section is defined as the portion of freeway between two neighboring sensors. The resulting six sections, corresponding to the 5 located sensors on cells 2, 7, 9, 16 and 21 of our freeway example, are shown in Figure 6. Note that, in this case, the boundaries of a section correspond with the location of the sensors. Here, two phantom sensors are considered to be located, one at the beginning of the freeway segment and the other at its end (these are not shown in Figure 6). That is, the first section has the first phantom sensor as upstream sensor, while the last section has the second phantom sensor as downstream sensor. The speed associated with each section i is the average between the speed detected by the upstream the downstream sensor of the section. This is not true for the first and the last section, whose associated speeds correspond with the speed of the real sensor associated with the downstream boundary and the speed of the real sensor associated with the upstream boundary, respectively.

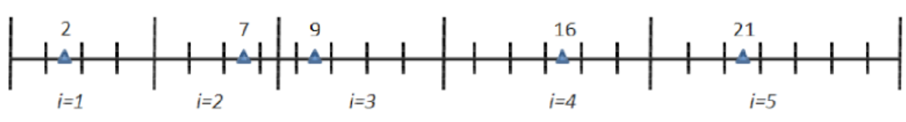

Figure 5: Association between sensors and sections of the freeway obtained by applying the Zone of Influence (ZOI) method.

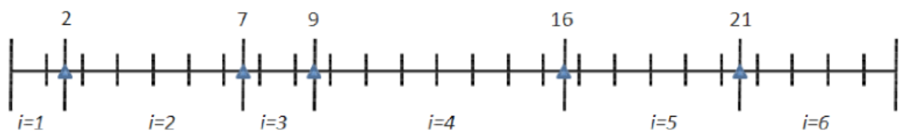

Figure 6: Association between sensors and sections of the freeway obtained by applying the Neighborhood Sensor method.

- *Midpoint* method: the freeway is first partitioned into a fixed number of sections (equal to the total number of sensors that are to be located) and, successively, a sensor is located in the middle point of each section (see Figure 7). The speeds v_{jt}^{m} associated with section i are the speeds monitored by the corresponding sensor that is located in cell j of the section.

- *Optimal Placement* method: the freeway is first partitioned into a fixed number of sections (equal to the total number of sensors that are to be located) and, successively, a sensor is located in the cell of the section that results in the best travel time estimate for the section [25]. In particular, for each cell j of the section a measure ε_j of the error in the estimation is evaluated (different error measurements can be defined, and they are explained in subsection 2.4). A sensor is located on the cell j^* of the section that shows the minimum estimation error ε_{j^*}.

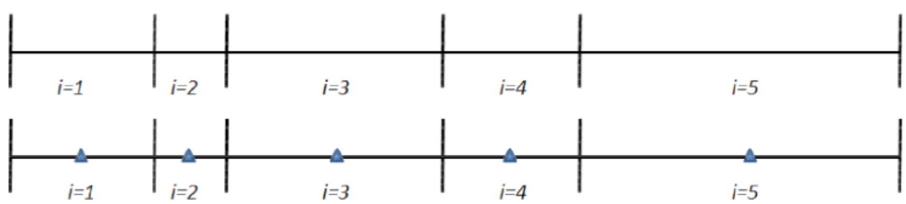

Figure 7: Association between sensors and sections of the freeway obtained by applying the Midpoint method.

2.3 Travel Time Estimation Methods

There are four different estimation methods to compute travel time on a section i of a freeway: Instantaneous model, Time Slice model, Dynamic Time Slice model and Linear model. Let us consider a study period $(0, T)$ and a set of M vehicles traveling on a freeway segment. Each vehicle will start its trip at a given time $t \in (0, T)$. Each model computes for each section the value $\hat{\tau}_i^m$ that is the estimated travel time on section i of vehicle m. One of the most commonly used assumptions is the instantaneous model:

$$\hat{\tau}_i^m = \frac{\ell_i}{v_{it}} \tag{1}$$

where, travel time $\hat{\tau}_i^m$ is computed as the ratio between the length ℓ_i of the section and the speed v_{it} associated with section i at time t. The travel time estimation obtained with this model is a rough estimate of the actual travel time, since this model assumes: (i) travel time conditions on a given section are not changing, and (ii) speeds will not change dramatically over the time it takes for each vehicle to traverse the entire freeway segment. More complex models exist that take into account both the variations of speed within the sections and the variations of speed over the time. However, for demonstration purposes, in the following we will use the instantaneous model in the approaches that will be presented in section 2.5. Some of the more complex travel time estimation methods are not considered here, the interested reader can refer to [28] for an introduction to them.

For each vehicle m, the estimation of its travel time on each section can be computed by (1) and the estimation of travel time on the entire freeway segment is obtained as the sum of the estimation of the travel time on each of the sections as follows:

$$\hat{\tau}^m = \sum_{i=1}^{S} \hat{\tau}_i^m \tag{2}$$

where S is the total number of sections defined on the freeway segment. In the discussion to follow we will denote τ_i^m to be the ground-truth travel time of vehicle m on section i, $\hat{\tau}_i^m$ the estimated travel time of vehicle m on section i; while, τ^m and $\hat{\tau}^m$ are the ground-truth travel time and the estimated travel time of vehicle m on the entire freeway segment, respectively.

2.4 Estimation Error Evaluation

To evaluate the quality of a given set of sensor locations, a measure of the travel time estimation error associated with the location set is needed. There are different error functions that are proposed in the literature, we review some of them below.

- The *Average Absolute Error (AAE)* [25] is the average, for the entire set of the M vehicles, of the absolute values of the difference between the estimated travel time and the ground-truth travel time on the freeway segment:

$$AAE = \frac{\sum_{m=1}^{M} |\hat{\tau}^m - \tau^m|}{M} \tag{3}$$

- The *Cumulative Relative Error (CRE)* [31] is the average, for the M vehicles, of the absolute value of the relative difference between the estimated travel time and the ground-truth travel time on the freeway segment:

$$CRE = \sum_{m=1}^{M} \left|\frac{\hat{\tau}^m - \tau^m}{\tau^m}\right| \tag{4}$$

Both indices (3) and (4) evaluate the difference between the estimated travel time and the ground-truth travel time for the entire freeway segment. Travel time estimation errors at the section level are not considered. A drawback of these indices is that the overestimation or underestimation of the error at the section level could cancel each other resulting in a lower value of the index, but with the possibility of large estimation errors at the section level. To quantify the quality of the estimates at the section level, the following indices can be used:

- *Error Uniformity Index (EUI)* [25]:

$$EUI = \frac{\sum_{m=1}^{M} \sum_{i=1}^{S} \ell_i \left|\frac{\hat{\tau}_i^m - \tau_i^m}{\tau_i^m}\right|}{LM} \tag{5}$$

where ℓ_i is the length of section i and L is the length of the entire freeway.

- *Mean Square Error (MSE)* [15, 20, 21]:

$$MSE = \frac{\sum_{m=1}^{M} \sum_{i=1}^{S} (\hat{\tau}_i^m - \tau_i^m)^2}{M} \tag{6}$$

In the discussions to follow we will use the following notation: ε_i^m will denote the estimation error of travel time of the single vehicle m on section i and $\varepsilon_i = \frac{\sum_{m=1}^{M} \varepsilon_i^m}{M}$ will denote the estimation error of travel time on section i.

2.5 Sensor Location Methods to Minimize Estimation Errors

We can distinguish two different modeling approaches to optimally locating sensors on a freeway to estimate travel time. The first approach is a shortest path approach [20, 24, 31] and the second approach is a clustering approach [21, 25]. Both are described next.

2.5.1 Shortest Path Approaches

Shortest path approaches proposed in the literature to solve the sensor location problem on a freeway to estimate travel time are based on a graph representation of the problem, and reduce it either (a) to finding a resource constrained shortest path on a general graph [24, 31] or (b) to finding a shortest path on an acyclic graph [20]. These two methods differ in one main aspect. The first one follows a *Locate & Divide* strategy: it focuses first in selecting the cells of the freeway where to locate sensors, and successively, it applies either the ZOI or the Neighborhood Sensor method to define the sections. The second one follows a *Divide & Locate* strategy: it focuses first in defining the boundary of the sections, and successively, in locating sensors in each determined section by applying either the Midpoint or the Optimal Placement method.

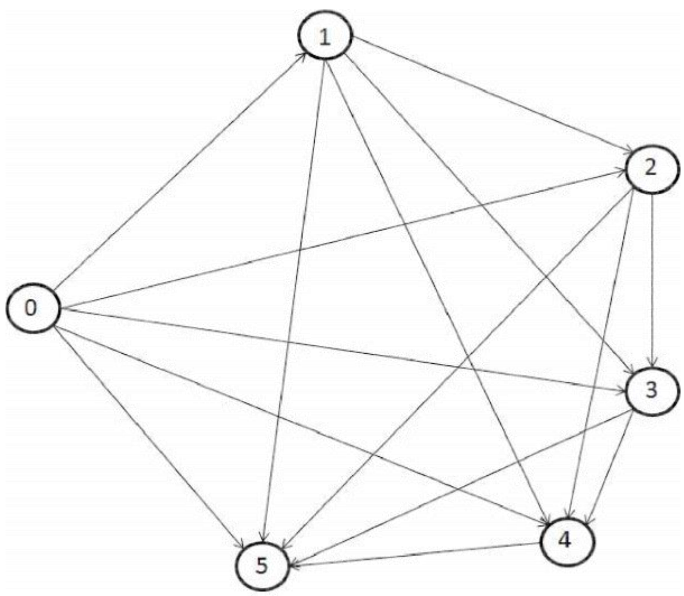

Figure 8: Graph associated with a freeway with $n = 4$ cells. The location of k sensors on the freeway corresponds to an oriented path of $k + 2$ nodes starting at node 0 end ending at node 5.

Let us discuss the first approach. Given a freeway of length L that is divided into n cells of length δ_L such that $L = n\delta_L$, a graph $G = (V, A)$, associated with it, is defined as follows. Each node $i \in V$ corresponds to the i-th cell of the freeway, $i = 1, \ldots, n$, and there exists an arc

(i, j) for each pair of nodes such that $i < j$. Two phantom nodes are added, namely node 0 and node $n + 1$, that correspond to the starting and the ending points of the freeway segment. Hence, we consider the node set V to contain also these two nodes, i.e., $V = \{0, 1, 2, \dots, n + 1\}$. Figure 8 shows the resulting graph associated with a freeway with $n = 4$ cells. This graph has 6 nodes and each node i is associated only with a node j such that $i < j$. Let us associate with each node $i \in V$ a weight c_i that defines the cost of installing a sensor on the cell corresponding to node i, and let C_{max} be the available budget. We associate with each arc (i, j) a weight ε_{ij} that is a measure of the estimation error that results after sensors are located; it is defined according to the adopted sensor-section association, the ZOI method or the Neighborhood Sensor method. For example, if the Neighborhood Sensor method is adopted then each arc $(i, j) \in A$ corresponds to a section, and hence ε_{ij} is the estimation error associated with the section represented by the arc (i, j). If the ZOI method is adopted then each arc $(i, j) \in A$ corresponds to a segment of the freeway between the sensor located at cell i and the sensor located at cell j. The first half of this segment is assumed to be traversed by vehicle m at the speed detected by the sensor located at cell i, while the second half is assumed to be traversed by vehicle m at the speed detected by the sensor located at cell j. If the MSE error evaluation method is used then we could define:

$$\varepsilon_{ij} = \frac{\sum_{m=1}^{M} (\hat{\tau}_{ij}^{m} - \tau_{ij}^{m})^2}{M}$$

where $\hat{\tau}_{ij}^{m}$ and τ_{ij}^{m} are, respectively, the estimated and the ground-truth travel time of arc (i, j) of vehicle m. Note that such weights can be computed in advance once the measurement error function, the estimation method and sensor-section association have been defined. Associated with the location of k sensors on the freeway is a directed path consisting of $k + 2$ nodes on G starting at node 0 end ending at node $n + 1$. The total weight associated with the path is the total estimation error associated with the location of the sensors, and the sum of the costs of the nodes of the path is the total installation cost. The problem of locating sensors on the freeway to minimize the total estimation error respecting a given budget limit can then be formulated as a resource constrained shortest path problem as follows.

$$\min \sum_{(i,j)\in A} \varepsilon_{ij} x_{ij} \tag{7}$$

$$s.t. \qquad \sum_{j=1}^{n+1} x_{0j} = 1 \tag{8}$$

$$\sum_{i=0}^{n} x_{in+1} = 1 \tag{9}$$

$$\sum_{j=i+1}^{n+1} x_{ij} = \sum_{j=0}^{i-1} x_{ji} \qquad \forall \quad i = 1, \dots, n \tag{10}$$

$$\sum_{j=i+1}^{n+1} x_{ij} = y_i \qquad \forall \quad i = 1, \dots, n \tag{11}$$

$$y_0 = y_{n+1} = 1 \tag{12}$$

$$\sum_{i=1}^{n} c_i y_i \le C_{max} \tag{13}$$

$$x_{ij} \in \{0,1\} \qquad \forall \quad (i,j) \in A \tag{14}$$

$$y_i \in \{0,1\} \qquad \forall \quad i \in V \tag{15}$$

The binary variable x_{ij} associated with arc $(i,j) \in A$ assumes value equal to 1 if the arc is selected to be in the shortest path and is equal to 0 otherwise. The binary variable y_i associated with node i assumes value equal to 1 if node $i \in V$ (the cell) is selected to be in the shortest path and is equal to 0 otherwise. The objective function (7) to be minimized is the total weight of the path (and hence the total estimation error). Constraints (8) and (9) ensure the shortest path starts at vertex 0 and ends at vertex $n + 1$. Constraints (10) are usual flow conservation constraints to ensure connectivity. Constraints (11) link the binary variables and ensure that if one arc (i,j) outgoing from node i is selected then node i belongs to the shortest path, that is $y_i = 1$. Constraints (12) assume the location of two phantom sensors on the two corresponding phantom cells. Finally, constraint (13) is the budget constraint which requires the total installation cost to be no more than the available budget C_{max}. Note that the above model solves the weighted version of the shortest path problem which provides the optimal locations limited by a predefined budget. The model can, of course, be used to solve also the non-weighted version of the problem, that is, optimally locate a given number k of sensors to be installed on the freeway to minimize the total estimation error, by setting $C_{max} = k$ and $c_i = 1$, $\forall i \in V \backslash \{0, n+1\}$. Danczyk and Liu [24] proposed a branch and bound method to solve model (7)-(15) when the *Neighborhood Sensor* method is used to define sections and the MSE is selected as the error evaluation function. They tested their approach by simulating traffic conditions on the Interstate I-94 in Minneapolis, Minnesota, with a total number of cells $n = 87$. They compared their strategy with the uniform spacing strategy, locating sensors at spacing of 0.5 miles. Edara et al. [31] proposed a genetic algorithm to solve model (7)-(15) when the ZOI method is used to define sections and when CRE is selected as the error evaluation function. They applied their strategy on real data from 11 miles section of I-66 in Northern Virginia and a portion of I-64 and I-95 in Richmond.

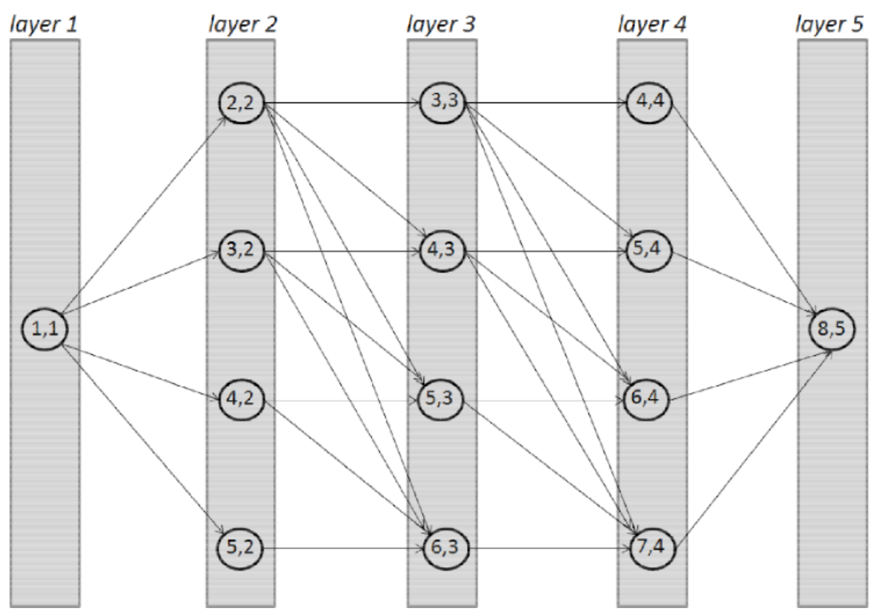

Figure 9: The layered graph associated with a freeway with $n + 7$ cells, where a total of $k = 4$ sensors need to be located.

Ban et al. [20] addressed the location problem by means of a different graph construction where the problem can be solved in polynomial time since it reduces to find an s-t shortest path on an acyclic graph with positive weights and polynomial number of arcs and nodes. The problem is addressed by means of a Divide & Locate approach. It first determines the first and the last cell of each section i, $i = 1,2, \dots, k$. Once the sections are defined, the location of sensors can be determined by either applying the Midpoint method or the Optimal Placement method. The built graph is a layered graph where directed arcs connect nodes of layer i to nodes of the subsequent layer $i+1$. If k sensors can be located on the freeway, then k sections need to be defined and the resulting graph contains $k+1$ layers. With each cell j of the freeway a set of nodes (j,i), is associated, where, node (j,i) in the graph represents a copy of cell j in layer i. In more detail, layer i contains nodes that correspond to copies of cells of the freeway whose upstream boundary could be the upstream boundary of section i (or equivalently, whose upstream boundary could be the downstream boundary of section $i-1$). For example, a directed arc between node (j,i) of layer i and node $(h,i+1)$ of layer $i+1$ corresponds to a possible section i that starts at the beginning of cell j and ends at the beginning of cell h. Of course, such an arc can exist only if $j \leq h$. Moreover, since k sections need to be determined, each layer i contains a total number of nodes equal to $n-k+1$; indeed it contains nodes corresponding to cells in the freeway with index that varies from i to $n-k+i$. Finally, the first and the last layer of the graph contain only one node, respectively node $(1,1)$ and node $(n+1,k+1)$, corresponding to the cell 1 and to the phantom cell $n+1$, respectively. In this acyclic graph, any path from node $(1,1)$ to node $(n+1,k+1)$ is a path of length k that determines k sections on the freeway. Let us consider the simple example in Figure 9 where the layered graph is built for a freeway containing $n=7$ cells and for $k=4$ sensors that have to be located. The resulting graph has $k+1=5$ layers. The first layer contains node $(1,1)$ corresponding to cell 1 of the freeway, while the last layer contains node $(8,5)$ corresponding to a phantom cell at the end of the freeway segment. Each node (j,i) of layer i is connected to a node $(h,i+1)$ of layer $i+1$ such that $j \leq h$. The second layer contains $n-k+1=4$ nodes corresponding to cells 2, 3, 4 and 5. The directed arc from node $(1,1)$ to node $(2,2)$ defines section 1 as starting at the beginning of cell 1 end ending at the beginning of cell 2. The directed arc from node $(1,1)$ to node $(4,2)$ defines section 1 as starting at the beginning of cell 1 end ending at the beginning of cell 4. Layer 2 of the graph does not contain nodes corresponding to cell 6 or 7 since otherwise a total of $k=4$ sections cannot be determined. Layer 3 contains $n-k+1=4$ nodes corresponding to cells 3, 4, 5 and 6. A path on this layered graph with nodes $(1,1),(3,2),(5,3),(7,4),(8,5)$ corresponds to the sections depicted in Figure 10; the path with nodes $(1,1),(4,2),(6,3),(7,4)$ corresponds to sections depicted in Figure 11.

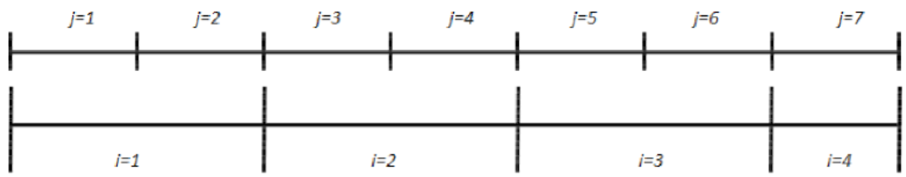

Figure 10: Sections of the freeway corresponding with the path with nodes $(\mathbf{1},\mathbf{1}),(\mathbf{3},\mathbf{2}),(\mathbf{5},\mathbf{3}),(\mathbf{7},\mathbf{4}),(\mathbf{8},\mathbf{5})$ on the layered graph of Figure 9.

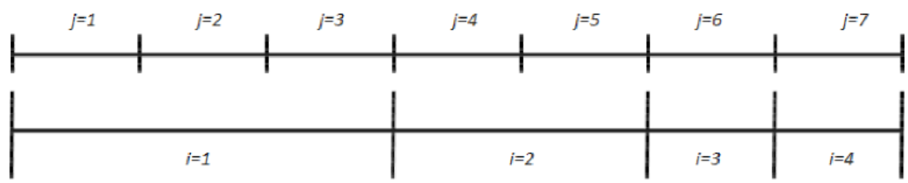

Figure 11: Sections of the freeway corresponding with a path of the graph in Figure 9 with nodes $(\mathbf{1},\mathbf{1}),(\mathbf{4},\mathbf{2}),(\mathbf{6},\mathbf{3}),(\mathbf{7},\mathbf{4})$ on the layered graph of Figure 9.

With each arc from node (j,i) to node $(h,i+1)$ a weight ε_{jh} can be associated defining the estimation error of the corresponding section. Note that such weights can be computed in advance, once the measurement error function, the estimation method and the way to associate a sensor in the section have been defined. Hence, the problem of locating k sensors on the freeway to minimize the total estimation error reduces to finding a shortest path from node $(1,1)$ to node $(n+1,k+1)$ on this layered acyclic graph. Since (i) the resulting graph has a total number of nodes equal to $(n-k-1)(n-k+1)+2$ and a total number of arcs equal to $\frac{n(n+1)}{2}$, (ii) all the weights are non negative, and (iii) the graph is acyclic, the shortest path, and hence an optimal location set, can be found in polynomial time with respect to the total number of arcs. Ban et al. [20] proposed a dynamic programming approach to solve the problem when the Midpoint method is used to locate sensors inside sections and the MSE measurement error function is considered. They demonstrated their method by applying it to simulated data of a freeway segment of 8.7 miles for a total of $n=459$ cells, and a total number of vehicles $M=3586$, for a total number of sensors to be located ranging from $k=3$ to $k=25$.

2.5.2 Clustering Approaches

The second type of methods that addresses the problem of locating sensors on a freeway to minimize travel time estimation error are based on clustering approaches. Such a method was initially proposed by Bartin et al. [21], where the problem of locating k sensors on a freeway to minimize the total estimation error was defined as the problem of defining k clusters (composed of cells of the freeway) to minimize a minimum within-group distance.

Generally speaking, given a set of data points $X = \{\mathbf{x_1}, \mathbf{x_2}, \dots, \mathbf{x_n}\}$, a clustering problem consists in defining homogeneous groups of data points (referred to as *clusters*) according to a given homogeneity measure. In our context, the set of data points corresponds to the set of n cells of the freeway and the general methodology to solve the location problem consists mainly of the following steps: (i) a clustering algorithm to minimize a within group measure is applied to group the cells into a predefined number k of clusters, (ii) clusters are then converted into disjoint sections of the freeway, and finally, (iii) the Midpoint method or the Optimal Placement method are applied to locate sensors on each determined section. The clustering methods adopts the Divide & Locate approach where the main focus is on determining how to divide the freeway into sections. Note that the clusters returned by a clustering algorithm do not necessarily contain contiguous cells. When this happens, a single cluster is converted into more than one section by merging contiguous cells of the cluster to form a section. In this way, the total number of sensors to be deployed (one for each resulting section) may be greater than or equal to the total number of clusters. This drawback of the approach is overcome by properly looking for clusters containing contiguous cells as proposed by Bartin et al. [21] and, hence, resulting in each cluster corresponding to a section, and hence to a sensor.

Let us describe now how these methods can be applied to solve our location problem. The main idea is to associate each data point $\mathbf{x_j}$ in the data set $X = \{\mathbf{x_1}, \dots, \mathbf{x_n}\}$ with a cell of the freeway segment[5]. In particular, given a set of M trajectories on the freeway, each data point $\mathbf{x_j}$ corresponds with the *speed pattern* $\mathbf{v_j} = [v_j^1, v_j^2, \dots, v_j^M]$, where v_j^m, $m = 1,2, \dots, M$, is the speed of vehicle m at cell j [21]. Any cluster methodology groups the set of cells into a

[5] In this section, we will denote vectors of elements by boldface, that is $\mathbf{x} \in R^M$.

predefined number of clusters that have similar speed patterns. The existing approaches in the literature differ in the functions used to measure similarity between speed patterns and in the clustering algorithm adopted to form the clusters. Three clustering algorithms have been applied to group the cells into a predefined number k of clusters: *agglomerative hierarchical clustering algorithm*, *k-means clustering algorithm* and *global k-means clustering algorithm*.

The agglomerative hierarchical clustering algorithm, starts from a set of clusters, each containing a single data point, and, at each step, it merges the two most similar clusters into a single one. The algorithm stops when k clusters are obtained. The mains steps of the algorithm are given in Figure 12. The similarity between two clusters is measured by a defined *distance* metric which is based on a *linkage function* that indicates the similarity of two clusters as a function of the pairwise distances of data points in the clusters. Kianfar and Edara [25] applied this algorithm by considering the Euclidian distance between two points and the single-linkage function, resulting in the following similarity measure between two clusters, say C_1 and C_2, of cells:

$$f(C_1, C_2) = \min_{\mathbf{v_j} \in C_1, \mathbf{v_h} \in C_2} \sqrt{\sum_{m=1}^{M} [(v_j^m) - (v_h^m)]^2} \tag{16}$$

The k-means clustering algorithm and the global k-means clustering algorithm (the slight differences between them will be described shortly) both aim to partition the n cells into k clusters such that the following within-cluster distance criterion is minimized:

$$\sum_{h=1}^{k} \sum_{\mathbf{v_j} \in C_h} d(\mathbf{v_j}, {}^{\boldsymbol{h}}\overline{\mathbf{v}}) \tag{17}$$

where $d(\mathbf{v_j}, {}^{\boldsymbol{h}}\overline{\boldsymbol{v}})$ is the distance between the data point $\mathbf{v_j}$ and the cluster center (centroid) ${}^{\boldsymbol{h}}\overline{\boldsymbol{v}}$. In particular, Kianfar and Edara [25] and Bartin et al. [21] applied, respectively, the k-means clustering algorithm and the global k-means clustering algorithm, to minimize (17) where

$$d(\mathbf{v_j}, {}^{\boldsymbol{h}}\overline{\mathbf{v}}) = \sqrt{\sum_{m=1}^{M} [(v_j^m)^2 - ({}^{h}\overline{v}^m)^2]} \tag{18}$$

where ${}^{\boldsymbol{h}}\overline{\boldsymbol{v}}^m$ is the average speed of vehicle m in the cells belonging to cluster h (referred to as *centroid* of the cluster):

$${}^{h}\overline{v}^m = \frac{\sum_{v_j \in C_h} v_j^m}{|C_h|} \tag{19}$$

The k-means clustering algorithm is described in Figure 13. It starts by randomly generating k centroids and assigning each cell to the nearest centroid to form the initial set of k clusters. At each iteration, the centroid of each cluster is updated and new clusters are formed. The algorithm stops when the centroids do not change, and hence, the set of clusters remain the same.

The global k-means clustering algorithm iteratively applies the k-means clustering algorithm as a subroutine. In particular, in the first iteration the centroid of the entire data set is computed, let us denote it as ${}^{\mathbf{1}}\bar{\mathbf{v}}^{(\mathbf{1})}$. In the second iteration, the 2-means clustering problem is solved by applying the k-means clustering algorithm n times and choosing as candidate 2 centroids among the n-pairs of points $\{{}^{\mathbf{1}}\bar{\mathbf{v}}^{(\mathbf{1})}, \mathbf{v}_j\}$, $j = 1, \dots, n$. The best solution among the n pairs is then stored. Let us denote the resulting two centroids corresponding to the two clusters after iteration 2 as ${}^{\mathbf{1}}\bar{\mathbf{v}}^{(\mathbf{2})}$ ${}^{\mathbf{2}}\bar{\mathbf{v}}^{(\mathbf{2})}$. The 3-means clustering problem is solved at the third iteration by applying the 3-means clustering algorithm n times and choosing the best solution from the candidate 3 centroids among the n 3-tuples $\{{}^{\mathbf{1}}\bar{\mathbf{v}}^{(\mathbf{2})}, {}^{\mathbf{2}}\bar{\mathbf{v}}^{(\mathbf{2})}, \mathbf{v}_j\}$, $j = 1, \dots, n$. The algorithm proceeds in this way until k clusters are formed. The mains steps of the global k-means algorithm are given in Figure 14.

Note that the global k-means algorithm applied as it is to solve our location problem does not ensure the resulting clusters are composed of contiguous cells. The implementation of the algorithm provided by Bartin et al. [21] ensures the clusters to have such a property. In particular, the algorithm at each iteration determines a new boundary to be added to the boundaries defined in the previous iteration to obtain a new cluster. At the beginning, there is just one cluster corresponding to the entire set of cells with boundary b_0 and b_n, that is the upstream boundary of cell $j = 1$ and the downstream boundary of cell $j = n$, respectively. The algorithm then defines $k = 2$ clusters. To obtain these first two clusters, a new boundary b_1 is needed between b_0 and b_n, so that it will correspond to the downstream boundary of the first cluster and to the upstream boundary of the second cluster. Such a boundary is chosen by evaluating the downstream boundary of each cell $j = 1, \dots, n$ as the possible boundary to be added and the corresponding value of objective function is computed. The boundary location b_1 selected will be the downstream boundary of the cell where the computed objective function value is minimum. Two clusters are then obtained and are described by the boundaries: b_0, b_1 and b_n. When solving the problem to define three clusters in the next iteration, the solution to the $k = 2$ clustering problem of the previous step is considered. Let us suppose that boundary b_1 correspond to the downstream boundary of cell $j = 3$. To define the additional boundary b_2, the downstream boundary of each cell $j = 1, \dots, n$ (except $j = 3$ which is already chosen) is selected iteratively as a candidate boundary. The same procedure as in 2-clustering problem (i.e., $k = 2$) is applied and the cell with the minimum objective function value is chosen for boundary location b_2. Reordering the boundary indices, then the three clusters are defined by boundaries b_0, b_1, b_2 and b_n. The algorithm proceeds in this way until k clusters are obtained by determining boundaries $b_0, b_1, \dots, b_{k-1}, b_n$.

Agglomerative Hierarchical Clustering Algorithm

Step 1. Assign each cell to a different cluster and form n initial clusters:

$$C_1 = \{\mathbf{v_1}\}, C_2 = \{\mathbf{v_2}\}, \dots, C_n = \{\mathbf{v_n}\}$$

Step 2. Find the two most similar clusters.
Merge them to form a new cluster.
Decrease the total number of clusters by one.

Step 3. Repeat Step 2 until k clusters are formed.

Figure 12: Agglomerative Hierarchical Clustering Algorithm

k-means Clustering Algorithm

Step 1. Randomly choose k cells as the initial centroids.
Step 2. Assign each cell to the closest centroid and form k distinct clusters
Step 3. Compute the new centroids of the clusters by applying (19).
Step 4. If the centroids remain the same then stop, otherwise goto Step 2.

Figure 13: k-means Clustering Algorithm

Global k-means Clustering Algorithm

For $i = 1,2, \dots , k$
    For $j = 1,2, \dots , n$
        - Apply the i-means clustering algorithm by choosing as initial centroids the points:
        $({}^{\mathbf{1}}\overline{\mathbf{v}}^{(i-1)}, {}^{\mathbf{2}}\overline{\mathbf{v}}^{(i-1)}, \dots , {}^{\boldsymbol{i-1}}\overline{\mathbf{v}}^{(i-1)}, \mathbf{v_j})$, where ${}^{\mathbf{1}}\overline{\mathbf{v}}^{(i-1)}, {}^{\mathbf{2}}\overline{\mathbf{v}}^{(i-1)}, \dots , {}^{\boldsymbol{i-1}}\overline{\mathbf{v}}^{(i-1)}$
        is the optimum solution of the $(i-1)$-means clustering algorithm
        - Let $\left({}^{\mathbf{1}}\overline{\mathbf{v}}^{\mathrm{i,j}}, {}^{\mathbf{2}}\overline{\mathbf{v}}^{\mathrm{i,j}}, \dots , {}^{\boldsymbol{i-1}}\overline{\mathbf{v}}^{i,j}\right)$ be the resulting optimum solution
        Set $({}^{\mathbf{1}}\overline{\mathbf{v}}^{(i)}, {}^{\mathbf{2}}\overline{\mathbf{v}}^{(i)}, \dots , {}^{\boldsymbol{i-1}}\overline{\mathbf{v}}^{(i)}) = \min_{j=1,\dots,n}\{({}^{\mathbf{1}}\overline{\mathbf{v}}^{\mathrm{i,j}}, {}^{\mathbf{2}}\overline{\mathbf{v}}^{\mathrm{i,j}}, \dots , {}^{\boldsymbol{i-1}}\overline{\mathbf{v}}^{i,j})\}$

Figure 14: Global k-means Clustering Algorithm

3 Locating AVI Readers to Estimate Travel Time on Networks

Travel time estimation can also be conducted using technology such as Automatic Vehicle Identification (AVI) readers. A reader, installed on the roads (above lanes, or on roadsides), can univocally identify vehicles equipped with a passive tags every time they pass it. When a vehicle is intercepted by two different readers then a measurement of the vehicle's travel time needed to cover the distance between the two readers can be computed. An estimation of the travel time can then be obtained by using summary statistics of these measurements (e.g., coefficient of variation [17], median of the travel time [49]). Different technologies can be used for AVI purposes which include (a) machine vision for reading license plates [32, 33, 48], (b) sensors that read unique blue tooth signal from each vehicle that passes its vicinity [34, 35], (c) sensors that read radio electronic tags such as RFID [36].

Travel times estimated through a system equipped with AVI readers can be determined on links of the network, on specific routes or subroutes in the network, or between Origin-Destination (OD) pairs in the network. Decisions on where to locate AVI readers on the network depend on the optimality criterion and different factors including: (i) total number of AVI readers to be installed; (ii) total number of readings than can be obtained *(*a *reading* is obtained when the same vehicle is intercepted at two different locations); (iii) length of a reading; (iv) number of covered routes in the network; (v) number of covered OD pairs and (vi) reliability of the resulting travel time estimates.

In this second part of the paper, we review some of the existing location models in this context, where the above-mentioned factors are taken into account either in the objective function or in the set of constraints. Some of these models are also reviewed with a real case application in [37]. A summary of the models, which are differentiated with respect to their underlying modeling assumptions and the components considered in the objective function, is given in Table 1. The details of the mathematical formulations are given in the next section.

All the models are applied on a general network structure which could represent either arterial roads, freeways or urban roads. We kept the network definition very general and made specific comments when the underlying assumptions of the model make it applicable only on a specific type of network (such as model AVI_1 which is developed for freeways).

Model	Authors	Objective function components						Underlying assumptions
		Covered OD pairs	Number of readers	Number of readings	Length of reading	Traffic flow	Measure of surveillance effectiveness	
AVI_1	Sherali et al. [17]	X						- sensors located on links - single route between each OD pair - an OD pair is considered to be covered if two readers are located at the upstream and the downstream end-points of the route connecting it - multiple location of readers on the same link is not allowed
AVI_2	Teodorovic et al. [40]	X		X				- sensors located on links - multiple routes between each OD pair - an OD pair is considered to be covered if at least two devices are installed on each route connecting it - multiple location of readers on the same link is allowed
AVI_3	Chen et al. [41]		X					- sensors located on links - multiple routes between each OD pair - an OD pair is considered to be covered if at least two devices are installed on each route connecting it - multiple location of readers on the same link is allowed
AVI_4	Chen et al. [41]	X						- sensors located on links - multiple routes between each OD pair - an OD pair is considered to be covered if at least two devices are installed on each route connecting the OD pair - multiple location of readers on the same link is allowed
AVI_5	Mirchandani et al. [42]				X	X		- sensors located on links - multiple routes between each OD pair - multiple location of readers on the same link is not allowed
AVI_6	Li, Ouyang [43]						X	- sensors located on links - multiple routes between each OD pair - multiple location of readers on the same link is not allowed
AVI_7	Asudechi, Aghani [44]	X	X			X	X	- sensors located on nodes - multiple routes between each OD pair - an OD pair is considered to be covered if at least two devices are installed on at least one of the routes connecting the OD pair - multiple location of readers on the same node is not allowed
AVI_8	Sánchez-Cambronero et al. [49]		X			X		- sensors located on links - multiple routes between each OD pair - multiple location of readers on the same link is not allowed

Table 1: Summary of the models for the optimal location of AVI readers on a network.

3.1 Mathematical Models

A traffic network is represented by a graph $G = (V, A)$ where the set of nodes V represents intersections in the network and the set of links A, joining node pairs, represents roads. We denote by W the set of OD pairs of the network and by R_w the set of routes connecting the OD pair $w \in W$. R denotes the entire set of routes and we have $R = \cup_{w \in W} R_w$. We denote by h^w the average number of trips connecting the OD pair w for the period of study, and by f_r

the average flow volume on route r. Additional notation will be introduced when needed in the developments to follow.

The first model AVI_1 is due to Sherali et al. [17]. It works under the assumption there is a single route used in connecting each OD pair and aims at locating a predefined number of readers (or a total number of readers whose installation cost is less than or equal to a predefined available budget C_{max}) to maximize the total benefit accrued by the coverage of the OD pairs. A route is said to be covered when sensors are located both on its upstream and its downstream links. An OD pair is considered to be covered if its connecting route is covered. Note that, the restricted assumption of having exactly one route per each OD pair is realistic when the underlying network is a sparse freeway or highway networks. Hence this model, can be related to the models presented in the previous section which are focused on such network types. However, we decided to include the description of the model in this section for clarity in the exposition and in the organization of the paper.

More formally, let us define r_{ij} to be the used path connecting the OD pair w from link i to link j. If readers are located both on link i and on link j of the route then r_{ij} is defined to be covered and a benefit u_{ij} is obtained. Let c_j be the cost of installing a reader on link j. We define z_j to be a binary variable associated with each link $j \in A$ assuming value equal to 1 if a reader is located on the link and 0 otherwise. The AVI_1 model is the following nonlinear optimization problem:

(AVI_1)

$$\max \sum_{r_{ij} \in R} u_{ij} z_i z_j \tag{20}$$

$$s.t. \qquad \sum_{j \in A} z_j \leq k \tag{21}$$

$$\sum_{j \in A} c_j z_j \leq C_{max} \tag{22}$$

$$z_j \in \{0,1\} \qquad \forall \quad j \in A \tag{23}$$

The total benefit obtained by the location is maximized by the objective function (20). Constraints (21) and (22) limit the total number of sensors that can be installed and the total budget that can be spent, respectively. Sherali et al. [17] defined the benefit factors u_{ij} associated with each OD pair as a function of the coefficient of variation of the travel time associated with each link in the route. In particular,

$$CV_j = \frac{\sigma_j}{\mu_j} \tag{24}$$

is the coefficient of variation associated with each link j, where μ_j and σ_j are estimates of the mean and standard deviation of travel times on link j, respectively. For each route r_{ij}, the corresponding benefit factor u_{ij} is computed as:

$$u_{ij} = CV_{ij} = \sqrt{\frac{\sum_{a \in r_{ij}} \sigma_a^2}{\sum_{a \in r_{ij}} \mu_a^2}}. \tag{25}$$

Model AVI_1 was solved by Sherali et al. [17] through the Reformulation Linearization Technique [38, 39] and was applied to data from Interstate-35 Freeway, San Antonio, Texas, USA.

Model AVI_2 [40] was the first location model proposed in this context. It was initially formulated under the assumption of a single route connecting each OD pair. We extend the initial formulation to consider the more realistic scenarios of multiple routes between each OD pair. In this model, and in models $AVI_3 - AVI_4$ to follow, an OD pair is said to be covered if there are at least two readers located on each of the routes used in connecting the OD pair, without any restriction on where on the routes they are located. Model AVI_2 locates k readers on the links of a network to maximize the weighted sum of the total number of readings and of the total number of covered OD pairs. Multiple readers can be located on any link and the decision variable z_j denote the number of readers located on link j, where we can assume the maximum number of readers that can be installed on each link to be related to the spacing requirement between pairs of readers. Model AVI_2 is:

$$(AVI_2) \qquad \max \frac{h_1}{H} \sum_{r \in R} f_r \left(\sum_{j \in r} z_j - 1 \right) + \frac{h_2}{|W|} \sum_{w \in W} y_w \tag{26}$$

$$s.t. \qquad y_w \leq max\{0, \sum_{j \in r} z_j - 1\} \qquad \forall \quad r \in R_w, \forall \quad w \in W \tag{27}$$

$$\sum_{j \in A} z_j \leq k \tag{28}$$

$$z_j \in \{0,1,2,\ldots\} \qquad \forall \quad j \in A \tag{29}$$

$$y_w \in \{0,1\} \qquad \forall \quad w \in W \tag{30}$$

where y_w is a binary variable assuming value equal to 1 if the OD pair w is covered. The objective function (26) maximizes the weighted sum of (i) the ratio between the total number of readings obtained from the locations of the readers and the total trips $H = \sum_{w \in W} h^w$ in the network, and of (ii) the ratio between the total number of covered OD pairs and the total number of OD pairs in the network. Weights h_1 and h_2 are used to model the relative importance of the two components. If a weight u_w is assigned to each OD pair w then the second component of the objective function could also be written as: $\frac{h_2}{\sum_{w \in W} u_w} \sum_{w \in W} u_w y_w$. Constraints (27) are used to model coverage of an OD pair w by forcing y_w to be equal to 0 unless there are at least two readers located on each route connecting w. The total number of readers that can be installed on the network is limited by constraints (28).

Model AVI_3 [41] that follows, aims at minimizing the total number of readers to be located on the network to ensure the coverage of all the OD pairs.

$$(AVI_3) \qquad \min \sum_{j \in A} z_j \tag{31}$$

$$s.t. \qquad \sum_{j \in r} z_j \geq 2 \qquad \forall \ r \in R_w \qquad \forall \ w \in W \tag{32}$$

$$z_j \in \{0,1,2\} \qquad \forall \ j \in A \tag{33}$$

The total number of located readers is minimized in the objective function (31). Constraints (32) ensure the coverage of all the routes connectig each OD pair and hence the coverage of each OD pair. Note that multiple location of readers on the same link is allowed, however, since requiring more than two readers to be located on each link does not affect the objective function we set this maximum value to 2.

If the total number of readers is limited, Chen et al. [41] proposed the following model AVI_4 where the objective is to maximize the total number of covered OD pairs.

$$(AVI_4) \qquad \max \sum_{w \in W} y_w \tag{34}$$

$$s.t. \quad y_w \leq max\{0, \sum_{j \in r} z_j - 1\} \quad \forall \quad r \in R_w \quad \forall \quad w \in W \tag{35}$$

$$\sum_{j \in A} z_j \leq k \tag{36}$$

$$z_j \in \{0,1,2\} \quad \forall \quad j \in A \tag{37}$$

$$y_w \in \{0,1\} \quad \forall \quad w \in W \tag{38}$$

The objective function (35) maximizes the total number of ODs covered. OD coverage is modeled by constraints (35) which force, for each OD pair w, variable y_w to 0 unless there are at least two readers located on each route connecting w. No more than k readers can be installed on the network as ensured by constraint (36). As for the previous model AVI_3, since locating more than two readers on a link does not improve the objective function, constraints (37) impose variable z_j to be at maximum two.

Model AVI_5 and AVI_6 to follow both take into account (unlike the previous models) the distance between two located readers (that is, the length of a reading). Specifically, in model AVI_5 [42] this measure is considered in the objective function which optimizes the total vehicle-miles monitored, while in model AVI_5 [43, 46] it is considered in the objective function which optimizes a general benefit factor accrued by the locations.

Let d_j^r denote the distance from the origin of route r to a potential reader located on link j, for each link $j \in A$ and each route $r \in R$. Decision variables of model AVI_5 are defined as follows: z_j is a binary variable which is equal to 1 if a reader is located on link j and 0 otherwise; y_{rj} is a binary variable which is equal to 1 if link j is the most downstream reader located on route r and 0 otherwise; x_{rj} is a binary variable which is equal to 1 if a reader on link j is the most upstream reader located on route r and 0 otherwise. The mathematical formulation is the following:

$$(AVI_5) \qquad \max \sum_{r \in R} \sum_{j \in r} (y_{rj} - x_{rj}) d_j^r f_r \tag{39}$$

$$s.t. \qquad \sum_{j \in A} z_j \leq k \tag{40}$$

$$y_{rj} \leq z_j \qquad \forall \quad r \in R \qquad \forall \quad j \in A \tag{41}$$

$$x_{rj} \leq z_j \qquad \forall \quad r \in R \qquad \forall \quad j \in A \tag{42}$$

$$\sum_{j \in r} y_{rj} \leq 1 \qquad \forall \quad r \in R \tag{43}$$

$$\sum_{j \in r} x_{rj} \leq 1 \qquad \forall \quad r \in R \tag{44}$$

$$\sum_{j \in r} y_{rj} - \sum_{j \in r} x_{rj} = 0 \qquad \forall \quad r \in R \tag{45}$$

$$z_j \in \{0,1\} \qquad \forall \quad j \in A \tag{46}$$

$$y_{rj} \in \{0,1\} \qquad \forall \quad r \in R \qquad \forall \quad j \in A \tag{47}$$

$$x_{rj} \in \{0,1\} \qquad \forall \quad r \in R \qquad \forall \quad a \in A \tag{48}$$

The total vehicle-miles monitored are maximized in the objective function (39). Constraint (40) requires locating no more than k readers on the network. Constraints (41) and (42) are logical constraints linking the variables and ensuring that there cannot be a most upstream (or a most downstream) reader on a route if no reader is located on it. At most one most upstream reader and one one most downstream reader are allowed on each route by constraints (43) and (44) Finally, constraint (45) ensure that if on a route there is only one reader installed, then it is both the most upstream and downstream reader on that route, and the vehicle miles monitored on that route is zero.

The objective function of AVI_5 can be modified so that the model tries to locate AVI readers to obtain travel time statistics and improve predictability of estimated travel times. The interested reader can refer to [37, 42] for additional details.

Model AVI_6 aims at maximizing a generalized surveillance benefit function. Consider a route r and assume s_r readers are located on it. As illustrated in Section 2, a route can be divided into $s_r - 1$ sections where each section has a reader on its upstream boundary link and its downstream boundary link. Let us assume, without loss of generality, for each route two virtual readers are located at positions 0 and $s_r + 1$ on two virtual links b and t, respectively, at the very beginning and at the very end of the route. We say the portion r_{jh} of the route between link j and link h is *monitored* if (i) a reader is located on link j and on link h, and (ii) there is no reader located in between. In this case the two links j and h are said to be *paired* on route r.

Consider for example Figure 15 where a route r with 5 links, that is $j = 1,2,3,4,5$, is shown. Dotted lines in the figure denote the two virtual links b and t (which are added to the route) and where two virtual readers are assumed to be located (white triangles in the figure). Assume $s_r = 3$ readers are located on the route, on links 1, 3, and 5 (solid triangles in the figure). In this case, the monitored portions of the route are: r_{b1}, r_{13}, r_{35} and r_{5t}. The virtual link b is paired with link 1 which is also paired with link 3, which in turn is paired with link 5, and so on.

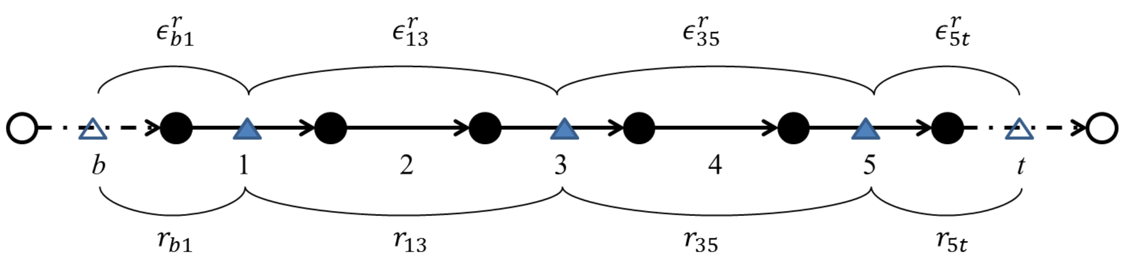

Figure 15: ***Monitored portions of a route and corresponding benefit factors.***

A benefit ε_{jh}^{r} is associated with each monitored route portion r_{jh} (see example in Figure 15) and the total benefit obtained with the given locations of the readers is then:

$$\varepsilon = \sum_{r \in R} \sum_{i=0}^{s_r} \varepsilon_{j_i j_{i+1}}^{r} \tag{49}$$

where $\varepsilon_{j_i j_{i+1}}^{r}$ is the benefit associated with the monitored route portion between the i-th reader and the $(i+1)$-th reader located on the route.

Let y_{jh}^{r} be a binary variable which is equal to 1 if the portion of route r between two readers located on link j and link h is monitored and 0 otherwise.The mathematical formulation for AVI_6 is as follows [43, 46]:

$$(AVI_6) \qquad \max \sum_{r \in R} \sum_{j \in F_{t-}^r} \sum_{h \in F_{j+}^r} y_{jh}^r \varepsilon_{jh}^r \tag{50}$$

$$s.t. \qquad \sum_{j \in A} z_j \leq k \tag{51}$$

$$\sum_{h \in F_{j+}^r} y_{jh}^r = z_j \qquad \forall \quad j \neq t \in r \qquad \forall \quad r \in R \tag{52}$$

$$\sum_{j \in F_{h-}^r} y_{jh}^r = z_h \qquad \forall \quad h \neq b \in r \qquad \forall \quad r \in R \tag{53}$$

$$z_b = z_t = 1 \tag{54}$$

$$z_j \in \{0,1\} \qquad \forall \quad j \in A \tag{55}$$

$$y_{jh}^r \in \{0,1\} \qquad \forall \quad h, j \in r \qquad \forall \quad r \in R \tag{56}$$

where for a given route r and a given link $j \in r$: (i) F_{j+}^r is the set of links of the route *after* link j where a reader can be located; (ii) F_{j-}^r is the set of links of the route *before* link j where a reader can be located; and (iii) F_{jh}^r is the set of links of route r between link j and link h where a reader can be located. The objective function (50) maximizes the total benefit. No more than k readers can be installed as imposed by constraint (51). Constraints (52) and (53) are used to pair links. If a reader is located on link j $(\text{i.e.}, \ z_j = 1)$ it can be paired with exactly a single link in the route that follows j where a reader is located and with exacly a single link in the route that precedes j where a reader is located. For possible examples of how benefit factors ε_{jh}^r can be defined, the reader can refer to [43].

Model AVI_7 [44] differs from the previous models in the fact that readers are assumed to be located on the nodes of the networks instead of the links of the network. Therefore, a link can be monitored if both its end nodes have a reader. Additionally, several factors can be simoultaneously optimized in the objective function including: the number of located readers, the number of covered ODs, the monitored flow, error in travel time estimation, travel time estimation variability. Let z_i be a binary variable, associated with each node $i \in V$, which is equal to 1 if a reader is located on node i and 0 otherwise; let y_r be a binary variable, associated with each route $r \in R$, which is equal to 1 if at least two readers are located on route r; and finally, let x_{ij} be a binary variable, associated with link $(i,j) \in A$, which is equal to 1 if a reader is located both on node i and on node j. Finally, let γ_{ij}^r a parameter equal to 1 if link (i,j) belongs to route r. The mathematical formulation of model AVI_7 is as follows:

$$(AVI_7) \qquad \max f(z_j, y_r, x_{ij}) \tag{57}$$

$$s.t. \qquad \sum_{i \in V} c_i z_i \leq C_{max} \tag{58}$$

$$y_r \leq min\{1, \sum_{i \in V} \sum_{j \in \delta(i)} \gamma_{ij}^r x_{ij}\} \qquad \forall \quad r \in R \tag{59}$$

$$x_{ij} \leq max\{0, z_i + z_j - 1\} \qquad \forall \quad (i,j) \in A \tag{60}$$

$$z_i \in \{0,1\} \qquad \forall \quad i \in V \tag{61}$$

$$y_r \in \{0,1\} \qquad \forall \quad r \in R \tag{62}$$

$$x_{ij} \in \{0,1\} \qquad \forall \quad (i,j) \in A \tag{63}$$

The objective function (57) has the following expression $f(z_j, y_r, x_{ij}) = \alpha_1 f_1 + \alpha_2 f_2 + \alpha_3 f_3 +$

$\alpha_4 f_4 + \alpha_5 f_5$ where the weights $\alpha_i, i = 1, 2, \dots, 5$ consider importance, units and sign (minimizing vs. maximizing) for each objective:

- $f_1 = \frac{\sum_{i \in V} \sum_{j \in \delta(i)} f_{ij} x_{ij}}{\sum_{(i,j) \in A} f_{ij}}$ is the ratio between the monitored flow in the network divided by the total flow;
- $f_2 = \frac{\sum_{i \in V} \sum_{j \in \delta(i)} CV_{ij} x_{ij}}{\sum_{(i,j) \in A} CV_{ij}}$ is the ratio between the sum of the coefficients of variation of the monitored links divided by the sum of the coefficients of variation of all the links of the network (see equation (24) for the definition of the coefficient of variation on a link);
- $f_3 = \frac{\sum_{r \in R} \ y_r}{|R|}$ is the ratio between the total monitored routes divided by the total number of routes of the network;
- $f_4 = \frac{\sum_{i \in V} z_i}{|V|}$ is the ratio between the total number of located readers and the total number of nodes in the network;
- $f_5 = \frac{\sum_{i \in V} \sum_{j \in \delta(i)} \epsilon_{ij} x_{ij}}{\sum_{(i,j) \in A} \epsilon_{ij}}$ is the ratio between the sum of the travel time estimation error on the monitored links divided by the sum of the travel time estimation error on all the links of the network. Asudechi and Aghani defined the estimation error as $\epsilon_{ij} = |\frac{\hat{\tau}_{ij} - \ \tau_{ij}}{\tau_{ij}}|$ where τ_{ij} is the ground truth travel time on link (i, j) and $\hat{\tau}_{ij}$ is the estimated ravel time on link (i, j).

Constraint (58) limits the total budget available to locate readers on the network. Constraints (59) force variable y_r to zero if none of the links of route r are monitored. Constraints (60) force variable x_{ij} to zero if a reader is located in at most one node among nodes i and j.

Model AVI_8 [49] aims at determining travel time for a set of target subroutes in the network. The mathematical model locates readers on links of the network so that vehicles traveling on a given subroute can be univocally identified among all the vehicles traveling in the network. The travel time on the subroute is then computed as the median of the travel time of the identified vehicles.

Before providing the mathematical formulation, let us introduce two important concepts which contitute the foundation of the developed mathematical model: the concept of *extender links* of a subroute, and the concept of *conflicting subroutes*. Consider the network in Figure 15, which is composed of 12 nodes and 13 links.

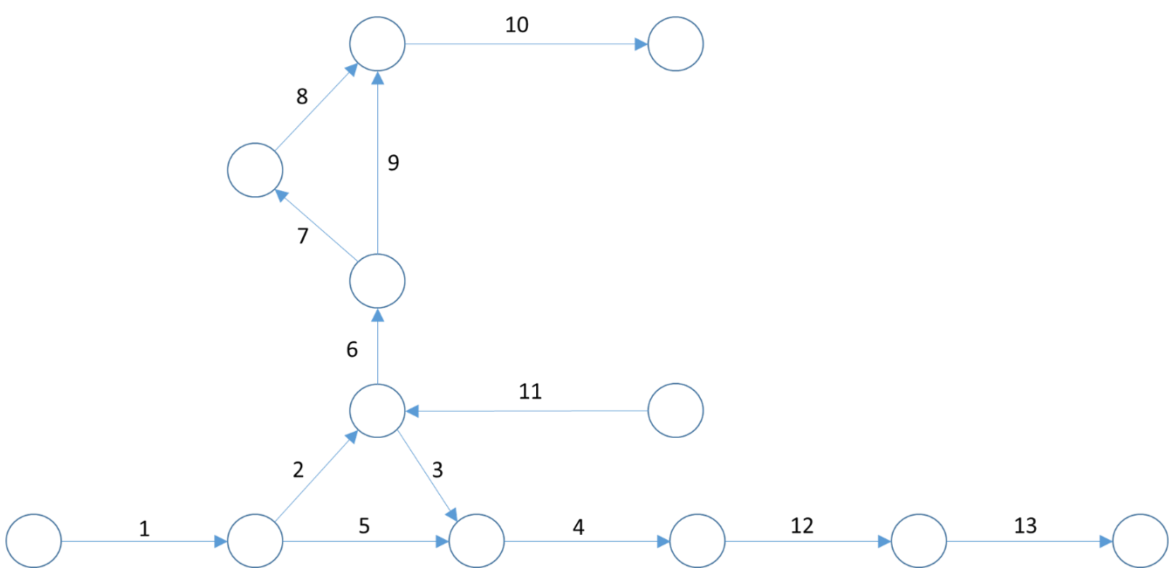

Figure 15. A network example for Model AVI_8

Assume, there are three underlying routes on this network: $R_1 = \{1,2,3,4,12,13\}$, $R_2 = \{1,5,4,12,13\}$, and $R_3 = \{6,9,10\}$, where a route is described as the list of its links. Let us assume we are interested in determining the travel time on the two subroutes $S_1 = \{1, 2, 3\}$ and $S_2 = \{6\}$. Assume also that readers can be located only at the beginnings of a link. This implies that for each subroute two readers are needed to obtain travel times; if a subroute S_i starts at link a and ends at link b, one reader needs to be located at the beginning of link a, and one at the beginning of any adjacent link c that follows link b in any route containing the subroute S_i. We refer to link c as an *extender* link for the subroute S_i. For example, to determine the travel time of the subroute $S_1 = \{1, 2, 3\}$, two readers would need to be located, one at the beginning of link 1, and the other at the beginning of link 4. Indeed the subroute S_1 is contained in route R_1, and link 4 follows the last link (that is, link 3) of subroute S_1 in R_1. Link 4 is then an extender link for subroute S_1. On the other hand, to determine the travel time on the subroute $S_2 = \{6\}$, two readers would need to be located. One on link 6, and the other on link 9, which is an extender link for subroute S_2. Indeed, link 9 comes after link 6 in route R_3 which contains S_2. Note that, since there does not exist any route in the network which contains links 7 just after link 6, then locating readers on links 6 and 7, would not be useful to estimate the travel time of S_2. Indeed, there is no vehicle actually traveling on both the links. That is, link 7 is not an extender link for subroute S_2. It is worth nothing that, there could be multiple extender links associated with any given subroute S_i, one for each route containing S_i.

The concept of extender links associated with a given set of target subroutes is important to determine a subset of links where it is *necessary* to locate readers to get the travel time on *all* the target subroutes. Indeed, given a set of target subroutes, a necessary set of links where to locate readers, in order to determine the corresponding subroutes' travel time, can be obtained as the union of the links corresponding to the first link of each target subroute and any of the extender links associated with each target subroute. In our example, a necessary set of links is the set $L = \{1,4,6,9\}$. Note that this set is necessary but not sufficient to determine the travel time of *all* the target subroutes. Indeed, this set, on the one hand, allows to univocally identify the vehicles traveling on S_2, but, on the other hand, fails to univocally identify vehicles traveling on S_1. Indeed, vehicles intercepted on links 1 and 4 (which are the first and the extender link for subroute S_1, respectively) can either be traveling on the subroute $H_1 = \{1,2,3,4\} \supset R_1$ (and hence, traveling on subroute S_1), or on the subroute $H_2 = \{1,5,4\}$. To estimate the travel time of S_1 we need to distinguish those vehicles traveling on H_1 from those traveling on H_2. We say the two subroutes H_1 and H_2 are in *conflict* to identify S_1. To identify vehicles traveling on S_1, we need to resolve the conflict. There are two methods to resolving this conflict. The first method (*M1*) is to locate one reader on a link which is in either one of the two conflicting subroutes. The second method (*M2*) will be explained later with another example. If we apply method M1 to resolve the conflict between H_1 and H_2, the location of an additional reader on link 3[6] would serve this purpose. Indeed, any vehicle which is intercepted on links 1, 3 and 4 is traveling on subroute H_1, and any vehicle which is intercepted on links 1 and 4 but not on link 3 is traveling on route H_2. Formally, given a subroute S_i and one of its extender links, say j, all the subroutes with the same first link of S_i, and with last link the extender link j, are in conflict to identify S_i. Note that, in our example, there are no conflicting subroutes to identify S_2, whose travel time can be then estimated with the location of readers on links 6 and 9.

[6]Or equivalently on link 2 or on link 5.

To explain how to resolve a conflict between conflicting subroutes by method M2, consider the case where the set of routes in the network is composed of routes R_1, R_2, R_3 and the two additional routes $R_4 = \{1, 2, 6, 7, 8, 10\}$ and $R_5 = \{11, 6, 9, 10\}$. Assume we want to determine travel time on the target subroutes $S_1 = \{1, 2, 3\}$ and $S_3 = \{6, 9\}$. Each subroute has one associated extender link. Specifically link 4 is extender for S_1, and link 10 is the extender for S_3. Hence, a set of necessary links where to locate readers to get subroutes' travel time is $L = \{1, 4, 6, 10\}$. This set is not sufficient to univocally determine the vehicles traveling on the targeted subroutes (and hence to obtain the corresponding travel time). Indeed, as before, subroutes H_1 and H_2 are in conflict to identify S_1, and, additionally, subroute $H_3 = \{6, 9, 10\}$ and $H_4 = \{6, 7, 8, 10\}$ are in conflict to identify subroute S_3. One possible way to resolve these conflicts, as we have already seen, is to apply method M1 and locate an additional reader on links belonging to one of the two conflicting subroutes. We already know, according to this method, that locating a sensor either on link 2, or 3 or 5, would resolve the conflict between H_1 and H_2. Similarly, an additional reader either on link 7 or 8 or 9 would resolve the conflict between H_3 and H_4.

Note that, vehicles traveling on H_3 are either vehicles using route R_3 or vehicles using route R_5, while vehicle traveling on H_4 are those vehicles using route R_4. Conflict between H_3 and H_4, can be resolved by using this information which is the basis for method M2. Indeed, if we locate readers either (i) to univocally identify vehicles of at least one of the two routes R_3 or R_5, or (ii) to univocally identify vehicles of route R_4, then the conflict between H_3 and H_4 can be resolved. Let us examine these two cases.

In order to univocally identify vehicles of at least one of the two routes R_3 or R_5, then at least one reader must be located on a link which belongs to either R_3 or R_5, but which does not belong to R_4. In our example, locating a reader on link 11 would univocally identify route R_5.

In order to univocally identify vehicles of route R_4, we either locate a reader on a link which belongs to both R_3 and R_5, but which does not belong to R_4 (in our example, locating a reader on link 9 would serve this purpose), or we locate a reader on a link which belongs to R_4 and does not belong to neither R_3 nor R_5 (in our example, this could be achieved by locating a reader on either link 1 or2).

ID	Necessary set	Resolving conflict between H_1 and H_2		Resolving conflict between H_3 and H_4		Final Set
		Additional link	Applied method	Additional link	Applied method	
1	{1, 4, 6, 10}	2	M1	7	M1	{1, 2, 4, 6, 7, 10}
2	{1, 4, 6, 10}	2	M1	8	M1	{1, 2, 4, 6, 8, 10}
3	{1, 4, 6, 10}	2	M1	9	M1 or M2	{1, 2, 4, 6, 9, 10}
4	{1, 4, 6, 10}	2	M1	11	M2	{1, 2, 4, 6, 10, 11}
5	{1, 4, 6, 10}	2	M1	1	M2	{1, 2, 4, 6, 10}
6	{1, 4, 6, 10}	2	M1	2	M2	{1, 2, 4, 6, 10}
7	{1, 4, 6, 10}	3	M1	7	M1	{1, 3, 4, 6, 7, 10}
8	{1, 4, 6, 10}	3	M1	8	M1	{1, 3, 4, 6, 8, 10}
9	{1, 4, 6, 10}	3	M1	9	M1 or M2	{1, 3, 4, 6, 9, 10}
10	{1, 4, 6, 10}	3	M1	11	M2	{1, 3, 4, 6, 10, 11}
11	{1, 4, 6, 10}	3	M1	1	M2	{1, 3, 4, 6, 10}
12	{1, 4, 6, 10}	3	M1	2	M2	{1, 2, 3, 4, 6, 10}
13	{1, 4, 6, 10}	5	M1	7	M1	{1, 4, 5, 6, 7, 10}
14	{1, 4, 6, 10}	5	M1	8	M1	{1, 4, 5, 6, 8, 10}
15	{1, 4, 6, 10}	5	M1	9	M1 or M2	{1, 4, 5, 6, 9, 10}
16	{1, 4, 6, 10}	5	M1	11	M2	{1, 4, 5, 6, 10, 11}
17	{1, 4, 6, 10}	5	M1	1	M2	{1, 4, 5, 6, 10}
18	{1, 4, 6, 10}	5	M1	2	M2	{1, 2, 4, 5, 6, 10}

Table 2. Possible locations of readers which lead to the estimation of the travel time of the target subroutes $\boldsymbol{S_1 = \{1,2,3\}}$ and $\boldsymbol{S_3 = \{6,9\}}$ on the network in Figure 1, when we assume there are five routes on the network: $\boldsymbol{R_1 = \{1,2,3,4,12,13\}}$, $\boldsymbol{R_2 = \{1,5,4,12,13\}}$, $\boldsymbol{R_3 = \{6,9,10\}}$, $\boldsymbol{R_4 = \{1,2,6,7,8,10\}}$ and $\boldsymbol{R_5 = \{11,6,9,10\}}$.

The possible locations of readers, in our example, which lead to the estimation of the travel time of the target subroutes $S_1 = \{1,2,3\}$ and $S_3 = \{6,9\}$ are shown in Table 2. They are the result of the combination of method M1 and method M2 to resolve the conflicts between subroutes H_1, H_2 and H_3, H_4. The first column in the table is the index of the row of the table. The second column shows the necessary set of links where to locate readers given the two target subroutes. The third column shows the additional link which can be selected to resolve the conflict between H_1 and H_2, and the corresponding method used. The fourth column shows the additional link which can be selected to resolve the conflict between H_3 and H_4, and the method used. The last column shows the resulting final set. Note that, to resolve the conflict between H_1 and H_2 we can only apply method M1 and locate a reader either on link 2, 3 or 5. To resolve the conflict between H_3 and H_4, we can locate a reader either on links 7, 8, or 9 (by applying method M1), or either on links 1, 2, 9 or 11 (when applying method M2).

Observe that, the resulting final sets have different cardinality. Additionally, it is worth noting that the number of univocally identified vehicles on the target subrotes is different according to which method is used to resolve conflicts. Since the estimation of the travel time on a subroute is computed as the median of the travel time of the identified vehicles, then, the more vehicles are identified, the more accurate the resulting estimate.

Model AVI_8 addresses the following problem: given a set of target subroutes SS and a necessary set L where to locate readers, find the minimum number of links where to locate additional AVI readers so that vehicles traveling on the target subroutes are univocally identified. Let SJ be the set of subroutes obtained by adding to any target subroute in S each of its extender links. Let SQ be the set of subroutes in conflict with routes $H \in SJ$. Let $\mathcal{R}$ be the set of all the routes on the network and of their interception sets (pairs, triads, quarters, etc.) containing subroutes in $SJ \cup SQ$. Let us define parameter δ_j^H to be equal to 1 if link $j \in H$, and 0 otherwise, parameter δ_j^R to be equal to 1 if link $j \in R$, and 0 otherwise, and parameter λ_H^R to be equal to 1 if subroute $H \subseteq R$, and 0 otherwise. Finally, we define the following binary variables: (i) z_j associated with each link $j \in A$, which is equal to 1 if a reader is located on link j, and 0 otherwise; (ii) y_H^1 associated with every subroute $H \in SJ \cup SQ$, which is equal to 1 if vehicles of subroute H can be univocally identified by applying method M1, and 0 otherwise; (iii) y_H^2 associated with every subroute $H \in SJ \cup SQ$, which is equal to 1 if vehicles of subroute H can be univocally identified by applying method M2, and 0 otherwise; (iv) p_R associated with each route $R \in \mathcal{R}$, which is equal to 1 if the vehicles of R are univocally identified, and 0 otherwise. Model AVI_8 is then formulated as:

$$(AVI_8) \qquad \min \sum_{j \in A} z_j \tag{64}$$

$$s.t. \qquad z_j = 1 \qquad \forall \quad j \in L \tag{65}$$

$$\sum_{j \in A:\ \delta_j^{H_k} + \delta_j^{H_h} = 1} z_j \geq y_{H_k}^1 \qquad \forall \quad (H_k, H_h) \in SJ \cup SQ, h > k \tag{66}$$

$$\sum_{j \in A:\ \delta_j^{R_k} + \delta_j^{R_h} = 1} z_j \geq p_{R_k} \qquad \forall \quad (R_k, R_h) \in \mathcal{R}, h > k \tag{67}$$

$$\sum_{R \in \Re} \lambda_H^R p_R \geq y_H^2 \qquad \forall \quad H \in SJ \cup SQ \tag{68}$$

$$\sum_{H \in SJ \cup SQ} \alpha_H^S (y_H^1 + y_H^2) \geq 1 \qquad \forall \quad S \in SS \tag{69}$$

$$z_j \in \{0,1\} \qquad \forall \quad j \in A \tag{70}$$

$$y_H^1, y_H^2 \in \{0,1\} \qquad \forall \quad H \in SJ \cup SQ \qquad (71)$$

$$p_R \in \{0,1\} \qquad \forall \quad R \in \quad \mathcal{R} \qquad (72)$$

The objective function (64) provides a feasible solution that minimizes the total number of readers on the link set defined by contraints (65) – (69). Constraint (65) ensures a reader is installed on every link of the set L. Constraints (66) ensure that the set of selected links where to locate readers is able to distiguish (according to method M1) vehicles traveling on subroute H_k from those traveling on subroute H_h, if $y^1_{H_k}$= 1. Constraints (67) ensure that the set of selected links where to locate readers is able to distiguish vehicles traveling on route R_k from those traveling on subroute R_h, if $p_{R_k} =$ 1. Constraints (68) ensure that the vehicles traveling on a subroute H are univocally identified according to method M2. Finally, the last set of constraints (69) ensure that vehicles of each of the target subroutes are univocally identified either by applying method M1 or by applying method M2, where α_H^S is equal to 1 if $S \subseteq H$ and 0 otherwise.

Model AVI_8 can be modified to include constraints on the minimum number of vehicles to be identified on each of the target subroutes, or also to consider multiple objective functions, for example the maximization of the univocally identified vehicles [49].

4 Open Research Questions and Conclusions

This paper surveyed analytical models to optimally locate sensors on a network to estimate travel time, either on individual links or on paths of a network. The review is divided into two main parts: the first part reviews the existing contributions for the optimal location of counting sensors on a freeway, or on individual road segments, for travel time estimation. In particular, the paper reviews contributions that propose a general methodology to locate such sensors on freeways and highways, which are not only specific for a particular network but that can be applied in a general context. Models were classified into two main approaches: shortest-path based approaches and clustering based approaches.

The second part of the paper focused on the existing results related to the optimal location of AVI readers on the links of a network to get travel time information. Several factors may be taken into account when locating AVI readers on the network (number of AVI readers to locate, number of readings, number of OD pairs, travel time reliability, etc.). Different mathematical formulations were surveyed that consider one or more of these factors to optimally locate readers.

The models reviewed in this paper focused only on fixed sensors, that is, their locations are fixed with respect to the network infrastructure. There are also mobile sensors (such as helicopter or drones). Drones are becoming readily available and being deployed commercially. They can be equipped with video camera, geo-positioning sensors and communications hardware to transmit data to a central station and can be used to provide information on traffic conditions, road conditions and emergency response. Contributions in this area are rapidly appearing [45, 50-51]; contributions in this very prolific area have not been reviewed and such an effort would result in a different survey itself [52-54].

The rapid development of new technologies result in some interesting new location models worth addressing. Some of these are discussed below.

Locating fixed RSUs in connected-vehicle systems

With the advent of Vehicle-To-Infrastructure (V2I) and Vehicle-To-Vehicle (V2V) technology, the location of Roadside Equipment units on the traffic network offers great potential in improving mobility in transportation systems in general, and in improving accuracy in travel time estimation. The main elements of V2I and V2V technology are: On-Board Unit (OBU) and the Roadside Unit (RSU). OBUs are installed on a vehicle and record vehicle's activity data in certain time intervals (referred to as snapshot) which include, among other information, speed, position and timestamp of the vehicle. RSUs are installed on the network infrastructure and store vehicle's snapshots when a vehicle enters the RSU coverage area. The information captured by the RSU can be used to estimate travel time on a given link in a given time interval. The problem of locating RSU differs from the problems reviewed since data acquisition through an RSU is accomplished with some time lag. Indeed, the snapshots of a vehicle are not available until it reaches the coverage area of an RSU. Due to this time lag, some of the snapshots recorded on a given link might not be useful anymore for travel time estimation due to the time gap between the record time and the upload time. To the best of our knowledge only three papers in the literature focus on the optimal location of RSUs for travel time estimation. Specifically, Kianfar and Edara [58], and Sun and Yang [59], address the problem of optimally locating RSUs on urban arterial roads, while Olia et al. [60], address the problem for optimally locating RSUs on freeways. The three papers do not present an analytical approach for addressing the problem; they propose simulation-based heuristics to determine the optimal location and the optimal number of RSUs. Since the focus of this paper is on analytical models for the optimal location of sensors to estimate travel time, such contributions are not discussed further.

Travel time estimation on urban arterial roads

Travel time estimation on urban arterial roads is more complex than travel time estimation on a freeway due to traffic interruptions by signal control, curb parking, and speed limits among other factors. Despite the importance of these travel time estimates for traffic management, very few contributions exist in the literature that address the problem to optimally deploy sensors for travel time estimation on urban arterial roads. The authors found two papers which analyze the optimal location of fixed passive sensors on urban roads [61-62], and the two papers mentioned above [58-59] which consider the location of RSU. Again, these are not included in the review since they do not provide any general analytical model. They instead use simulation models to evaluate the performance of preselected potential sensor locations on a specific network under study. However, due to the prohibitive cost of RSUs it would be worth investigating the deployment of RSUs on a network to optimize their functions while integrating the information provided by fixed counting detectors.

Integration of different types of sensors

The study of how to optimally deploy different types of sensors would be beneficial for link travel time estimation not only on urban arterial roads, but also for freeways and long arterials. The authors found one contribution which focuses on this specific topic. Xing et al. [55] address the problem of optimally locating fixed passive sensors and AVI readers, including the use of probe vehicles, to reduce error in travel time estimation.

Sensor failures

The models reviewed in this paper assume sensors to indefinitely perform successfully. What happens if a sensor fails? What is the best way to deploy sensors under this assumption? Development of analytical models which account for sensor failure are important directions of future research. This topic has been addressed by Li and Ouyang [43, 46], who present mathematical models to optimally locate AVI readers on the links of a network, and by Danczyk et al. [56] who extend their deterministic model, presented in section 2.5, to account for probabilistic sensor failures.

Uncertainty in traffic conditions

Optimal location of sensors to minimize travel time estimation error under non-recurring traffic conditions is another topic which deserves more investigation. A first attempt in this direction was performed by Xing et al. [55] and Park and Haghani [57]. Xing et al. [55] proposed a Kalman-filter based approach to optimally locate sensors considering both recurring and non-recurring traffic conditions. Also, Park and Haghani [57] proposed a two-stage stochastic formulation to optimally re-locate portable sensors to account for dynamic traffic patterns.

Adaptive and proactive signal control systems

Travel time estimation and prediction is also useful for adaptive and proactive signal control systems that have embedded in the system algorithms that estimate arrival of vehicles at the stop bar after they pass an upstream counting detector [63, 64]. In such systems the detector locations are either already given, or new locations are heavily constrained due to physical considerations at the intersections, such as availability of electric power and conduits for them. Generally, in such control systems there exist tradeoffs between prediction horizon and estimation accuracy, and a useful problem, not researched so far, would be for optimally locating detectors to consider such trade off.

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