Probabilistic Theory of Plastic Mechanism Control for Steel Moment Resisting Frames

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ABSTRACT

This work aims at the development of an advanced method for the seismic design of Moment Resisting Frames (MRFs) based on a target value of the failure probability in the attainment of a collapse mechanism of global type for stochastic frames (considering the aleatoric uncertainty on the material properties). Therefore, the method herein presented constitutes the probabilistic version of the Theory of Plastic Mechanism Control (TPMC) already developed for frames with deterministic material properties. With reference to MRFs whose members have random values of the yield strength, when structural collapse is of concern, the failure domain is related to all the possible collapse mechanisms. Within the probabilistic TPMC, the term “failure” does not mean the attainment of a structural collapse, but the development of a collapse mechanism different from the global one. The design requirements normally needed to prevent undesired collapse mechanisms are probabilistic events within the framework of the kinematic theorem of plastic collapse. The limit state function corresponding to each event is represented by a hyperplane in the space of random variables, so that the failure domain is a surface resulting from the intersection of the hyperplanes corresponding to the limit states representing the single failure events. Since plastic hinges in frame’s members are common to many different mechanisms, the single limit state events are correlated. Therefore, by applying the theory of binary systems and considering that the limit states are events located in series, the probability of failure can be computed by means of Ditlevsen bounds. This approach has been validated by means of Monte Carlo simulations. In order to achieve a predefined level of reliability in the attainment of the design goal, the reliability analysis is repeated for increasing values of the overstrength factor of the dissipative zones to be used in TPMC, aiming to its calibration. Finally, on the basis of the results of a parametric analysis, a simple relationship to compute the value of the overstrength factor needed to include the influence of random material variability in the application of TPMC is proposed.

Keywords: collapse mechanisms, stochastic frames, material uncertainty, probability of failure, binary systems

1 INTRODUCTION

It is well known that the control of the collapse mechanism is of primary importance in the seismic design of structures to assure adequate global ductility and energy dissipation capacity. Specifically, structures exhibiting soft storey or partial mechanisms are not able to exploit their plastic reserves and are subjected to damage concentration phenomena. For this reason, it is universally recognised that the optimum seismic performances are obtained when a collapse mechanism of global type occurs. Modern seismic codes (Eurocode 8, AISC) [1]-[2] provide simplified design rules to prevent unsatisfactory collapse mechanisms, such as the use of the so-called beam-column hierarchy criterion for Moment Resisting Frames (MRFs). However, this criterion is usually able to prevent soft storey mechanism only but it does not assure the development of a collapse mechanism

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of global type [3]-[4]. In order to overcome the drawbacks of code provisions, the Theory of Plastic Mechanism Control (TPMC) has been developed [3]-[4] by extending the kinematic theorem of plastic collapse to the concept of mechanism equilibrium curve. Such approach, including the influence of the second order effects, allowed the definition of the design conditions to prevent all the undesired collapse mechanisms, up to an ultimate displacement compatible with the local ductility supply of structural members.

Up to now, TPMC has been already applied to deterministic structures having different seismic resistant schemes [5]-[11] and structural material [12]. The reason of the success relies on the robustness of the theoretical background based on the kinematic theorem of plastic collapse and on second order rigid plastic analysis. However, even in structures designed by TPMC, undesired collapse mechanisms could occur when the effects of random material variability are taken into account. This is the case of stochastic steel frames, whose members have random plastic moments due to the aleatoric uncertainty of the yield strength of steel.

The problem of undesired effects in plastic mechanism control, due to random material uncertainty, are also recognised by modern seismic codes that, aiming to compensate such effects, suggest the use of overstrength factors for the evaluation of the ultimate resistance in dissipative zones within hierarchy criteria. As an example, ANSI/AISC 341-10 [2] computes the ultimate resistance of dissipative zones covering the effects of uncertainty of yield strength by means of a factor given by the ratio between the average yield strength of steel and its nominal value. Conversely, Eurocode 8 (EC8)[1] provides an overstrength factor γ_{ov} ranging between 1.0 and 1.25. However, these values, applied within the beam-column hierarchy criterion, are not based on a probabilistic assessment aimed at a specific collapse mechanism.

A rigorous application of the capacity design principles aimed to control the collapse mechanism requires, for stochastic structures, i.e. for structures whose yield strength values are randomly distributed, the combination of a rigorous deterministic approach with the a structural reliability analysis. Therefore, this work aims at the development of an advanced method for the seismic design of stochastic steel MRFs. The proposed method evaluates the failure probability in the attainment of a collapse mechanism of global type. It constitutes the probabilistic version of the Theory of Plastic Mechanism Control (TPMC) already developed for frames with deterministic material properties [3]-[4],[12]. Stochastic frames have random values of the yield strength of members, so that the random variables are the plastic moments of both beam and column sections, which randomly affect the collapse mechanism under seismic horizontal forces. The failure domain derives from all the possible collapse mechanisms leading to a manifold failure surface. It is important to underline that, within structural reliability analysis and within the context of failure mode control, the term “failure” denotes the attainment of a collapse mechanism different from the global one. The requirements to be satisfied to prevent undesired collapse mechanisms represent probabilistic events. A limit state function, representing a hyperplane in the space of the random variables, is defined for each event. It follows that the failure domain is a manifold surface resulting from the intersection of the hyperplanes corresponding to the limit states of the events. The limit state events are characterised by a correlation in a function of the plastic hinges belonging to the different mechanisms affecting the same structure. The probability of failure is computed by means of Ditlevsen bounds [13]-[14] applying the theory of binary systems [14]-[16] and taking into account that the limit states are probabilistically events in series.

2 COLLAPSE MECHANISMS OF MRFs

With reference to MRFs, subjected to seismic horizontal forces, dissipative zones are located at the beam ends where the development of plastic hinges is expected. Conversely, column sections are non-dissipative zones with the only exception of base sections of the first storey columns. This pattern of yielding corresponds to the global mechanism which provides the highest energy dissipation capacity, because all the dissipative zones are involved in plastic range.

Within the framework of rigid-plastic analysis, the attention is focused on the condition exhibited by the structure in the collapse state by neglecting each intermediate condition. Regarding the possible collapse mechanisms of MRFs under seismic horizontal forces, it is possible to observe that a kinematic mechanism can develop according to patterns of yielding corresponding to two mechanism typologies, namely "upper partial" and "shear band". These mechanisms are depicted in Fig. 1 for given mechanism indexes i_b and i_t; in addition, in the same figure, it is also pointed out that the global mechanism is a particular case of the upper partial mechanisms occurring when $i_b = 1$. The deterministic Theory of Plastic Mechanism Control (TPMC) identifies as undesired mechanisms only type-1, type-2 and type-3 mechanisms [3],[4] because they are the only significant ones from the design point in case of deterministic yield strength of members (Type-1 mechanisms are specific cases of shear band mechanisms occurring for $i_b = 1$; Type-2 mechanisms are herein referred as upper partial mechanisms; finally, Type-3 mechanisms are specific cases of shear band mechanisms occurring for $i_b = i_t$, i.e. soft storey mechanisms). Conversely, as for stochastic frames where yield strength values are randomly distributed for the different structural members, other undesired shear band mechanisms [17] can potentially develop.

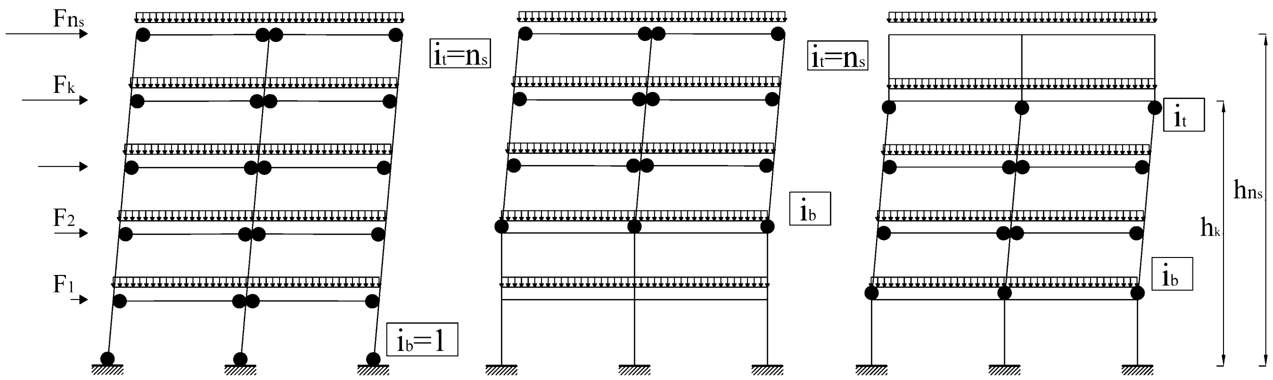

Fig. 1. Collapse mechanism typologies

Given the number of storeys, n_s, it is possible to demonstrate that the total number of possible mechanisms, with the exclusion of the global one, is given by:

$$N_{tot} = \frac{n_s(n_s + 1)}{2} + n_s - 1 \tag{1}$$

3 FAILURE EVENTS ACCORDING TO TPMC

Starting from the deterministic version of TPMC, it is easy to rewrite the design conditions needed to prevent undesired collapse mechanisms in a new form which, conversely, represents the occurrence of any failure event. The term "failure" herein means that the mechanism equilibrium curve corresponding to any undesired mechanism is located below that corresponding to the global mechanism. The *failure events*, thus identified, are probabilistic events constituting a series system of binary components. Each failure event can be expressed by means of the following safety margin parameter $E^{(t)}_{i_b.i_t}$:

$$E^{(t)}_{i_b.i_t} = (\alpha^{(t)}_{0.i_b.i_t} - \gamma^{(t)}_{i_b.i_t}\delta_u) - (\alpha^{(g)}_0 - \gamma^{(g)}\delta_u) < 0 \tag{2}$$

where all the possible mechanisms different from the global one have to be considered, so that in case of upper partial mechanisms $(t = up)$ $i_t = n_s$ and $i_b = 2, 3, \dots, n_s$, while in case of shear band mechanisms $(t = sb)$ $i_t = 1, 2, \dots, n_s$ and i_b ranges from 1 to i_t. The parameters $\alpha^{(g)}_0$ and $\alpha^{(t)}_{0.i_b.i_t}$ are the first order collapse mechanism multipliers of horizontal forces evaluated, respectively, for the global mechanism and for the generic mechanism. The parameters, $\gamma^{(g)}$ and $\gamma_{i_b.i_t}^{(t)}$, are the slopes of the mechanism equilibrium curves for the global and the generic mechanism, respectively, while δ_u is the design displacement compatible with the ductility supply of the structure. To this scope, in the following, the plastic rotation capacity of beams is assumed equal to 0.04 rad so that $\delta_u = 0.04\, h_{ns}$ where h_{ns} is the height of the structure [4]. The safety margin is negative when "failure" occurs, i.e. an undesired mechanism develops. Fig. 2 depicts failure events, success events and the limit state event. As illustrated in Fig. 2a, an undesired collapse mechanism occurs even for $\delta = 0$ (first order rigid-plastic analysis). Fig. 2b shows an undesired collapse mechanism occurred due to the influence of second order effects which are detected by means of second order rigid-plastic analysis. The limit state event where undesired collapse mechanisms are prevented up to a displacement level $\delta = \delta_u$ (i.e. the design displacement) is depicted in Fig. 2c. Finally, success events are those corresponding to Fig. 2d. The number of the considered inequalities is equal to N_{tot}, i.e. the number of undesired collapse mechanisms.

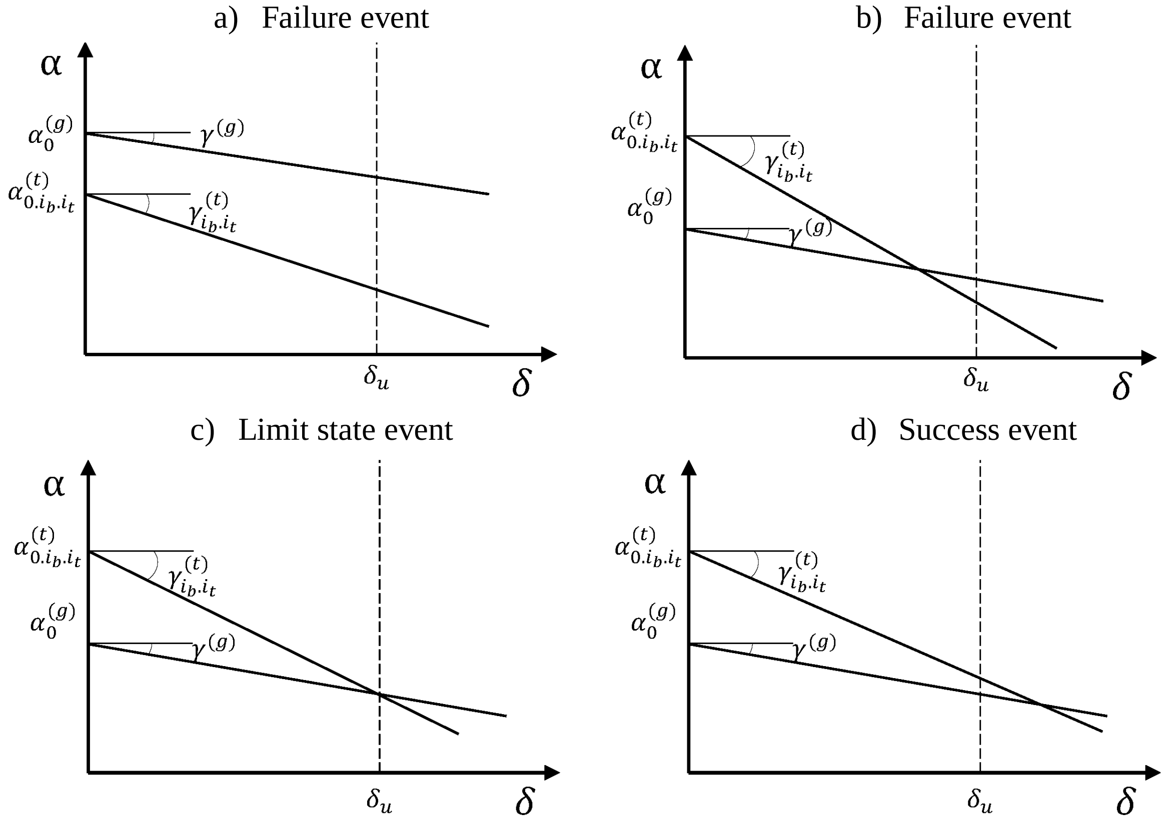

Fig. 2. Failure events according to the location of collapse mechanism equilibrium curves.

Starting from the above consideration, in order to apply First Order Reliability Method (FORM) [18], some preliminary assumptions are required:

1) plastic moments of members are jointly *Gaussian* random variables because of the random variability of the steel yield strength;

2) second order rigid-plastic analysis is carried out to include the influence of the second order effects;

3) horizontal seismic forces are deterministically distributed according to the first vibration mode of the structure and evaluated according to the lateral force method given in code provisions [1], neglecting the effects of the higher modes;

4) vertical loads are assumed as deterministic quantities and their magnitude is such that beam plastic hinges develop at the beam ends [3];

5) the plastic moment of columns are independent of the axial load.

With reference to upper partial mechanisms, ($i_t = n_s$), the first order kinematically admissible multiplier of horizontal forces is given by [3]:

$$\alpha_{0.i_b.i_t}^{(up)} = \frac{\sum_{i=1}^{n_c} M_{c.i.i_b} + 2\sum_{k=i_b}^{n_s}\sum_{j=1}^{n_b} M_{b.j.k}}{\sum_{k=i_b}^{n_s} F_k (h_k - h_{i_b-1})} \tag{3}$$

where $M_{c.i.k}$ is the plastic moment of i-th column of k-th storey, $M_{b.j.k}$ is the plastic moment of the beam of j-th bay at k-th storey, F_k is the seismic force applied at k-th storey, h_k is the height of k-th storey with respect to the foundation level, n_b is the number of bays, $n_c = n_b + 1$ is the number of columns and n_s is the number of storeys. Conversely, the slope of mechanism equilibrium curve applies [3]:

$$\gamma_{i_b.i_t}^{(up)} = \frac{1}{h_{n_s} - h_{i_b-1}} \frac{\sum_{k=i_b}^{n_s} V_k (h_k - h_{i_b-1})}{\sum_{k=i_b}^{n_s} F_k (h_k - h_{i_b-1})} \tag{4}$$

where V_k is the total gravity load applied at k-th storey.
Similarly, with reference to shear band mechanisms, the first order kinematically admissible multiplier of horizontal forces is given by:

$$\alpha_{0.i_b.i_t}^{(sb)} = \frac{\sum_{i=1}^{n_c} M_{c.i.i_b} + 2\sum_{k=i_b}^{i_t-1}\sum_{j=1}^{n_b} M_{b.jk} + \sum_{i=1}^{n_c} M_{c.i.i_t}}{\sum_{k=i_b}^{i_t-1} F_k (h_k - h_{i_b-1}) + (h_{i_t} - h_{i_b-1})\sum_{k=i_t}^{n_s} F_k} \tag{5}$$

and the slope of mechanism equilibrium curve is expressed as:

$$\gamma_{i_b.i_t}^{(sb)} = \frac{1}{h_{i_t} - h_{i_b-1}} \frac{\sum_{k=i_b}^{i_t-1} V_k (h_k - h_{i_b-1}) + (h_{i_t} - h_{i_b-1})\sum_{k=i_t}^{n_s} V_k}{\sum_{k=i_b}^{i_t-1} F_k (h_k - h_{i_b-1}) + (h_{i_t} - h_{i_b-1})\sum_{k=i_t}^{n_s} F_k} \tag{6}$$

With reference to “upper partial mechanisms”, by substituting Eq. (3) and (4) into Eq. (2), the safety margin parameter corresponding to the i_b-th mechanism of this typology (being $i_t = n_s$) is given by:

$$-\vartheta_{i_b}\sum_{i=1}^{n_c} M_{c.i.1} + \sum_{i=1}^{n_c} M_{c.i.i_b} + 2(1-\vartheta_{i_b})\sum_{k=i_b}^{n_s}\sum_{j=1}^{n_b} M_{b.jk} - 2\vartheta_{i_b}\sum_{k=1}^{i_b-1}\sum_{j=1}^{n_b} M_{b.jk} + \left(\gamma^{(g)} - \gamma_{i_b.i_t}^{(up)}\right)\delta_u \vartheta_{i_b} < 0 \tag{7}$$

where ϑ_{i_b} applies:

$$\vartheta_{i_b} = \frac{\sum_{k=i_b}^{n_s} F_k (h_k - h_{i_b-1})}{\sum_{k=1}^{n_s} F_k h_k} \tag{8}$$

While, as for “shear band mechanisms”, by substituting Eq. (5) and (6) into Eq. (2), the safety margin parameter corresponding to the mechanism defined by the indexes i_b and i_t for this typology is given by:

$$\begin{aligned} &-\tau_{i_b.i_t}\sum_{i=1}^{n_c} M_{c.i.1} + \sum_{i=1}^{n_c} M_{c.i.i_b} + \sum_{i=1}^{n_c} M_{c.i.i_t} + 2(1-\tau_{i_b.i_t})\sum_{k=i_b}^{i_t-1}\sum_{j=1}^{n_b} M_{b.jk} + \\ &-2\tau_{i_b.i_t}\sum_{k=1}^{i_b-1}\sum_{j=1}^{n_b} M_{b.jk} - 2\tau_{i_b.i_t}\sum_{k=i_t}^{n_s}\sum_{j=1}^{n_b} M_{b.jk} + \left(\gamma^{(g)} - \gamma_{i_b.i_t}^{(sb)}\right)\delta_u \tau_{i_b.i_t} < 0 \end{aligned} \tag{9}$$

where $\tau_{i_b.i_t}$ can be expressed as:

$$\tau_{i_b.i_t} = \frac{\sum_{k=i_b}^{i_t-1} F_k (h_k - h_{i_b-1}) + (h_{i_t} - h_{i_b-1})\sum_{k=i_t}^{n_s} F_k}{\sum_{k=1}^{n_s} F_k h_k} \tag{10}$$

4 FIRST ORDER RELIABILITY METHOD FOR SERIES SYSTEMS

As soon as the failure events are properly identified, the failure probability of a series system composed by binary components can be computed by exploiting the First Order Reliability Methods (FORMs) [16]. The random variables are collected into a vector $\mathbf{x}$ whose expectation and covariance matrix are respectively given by:

$$\boldsymbol{\mu}_{\mathbf{x}} = E[\mathbf{x}] \tag{11}$$

$$\mathbf{C}_{\mathbf{x}} = E[(\mathbf{x} - \boldsymbol{\mu}_{\mathbf{x}})(\mathbf{x} - \boldsymbol{\mu}_{\mathbf{x}})^T] \tag{12}$$

When the limit state condition defines a polyhedral surface $\mathrm{G}(\mathbf{x}) = \mathbf{0}$ constituted by *n* hyperplanes (being *n* the number of failure events), it means that any failure event is defined by a linear limit state function. The safety margin vector G is a function of the random vector $\mathbf{x}$:

$$\mathrm{G}(\mathbf{x}) = \mathbf{a}_0 + \mathbf{B}^T\mathbf{x} \tag{13}$$

where $\mathbf{a}_0$ is a vector representing the known quantities and $\mathbf{B}$ is a matrix of deterministic coefficients.

The linearity of the expectation operator E, allows an easy determination of the first two moments and the expected value is given by:

$$\boldsymbol{\mu}_{\mathrm{G}} = E[\mathrm{G}] = \mathbf{a}_0 + \mathbf{B}^T E[\mathbf{x}] = \mathbf{a}_0 + \mathbf{B}^T \boldsymbol{\mu}_{\mathbf{x}} \tag{14}$$

where $\boldsymbol{\mu}_{\mathrm{G}}$ is the vector of the mean safety margin values and $\boldsymbol{\mu}_{\mathbf{x}}$ is the vector containing the expected values of the random vector $\mathbf{x}$. In addition, from Eq. (12), the covariance matrix $\boldsymbol{\sigma}_{\mathrm{G}}^2$ is given by:

$$\boldsymbol{\sigma}_{\mathrm{G}}^2 = E[(\mathrm{G} - \boldsymbol{\mu}_{\mathrm{G}})^2] = \mathbf{B}^T E[(\mathbf{x} - \boldsymbol{\mu}_{\mathbf{x}})(\mathbf{x} - \boldsymbol{\mu}_{\mathbf{x}})^T]\mathbf{B} = \mathbf{B}^T \mathbf{C}_{\mathbf{x}} \mathbf{B} \tag{15}$$

where $\mathbf{C}_{\mathbf{x}}$ is the matrix of the second order moments of the random variables. Given the above, the Cornell reliability indexes corresponding to the different failure events are collected in the vector $\boldsymbol{\beta}_{\mathbf{C}}$ [14]-[16],[19], which is computed as the ratio between the mean value and the standard deviation of G.

$$\boldsymbol{\beta}_{\mathbf{C}} = \frac{\boldsymbol{\mu}_{\mathrm{G}}}{\boldsymbol{\sigma}_{\mathrm{G}}} = \frac{\mathbf{a}_0 + \mathbf{B}^T \boldsymbol{\mu}_{\mathbf{x}}}{\sqrt{\mathbf{B}^T \mathbf{C}_{\mathbf{x}} \mathbf{B}}} \tag{16}$$

The theoretical justification lies in the fact that when the distribution of $\mathbf{x}$ is jointly *Gaussian* then G is also Gaussian, being a linear combination of Gaussian variables. In this case, the distribution of G is completely defined by $\boldsymbol{\mu}_{\mathrm{G}}$ and $\boldsymbol{\sigma}_{\mathrm{G}}$. For this reason, the vector $\boldsymbol{p}_f$ collecting the probability of the failure events given by Eq. (2), is expressed as:

$$\boldsymbol{p}_f = \Phi\left(-\frac{\boldsymbol{\mu}_{\mathrm{G}}}{\boldsymbol{\sigma}_{\mathrm{G}}}\right) = \Phi(-\boldsymbol{\beta}_{\mathbf{C}}) \tag{17}$$

which establishes a bi-univocal relationship between the Cornell indexes and the failure probabilities of the events constituting the series system.

However, because the events, i.e. each row of the **B** matrix, are correlated, it is needed to account for the correlation between each couple of events. To this scope, the correlation coefficients ρ_{ij} have to be computed:

$$\rho_{ij} = \frac{\mathbf{b}_i^T \mathbf{C}_{\mathbf{x}} \mathbf{b}_j}{\sqrt{(\mathbf{b}_i^T \mathbf{C}_{\mathbf{x}} \mathbf{b}_i)(\mathbf{b}_j^T \mathbf{C}_{\mathbf{x}} \mathbf{b}_j)}} \tag{18}$$

where $\mathbf{b}_i$ and $\mathbf{b}_j$ are the i-th and j-th row of **B** matrix. The values of ρ_{ij} range between -1 and 1.

With reference to the whole series system identified by the failure events given by Eq. (2), the probability of failure P_f can be approximated on the basis of the individual components failure probabilities P_{fi} which are the elements of the vector provided by Eq. (17) and of their intersection up to the second order by means of *upper* and *lower* bounds, so called Ditlevsen bounds [13], which are generally close enough to provide an acceptable estimate [13]-[14],[20]-[22].

The failure probability has the following *lower bound* [13]:

$$P_f \geq P_{f1} + \sum_{i=2}^{n} max\left\{P_{f.i} - \sum_{j=1}^{i-1} P_{f.ij}, 0\right\} \tag{19}$$

and the following *upper bound* [13]:

$$P_f \leq \sum_{i=1}^{n} P_{f.i} - \sum_{i=2}^{n} \max_{j<i}\{P_{f.ij}\} \tag{20}$$

where $P_{f.i}$ is the i-th component of the vector $\boldsymbol{p}_f$. However, the bounds require the calculation of the *joint probabilities* of every pair of elements *i* and *j* by means of the following equation by exploiting the Cornell index and the correlation factors:

$$P_{f.ij} = \Phi_2(-\beta_i; -\beta_j; \rho_{ij}) \tag{21}$$

The *joint probabilities* can be computed according to the following equation:

$$P_{f.ij}(y_1;\ y_2; \rho_{ij}) = \Phi(y_1)\Phi(y_2) + \int_0^{\rho_{ij}} \frac{exp\left[-\frac{y_1^2 + y_2^2 - 2y_1y_2\rho^2}{2(1-\rho_{ij}^2)}\right]}{2\pi\sqrt{1-\rho^2}} d\rho \tag{22}$$

where a change of base is needed according to Cholesky decomposition [16] of covariance matrix leading to:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{bmatrix} \begin{bmatrix} -\beta_1 \\ -\beta_2 \end{bmatrix} \tag{23}$$

5 PROBABILISTIC "TPMC"

With reference to Eq. (9), which collects all the failure events $E_{i_b.i_t}^{(t)}$ in the vector $\mathrm{G}(\mathbf{x})$, the vector of the random variables is:

$$\mathbf{x} = \{\boldsymbol{x}_c, \boldsymbol{x}_b\}^T \tag{24}$$

where $\boldsymbol{x}_c$ is a vector of dimension n_s collecting the sum of the plastic moments of columns at each storey:

$$\boldsymbol{x}_c = \left\{\sum_{i=1}^{n_c} M_{c.i.1}, \dots, \sum_{i=1}^{n_c} M_{c.i.n_s}\right\}^T \tag{25}$$

where $M_{c.i.k}$ represents the plastic moment of i-th column of k-th storey and n_c is the number of columns. Moreover, $\boldsymbol{x}_b$ is a vector of dimension n_s collecting the sum of the plastic moments of beams at each storey:

$$\boldsymbol{x}_b = \left\{\sum_{j=1}^{n_b} M_{b.j.1}, \dots, \sum_{j=1}^{n_b} M_{b.j.n_s}\right\}^T \tag{26}$$

The dimension of the matrix **B** is $N_{tot} \times 2n_s$. Such matrix can be partitioned as follows:

$$\mathbf{B}^T = \begin{bmatrix} \boldsymbol{B}_c^{(up)} & \boldsymbol{B}_b^{(up)} \\ \boldsymbol{B}_c^{(sb)} & \boldsymbol{B}_b^{(sb)} \end{bmatrix} \tag{27}$$

The dimension of the submatrix $\boldsymbol{B}_c^{(up)}$ is $(n_s - 1) \times n_s$ as it collects the coefficients that multiply the sum of the plastic moments of columns at each storey with reference to the $(n_s - 1)$ events related to the "upper partial" mechanisms. This matrix is obtained analysing Eq. (7) and applies:

$$\boldsymbol{B}_c^{(up)} = \begin{bmatrix} -\vartheta_2 & 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ -\vartheta_k & 0 & \cdots & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ -\vartheta_{n_s} & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix} \tag{28}$$

where the ϑ parameters are given by Eq. (8) for $i_b = 2, \dots, n_s$. The dimension of the submatrix $\boldsymbol{B}_b^{(up)}$ is $(n_s - 1) \times n_s$ as it collects the coefficients that multiply the sum of the plastic moments of beams at each storey with reference to the $(n_s - 1)$ events related to the "upper partial" mechanisms. This matrix is obtained analysing Eq. (7) and is given by:

$$\boldsymbol{B}_b^{(up)} = \begin{bmatrix} -2\vartheta_2 & 2(1-\vartheta_2) & \cdots & 2(1-\vartheta_2) & \cdots & 2(1-\vartheta_2) \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ -2\vartheta_k & -2\vartheta_k & \cdots & 2(1-\vartheta_k) & \cdots & 2(1-\vartheta_k) \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ -2\vartheta_{n_s} & -2\vartheta_{n_s} & \cdots & -2\vartheta_{n_s} & \cdots & 2(1-\vartheta_{n_s}) \end{bmatrix} \tag{29}$$

Regarding the submatrices $\boldsymbol{B}_c^{(sb)}$ and $\boldsymbol{B}_b^{(sb)}$ related to the $n_s(n_s + 1)/2$ events corresponding to undesired "shear band" mechanisms, they can be furtherly partitioned according to the following relationships:

$$\boldsymbol{B}_c^{(sb)} = \left[\boldsymbol{B}_c^{(sb1)}, \boldsymbol{B}_c^{(sb2)}, \boldsymbol{B}_c^{(sb3)}, \boldsymbol{B}_c^{(sb4)}\right]^T \tag{30}$$

$$\boldsymbol{B}_b^{(sb)} = \left[\boldsymbol{B}_b^{(sb1)}, \boldsymbol{B}_b^{(sb2)}, \boldsymbol{B}_b^{(sb3)}, \boldsymbol{B}_b^{(sb4)}\right]^T \tag{31}$$

where the submatrix $\boldsymbol{B}_c^{(sb1)}$, whose dimensions are $n_s \times n_s$, contains the coefficients corresponding to the events related to the undesired mechanisms occurring for $i_b = 1$ and $i_t = 1, \dots, n_s$. Such mechanisms are those usually referred to as type-1 mechanism [3]-[4] or "bottom partial mechanisms". Such submatrix is obtained analysing Eq. (9) and is given by:

$$\boldsymbol{B}_c^{(sb1)} = \begin{bmatrix} 2-\tau_{1.1} & 0 & \cdots & 0 & \cdots & 0 \\ 1-\tau_{1.2} & 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 1-\tau_{1.k} & 0 & \cdots & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 1-\tau_{1.n_s} & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix} \tag{32}$$

Similarly, the corresponding $\boldsymbol{B}_b^{(sb1)}$ matrix, having the same dimensions, is given by:

$$\boldsymbol{B}_b^{(sb1)} = \begin{bmatrix} -2\tau_{1.1} & -2\tau_{1.1} & \cdots & -2\tau_{1.1} & \cdots & -2\tau_{1.1} \\ 2(1-\tau_{1.2}) & -2\tau_{1.2} & \cdots & -2\tau_{1.2} & \cdots & -2\tau_{1.2} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 2(1-\tau_{1.k}) & 2(1-\tau_{1.k}) & \cdots & -2\tau_{1.k} & \cdots & -2\tau_{1.k} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 2(1-\tau_{1.n_s}) & 2(1-\tau_{1.n_s}) & \cdots & 2(1-\tau_{1.n_s}) & \cdots & -2\tau_{1.n_s} \end{bmatrix} \tag{33}$$

The submatrix $\boldsymbol{B}_c^{(sb2)}$, whose dimensions are $(n_s - 1) \times n_s$, contains the coefficients corresponding to the events related to undesired mechanisms occurring for i_b varying from 2 to n_s and $i_b = i_t$. These mechanisms are the so-called soft storey mechanisms or type-3 mechanisms [1] and their number is equal to $n_s - 1$, because the event corresponding to the soft storey mechanism at first storey is already accounted for in the first row of matrix (Eq. (32)). Such submatrix is given by:

$$
\boldsymbol{B}_c^{(sb2)} = \begin{bmatrix} -\tau_{2.2} & 2 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ -\tau_{k.k} & 0 & \cdots & 2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ -\tau_{n_s.n_s} & 0 & \cdots & 0 & \cdots & 2 \end{bmatrix} \tag{34}
$$

The corresponding $\boldsymbol{B}_b^{(sb2)}$ matrix, having the same dimensions, is expressed as:

$$
\boldsymbol{B}_b^{(sb2)} = \begin{bmatrix} -2\tau_{2.2} & \dots & -2\tau_{2.2} \\ \vdots & \ddots & \vdots \\ -2\tau_{n_s.n_s} & \dots & -2\tau_{n_s.n_s} \end{bmatrix} \tag{35}
$$

Submatrix $\boldsymbol{B}_c^{(sb3)}$, whose dimensions are $(n_s - 2) \times n_s$, contains the coefficients corresponding to the events related to undesired mechanisms occurring for i_b varying from 2 to $n_s - 1$ and $i_t = n_s$. The number is equal to $n_s - 2$, because the event corresponding to the soft storey mechanism at the top storey is already accounted for in the last row of matrix in Eq. (34). Such submatrix is given by:

$$
\boldsymbol{B}_c^{(sb3)} = \begin{bmatrix} -\tau_{2.n_s} & 1 & \cdots & 0 & \cdots & 0 & 1 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ -\tau_{k.n_s} & 0 & \cdots & 1 & \cdots & 0 & 1 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ -\tau_{n_s-1.n_s} & 0 & \cdots & 0 & \cdots & 1 & 1 \end{bmatrix} \tag{36}
$$

The corresponding $\boldsymbol{B}_b^{(sb3)}$ matrix, having the same dimensions, is given by:

$$
\boldsymbol{B}_b^{(sb3)} = \begin{bmatrix} -2\tau_{2.n_s} & 2(1-\tau_{2.n_s}) & \cdots & 2(1-\tau_{2.n_s}) & \cdots & 2(1-\tau_{2.n_s}) & -2\tau_{2.n_s} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ -2\tau_{k.n_s} & -2\tau_{k.n_s} & \cdots & 2(1-\tau_{k.n_s}) & \cdots & 2(1-\tau_{k.n_s}) & -2\tau_{k.n_s} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ -2\tau_{n_s-1.n_s} & -2\tau_{n_s-1.n_s} & \cdots & -2\tau_{n_s-1.n_s} & \cdots & 2(1-\tau_{n_s-1.n_s}) & -2\tau_{n_s-1.n_s} \end{bmatrix} \tag{37}
$$

Finally, all the remaining events are collected in the submatrix $\boldsymbol{B}_c^{(sb4)}$, whose dimensions are $[n_s(n_s + 1)/2 - 3(n_s - 1)] \times n_s$. It contains the coefficients corresponding to the events related to undesired "shear band" mechanisms occurring for $i_b \neq 1$, $i_t \neq n_s$ and $i_b \neq i_t$. Such submatrix is given by:

$$
\boldsymbol{B}_c^{(sb4)} = \begin{bmatrix} -\tau_{2.3} & 1 & \cdots & 1 & \cdots & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ -\tau_{2.k} & 1 & \cdots & 0 & \cdots & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ -\tau_{2.n_s-1} & 1 & \cdots & 0 & \cdots & 0 & \dots & 1 & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ -\tau_{k.i_t} & 0 & \cdots & 1 & \cdots & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ -\tau_{k.n_s-1} & 0 & \cdots & 1 & \cdots & 0 & \cdots & 1 & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ -\tau_{n_s-3.n_s-1} & 0 & \cdots & 0 & \cdots & 1 & 0 & 1 & 0 \\ -\tau_{n_s-2.n_s-1} & 0 & \cdots & 0 & \cdots & 0 & 1 & 1 & 0 \end{bmatrix} \tag{38}
$$

The corresponding $\boldsymbol{B}_b^{(sb4)}$ matrix, having the same dimensions, is given by:

$$\boldsymbol{B}_b^{(sb4)} = \begin{bmatrix} -2\tau_{2.3} & 2(1-\tau_{2.3}) & \cdots & -2\tau_{2.3} & \cdots & -2\tau_{2.3} & \cdots & -2\tau_{2.3} & -2\tau_{2.3} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ -2\tau_{2.i_t} & 2(1-\tau_{2.i_t}) & \cdots & 2(1-\tau_{2.i_t}) & \cdots & -2\tau_{2.i_t} & \cdots & -2\tau_{2.i_t} & -2\tau_{2.i_t} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ -2\tau_{2.n_s-1} & 2(1-\tau_{2.n_s-1}) & \cdots & 2(1-\tau_{2.n_s-1}) & \cdots & 2(1-\tau_{2.n_s-1}) & \dots & -2\tau_{2.n_s-1} & -2\tau_{2.n_s-1} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ -2\tau_{k.i_t} & -2\tau_{k.i_t} & \cdots & 2(1-\tau_{k.i_t}) & \cdots & -2\tau_{k.i_t} & \cdots & -2\tau_{k.i_t} & -2\tau_{k.i_t} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ -2\tau_{k.n_s-1} & -2\tau_{k.n_s-1} & \cdots & 2(1-\tau_{k.n_s-1}) & \cdots & 2(1-\tau_{k.n_s-1}) & \cdots & -2\tau_{k.n_s-1} & -2\tau_{k.n_s-1} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ -2\tau_{n_s-3.n_s-1} & -2\tau_{n_s-3.n_s-1} & \cdots & -2\tau_{n_s-3.n_s-1} & \cdots & 2(1-\tau_{n_s-3.n_s-1}) & -2\tau_{n_s-3.n_s-1} & -2\tau_{n_s-3.n_s-1} & -2\tau_{n_s-3.n_s-1} \\ -2\tau_{n_s-2.n_s-1} & -2\tau_{n_s-2.n_s-1} & \cdots & -2\tau_{n_s-2.n_s-1} & \cdots & 2(1-\tau_{n_s-2.n_s-1}) & 2(1-\tau_{n_s-2.n_s-1}) & -2\tau_{n_s-2.n_s-1} & -2\tau_{n_s-2.n_s-1} \end{bmatrix} \tag{39}$$

The vector $\boldsymbol{a}_0$ of Eq.(13) can be expressed in the following form:

$$\boldsymbol{a}_0 = \delta_u \left\{ \boldsymbol{\Gamma}^{(up)}, \boldsymbol{\Gamma}^{(sb)} \right\}^T \tag{40}$$

where $\boldsymbol{\Gamma}^{(up)}$ is the vector accounting for the difference between the influence of second order effects in case of global mechanism and in case of upper partial mechanisms. It has dimension $(n_s - 1)$ and it is given by:

$$\boldsymbol{\Gamma}^{(up)} = \left\{ \gamma^{(g)} - \gamma_{2.n_s}^{(up)}, \gamma^{(g)} - \gamma_{3.n_s}^{(up)}, \dots, \gamma^{(g)} - \gamma_{n_s-1.n_s}^{(up)} \right\}^T \tag{41}$$

while $\boldsymbol{\Gamma}^{(sb)}$ is the vector accounting for the difference between the influence of second order effects in case of global mechanism and in case of undesired “shear band” mechanisms. It has dimension $n_s(n_s+1)/2$ and applies:

$$\boldsymbol{\Gamma}^{(sb)} = \left\{ \boldsymbol{\Gamma}^{(sb1)}, \boldsymbol{\Gamma}^{(sb2)}, \boldsymbol{\Gamma}^{(sb3)}, \boldsymbol{\Gamma}^{(sb4)} \right\}^T \tag{42}$$

The vector $\boldsymbol{\Gamma}^{(sb1)}$, whose dimension is n_s, contains the coefficients corresponding to the events related to the undesired mechanisms occurring for $i_b = 1$ and i_t varying from 1 to n_s. Such mechanisms are those usually referred to as type-1 mechanism [3]-[4] or “bottom partial mechanisms”. Such vector is given by:

$$\boldsymbol{\Gamma}^{(sb1)} = \left\{ \gamma^{(g)} - \gamma_{1.1}^{(sb)}, \gamma^{(g)} - \gamma_{1.2}^{(sb)}, \dots, \gamma^{(g)} - \gamma_{1.k}^{(sb)}, \dots, \gamma^{(g)} - \gamma_{1.n_s}^{(sb)} \right\}^T \tag{43}$$

Similarly, the vector $\boldsymbol{\Gamma}^{(sb2)}$, whose dimension is $(n_s - 1)$, contains the coefficients corresponding to the events related to undesired mechanisms occurring for i_b varying from 2 to n_s and $i_b = i_t$. These mechanisms are the so-called soft storey mechanisms or type-3 mechanisms [3]-[4] and their number is equal to $n_s - 1$, because the event corresponding to the soft storey mechanism at first storey is already accounted for by first element of vector (Eq. (43)). Such vector is as follows:

$$\boldsymbol{\Gamma}^{(sb2)} = \left\{ \gamma^{(g)} - \gamma_{2.2}^{(sb)}, \gamma^{(g)} - \gamma_{3.3}^{(sb)}, \dots, \gamma^{(g)} - \gamma_{k.k}^{(sb)}, \dots, \gamma^{(g)} - \gamma_{n_s.n_s}^{(sb)} \right\}^T \tag{44}$$

The vector $\boldsymbol{\Gamma}^{(sb3)}$, whose dimension is $(n_s - 2)$, contains the coefficients corresponding to the events related to undesired mechanisms occurring for i_b varying from 2 to $n_s - 1$ and $i_t = n_s$. The number is equal to $n_s - 2$, because the event corresponding to the soft storey mechanism at the top storey is already accounted for by last element of vector (Eq. (44)). It is given by:

$$\boldsymbol{\Gamma}^{(sb3)} = \left\{ \gamma^{(g)} - \gamma_{2.n_s}^{(sb)}, \gamma^{(g)} - \gamma_{3.n_s}^{(sb)}, \dots, \gamma^{(g)} - \gamma_{k.n_s}^{(sb)}, \dots, \gamma^{(g)} - \gamma_{n_s-1.n_s}^{(sb)} \right\}^T \tag{45}$$

Finally, all the remaining events are taken into account by the vector $\boldsymbol{\Gamma}^{(sb4)}$ whose dimension is $[n_s(n_s+1)/2 - 3(n_s-1)]$. It contains the coefficients corresponding to the events related to undesired “shear band” mechanisms occurring for $i_b \neq 1$, $i_t \neq n_s$ and $i_b \neq i_t$. Such vector is given by:

$$\boldsymbol{\Gamma}^{(sb4)} = \left\{ \gamma^{(g)} - \gamma_{2.3}^{(sb)}, \dots, \gamma^{(g)} - \gamma_{2.n_s-1}^{(sb)}, \dots, \gamma^{(g)} - \gamma_{k.i_t}^{(sb)}, \dots, \gamma^{(g)} - \gamma_{k.n_s-1}^{(sb)}, \dots, \gamma^{(g)} - \gamma_{n_s-3.n_s-1}^{(sb)}, \gamma^{(g)} - \gamma_{n_s-2.n_s-1}^{(sb)} \right\}^T \tag{46}$$

As soon as the vector $\boldsymbol{a}_0$ and the matrix $\mathbf{B}$ are defined according to Eq. (40) and Eq. (27), respectively, the probability of “failure”, i.e. the probability of occurrence of a collapse mechanism different from the global one (probability of failure in the attainment of the design goal of TPMC), can be computed by applying FORM as described in Section 4.

6 DESIGN PROCEDURE FOR PROBABILISTIC TPMC

The probability of occurrence of a collapse mechanism different from the global one is computed by considering as failure events the ones occurring when, at the ultimate displacement, the mechanism equilibrium curve of an undesired mechanism is located below the one corresponding to the global mechanism. Therefore, the number of "failure" events is equal to the number of undesired mechanisms. In order to set up a design procedure assuring the collapse mechanism of global type with a given probability of success (target probability), it is needed to combine the deterministic theory of plastic mechanism control (TPMC) with the probabilistic approach previously described.

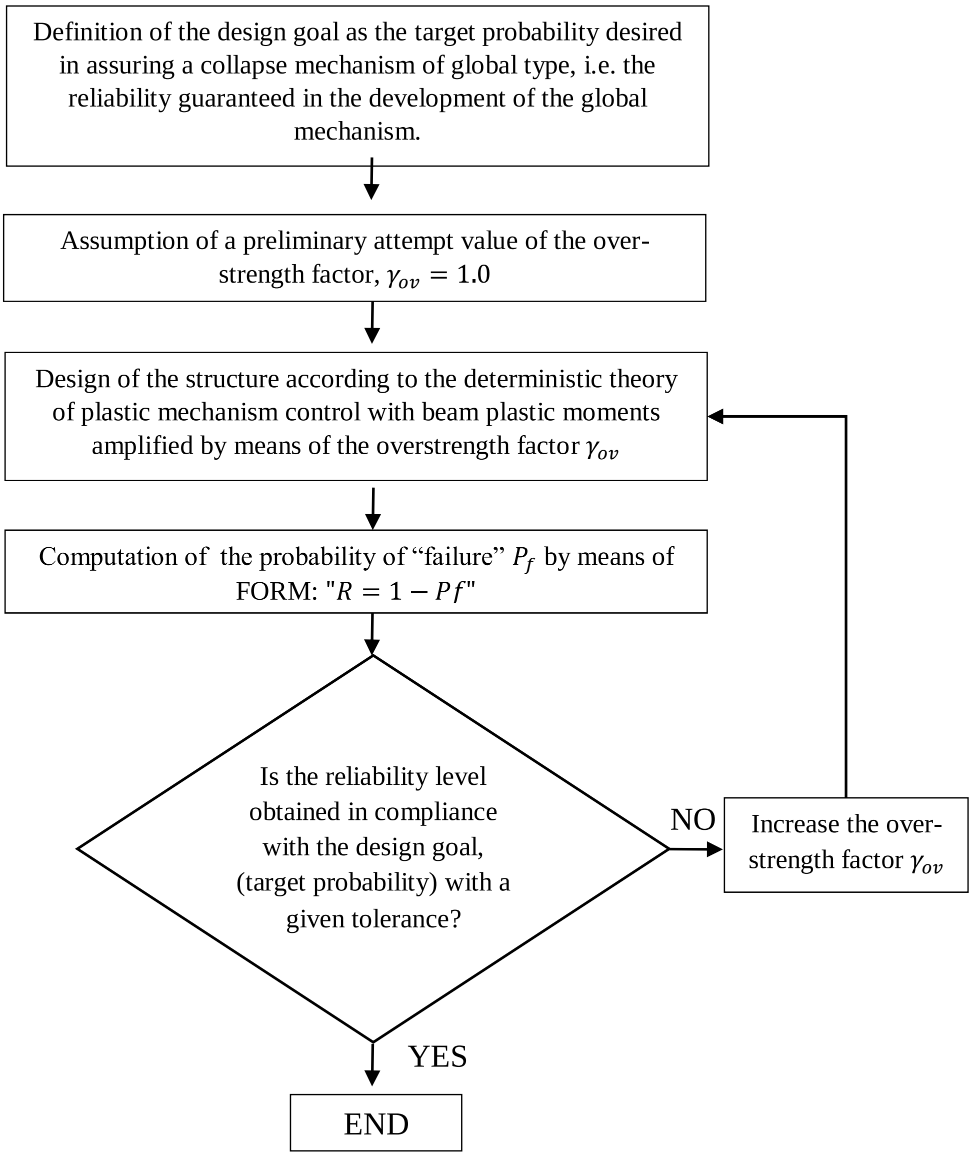

Fig. 3. Design procedure flowchart

To this scope, it is preliminarily needed to consider that, according to the second principle of capacity design, non-dissipative zones have to be designed considering the maximum internal actions which dissipative zones are able to transmit. It means that, for MRFs, the column sections have to be designed considering the maximum bending moments that the beams are able to transmit, including the effects of random material uncertainty. It follows that TPMC can be properly combined with FORM, provided that, the randomness in the yield strength is taken into account by means of a specific over-strength factor applied to the yield strength of the dissipative zones (beam plastic moments) and properly calibrated to lead to the target probability in assuring the global type mechanism.

It is useful to note that such approach is, in principle, already applied in seismic codes. In fact, the influence of random material uncertainty is accounted for in ANSI/AISC 341-10 [2] by means of an over-strength factor γ_{ov} whose value is assumed equal to the ratio between the average yield strength and the nominal yield strength of the steel grade adopted and, similarly, in Eurocode 8 [1] by means of the over-strength factor γ_{ov} whose value is typically assumed equal to 1.25. However, these values are essentially based on engineering judgement rather than on specific calibrations and, most of all, they are applied to a design procedure based on the beam-column hierarchy criterion which is not able to guarantee a collapse mechanism of global type.

As a result of the above observations, the following design procedure can be suggested starting from the assumption that, neglecting strain-hardening effects, the maximum bending moment which the beams are able to transmit to the columns is given by $\gamma_{ov}M_b$ where M_b is the nominal plastic moment and γ_{ov} is the over-strength coefficient to be calibrated according to the steps reported in the flowchart given in Fig. 3.

7 MONTE CARLO SIMULATIONS

In order to check the accuracy of the Probabilistic Theory of Plastic Mechanism Control (P-TPMC), Monte Carlo method has been applied to a 10 storeys-2 bays steel frame to compute the probability of failure. The accuracy of the probabilistic approach is, therefore, preliminarily investigated by comparing the probability of failure computed by means of Monte Carlo simulations with that resulting from the application of the relationships explained in Sections 4 and 5. Furthermore, as P-TPMC is also based on the First Order Reliability Method (FORM) [14],[16] and the estimation of the probability of failure by means of Ditlevsen bounds [13], Monte Carlo simulation is also used to check the narrowness and accuracy of such bounds.

In the following, the main steps of Monte Carlo simulation are reported:

a. Definition of the size of the sample n, i.e. the number of stochastic frames to be generated;
b. As the stochastic frames to be generated according to step a. are frames having the same geometrical configuration, but characterized by structural members with random values of the yield strength, the size of the sample of yield strength values to be generated is equal to $n\, n_s(n_b + n_c)$, being n_b the number of bays and n_c the number of columns;
c. By assuming that all the members are extracted from the same population which is distributed according to the normal distribution, the yield strength values of structural members are randomly generated from a Gaussian distribution characterized by the average value of the yield strength and by the corresponding coefficient of variation.
d. For each frame extracted from the generated sample, second order rigid-plastic analysis is carried out by computing the collapse multiplier of seismic horizontal forces corresponding to a given top sway ultimate displacement. Such collapse multiplier is derived by means of the kinematic theorem of plastic collapse extended to the concept of mechanism equilibrium curve [3]-[4]. Therefore, for each frame the following kinematically admissible multipliers of horizontal forces are computed:

$$\alpha_{i_b.i_t}^{(t)} = \alpha_{0.i_b.i_t}^{(t)} - \gamma_{i_b.i_t}^{(t)}\delta_u \tag{47}$$

Similarly, the kinematically admissible multiplier of horizontal forces corresponding to the global mechanism is also computed:

$$\alpha^{(g)} = \alpha_0^{(g)} - \gamma^{(g)}\delta_u \tag{48}$$

where all the possible mechanisms are considered, so that for upper partial mechanisms ($t = up$) $i_t = n_s$ and $i_b = 2, 3, \dots, n_s$ while for shear band mechanisms ($t = sb$) $i_t = 1, 2, \dots, n_s$ and i_b ranges from 1 to i_t. The parameters $\alpha_0^{(g)}$ and $\alpha_{0.i_b.i_t}^{(t)}$ are the first order collapse mechanism multipliers of horizontal forces corresponding to the of global mechanism and to the generic mechanism, respectively. Conversely, $\gamma^{(g)}$ and $\gamma_{i_b.i_t}^{(t)}$ are the slopes of the mechanism equilibrium curves for global and generic mechanisms, respectively, while δ_u is the ultimate design displacement as governed by the local ductility supply of the structure [3]-[4].

e. For each frame the occurrence of the following inequality is investigated for all the possible undesired mechanisms:

$$\alpha^{(g)} \leq \alpha_{i_b.i_t}^{(t)} \tag{49}$$

At each frame a Boolean [16] variable is associated. If the inequality (49) is satisfied for all the possible undesired mechanisms a value equal to 1 is assigned to the Boolean variable, because, it means that in such case the design goal has been achieved, i.e. a collapse mechanism of global type actually occurs. Conversely, if at least one inequality is not satisfied a value equal to 0 of the Boolean variable returns, because an undesired mechanism is developed.

f. To the frame sample, whose size is n, is associated to a Boolean vector whose elements denote the cases of success (global mechanism attained) and the cases of failure (an undesired mechanism developed). Therefore, dividing the number of failure cases n_0 by the size of the frame sample, the probability of failure is obtained :

$$P_f = \frac{n_0}{n} \tag{50}$$

8 VALIDATION OF P-TPMC BY MONTE CARLO SIMULATIONS

In order to investigate the accuracy of the Probabilistic Theory of Plastic Mechanism Control, the 10 storeys-2 bays moment resisting frame depicted in Fig. 4 has been investigated. In such figure, the beam sections selected from the standard shapes are given. Conversely, regarding the column sections, the plastic modulus, rather than the columns sections, are presented. This means that in order to stress the accuracy of P-TPMC ideal standard shapes are considered whose peculiarity is the availability of a plastic modulus exactly equal to the one required according to the application of TPMC. A triangular distribution of the horizontal forces has been considered according to the hypothesis of section 3. The steel grade considered in the analysis is S275. The 5% fractile of the yield resistance has been assumed equal to the nominal value $f_{yk} = 275\ MPa$. Moreover, Monte Carlo simulation has been repeated considering three values of the coefficient of variation cov equal to 0.05, 0.10 and 0.15, respectively. In addition, the influence of the frame sample size has been investigated and the corresponding results in terms of probability of failure are reported in Table 1.

Regarding the values obtained by means of the application of P-TPMC, in case of $cov = 0.05$, upper and lower Ditlevsen bounds coincide and are equal to 0.4960, while in case of $cov = 0.10$, Ditlevsen bounds have led to a lower bound value of the failure probability equal to an upper bound value equal to 0.5142. Increasing the coefficient of variation up to 0.15, the lower bound value of the failure probability is equal to 0.5334 while the upper bound value is equal to 0.5887. It is useful to note that the narrowness of the range corresponding to the prediction of the probability of failure by means of Ditlevsen bounds improves when the coefficient of variation reduces.

The values of the probability of failure computed by means of Monte Carlo simulations are compared with those resulting from P-TPMC and Ditlevsen bounds in Fig. 5-Fig. 7 for the values of the *cov* assumed. Such figures show that for low values of the size of the frame sample n, the values of the probabilities of failure computed by Monte Carlo simulations are unstable. As soon as the size of the sample n increases the probability tends to assume the actual value. Such actual value is between the two Ditlevsen bounds [13]-[14] confirming the accuracy of P-TPMC.

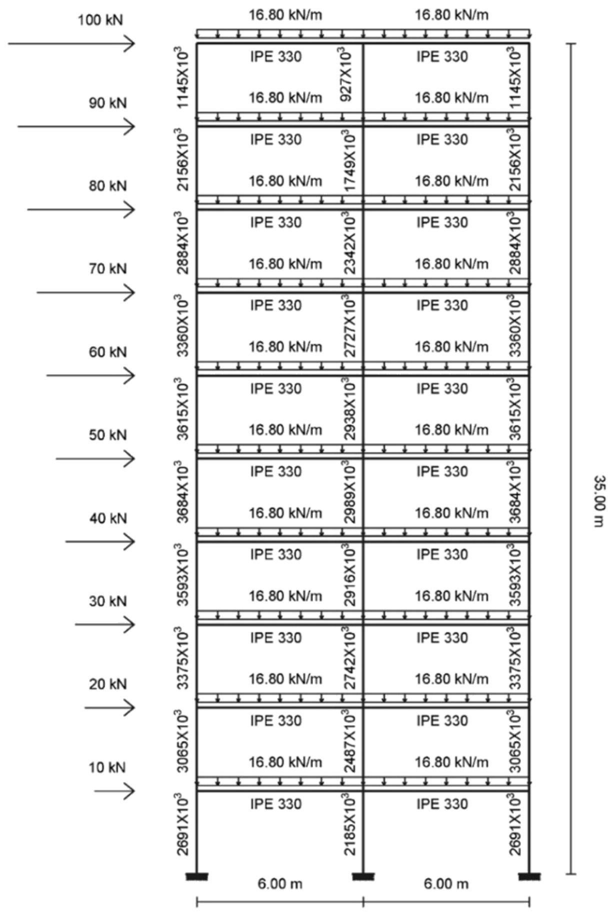

Fig. 4. 10 storey-2 bays moment resisting frame

Table 1. Results of Monte Carlo simulation

Frame sample size [n]	**Probability of failure** (P_f)		
	cov=0.05	**cov=0.10**	**cov=0.15**
100	0.5600	0.3900	0.5800
250	0.5040	0.5640	0.5280
300	0.4833	0.5200	0.5100
400	0.4950	0.4975	0.5450
500	0.4880	0.5160	0.5240
1000	0.5320	0.5090	0.5200
2000	0.4920	0.5090	0.5160
3000	0.5097	0.4943	0.5367
4000	0.4975	0.5082	0.5400
5000	0.4992	0.5044	0.5420
6000	0.4992	0.5074	0.5410
8000	0.4955	0.5103	0.5415

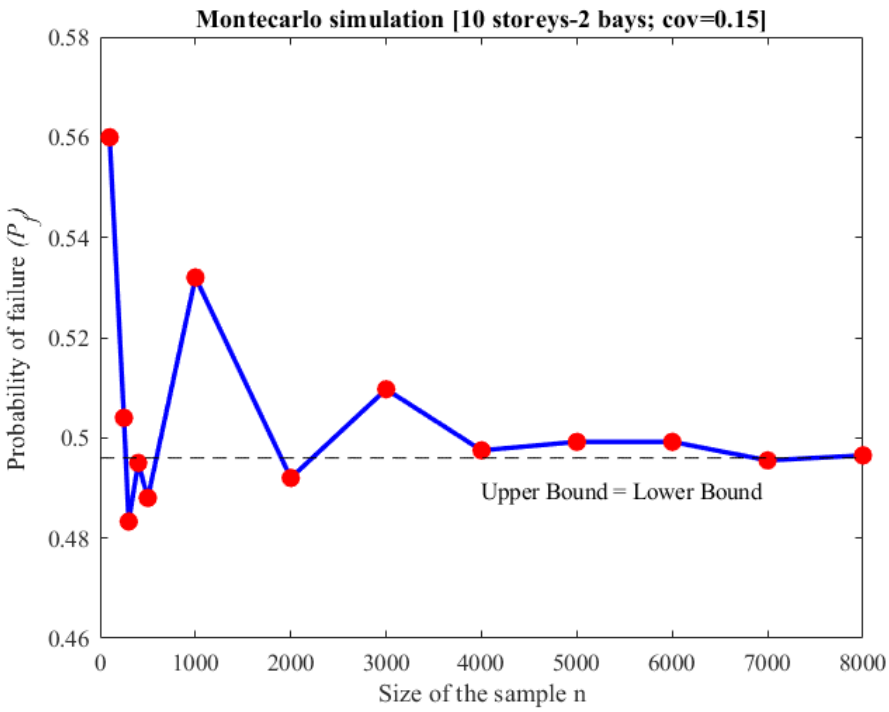

Fig. 5. Values of probability of failure obtained by Monte Carlo simulation for increasing values of the sample size n for cov=0.05

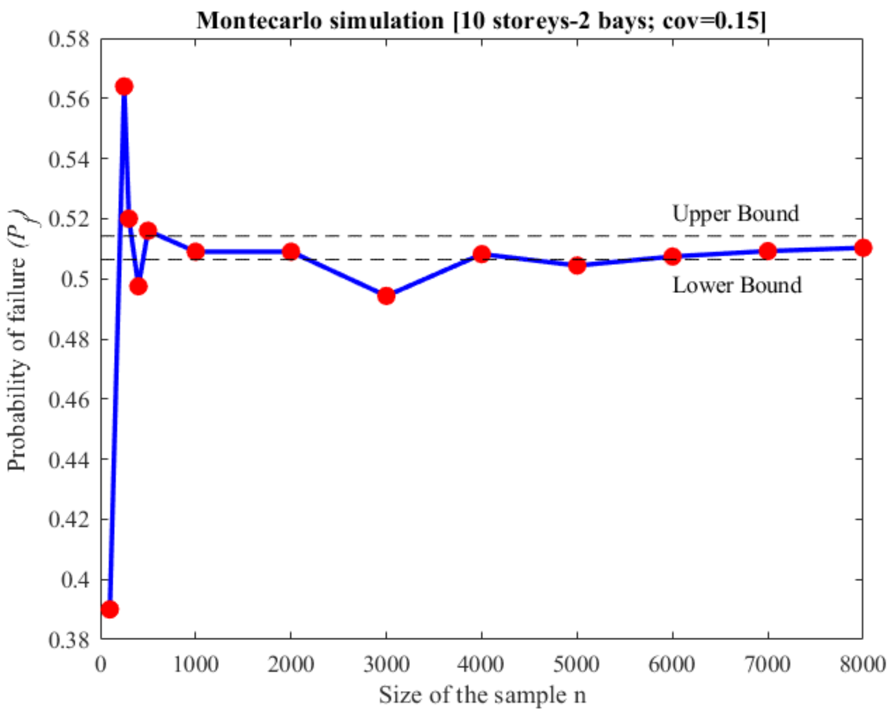

Fig. 6. Values of probability of failure obtained by Monte Carlo simulation for increasing values of the sample size n for cov=0.10

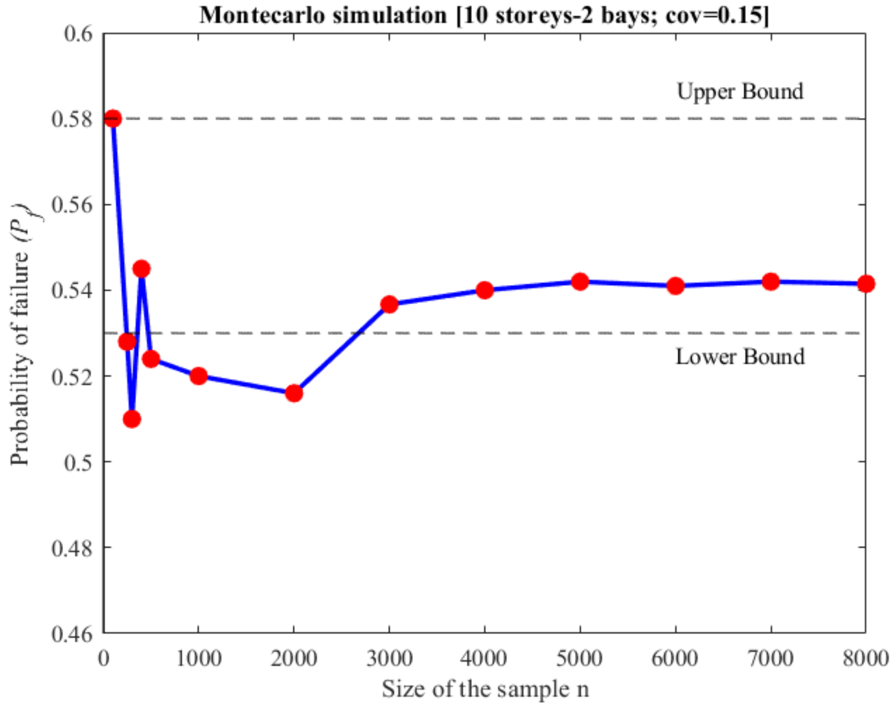

Fig. 7. Values of probability of failure obtained by Monte Carlo simulation for increasing values of the sample size n for cov=0.15

9 PARAMETRIC ANALYSIS TO ESTMATE OVERSTRENGTH FACTOR

The parametric analysis has been performed on 25 moment resisting steel frames with 4, 6, 8, 10 and 12 storeys and 2, 3, 4, 5 and 6 bays. The plastic moments of beams and columns are considered Gaussian random variables which depend on the aleatoric uncertainty of material properties. S275 steel grade has been considered. Three values of the coefficient of variation have been considered, equal to 0.05, 0.10 and 0.15, respectively.

The analyzed frames are preliminarily designed according to the deterministic Theory of Plastic Mechanism Control (TPMC) [3]-[4]. They are characterized by a bay span equal to 6.00 m and an interstorey height equal to 3.50 m. The beams are the same for each frame and they are designed in order to withstand distributed gravity loads whose values are $G_k = 15.00\ kN/m$ and $Q_k = 6.00\ kN/m$ for permanent and live load, respectively. With reference to the non-seismic load combination, the maximum gravity load acting on the beams is equal to $q_v = 1.35G_k + 1.5Q_k = 28.5\ kN/m$ [23]. Such value has been assumed as a deterministic value. IPE 330 sections have been selected for the beams. Since the results of the deterministic TPMC are not influenced by the magnitude of the seismic forces, but are only sensitive to their distribution, a triangular distribution of the seismic horizontal forces has been assumed. Successively, for each frame the plastic moments of the columns are defined according to TPMC.

It is well known that the influence of the material uncertainty can be accounted for within a deterministic design procedure by properly amplifying the plastic moment transmitted by the beams through an overstrength factor γ_{ov}.

As in this paper a rigorous design procedure (TPMC) assuring a collapse mechanism of global type is applied, also a rigorous calibration of the overstrength factor, γ_{ov}, is obtained by properly establishing the design goal in a probabilistic format by means of the P-TPMC. In this way, the resulting values of γ_{ov} are actually related only to the influence of material uncertainty without covering any possible imperfection resulting from unsatisfactory design procedures. To this scope, the deterministic TPMC [3]-[4] is applied for increasing values of the overstrength factor assigned to the beam plastic moments and the resulting frames are analyzed by means of the P-TPMC, thus computing the corresponding lower and upper bounds of the failure probability (i.e. the probability of developing an undesired collapse mechanism different from the global one). Therefore, the results gained by means of such parametric analysis provide the variation of the probability of failure as a function of the overstrength factor adopted for the beams in the deterministic design and of the coefficient of variation.

Table 2. Probability of failure (i.e. probability of developing a collapse mechanism different from the global one) for the 10 storey-2 bay frame

γ_{ov}	**cov=0.05**		**cov=0.10**		**cov=0.15**	
	Lower bound of P_f	**Upper bound of P_f**	**Lower bound of P_f**	**Upper bound of P_f**	**Lower bound of P_f**	**Upper bound of P_f**
1.00	0.4960	0.4960	0.5064	0.5142	0.5334	0.5887
1.05	0.1419	0.1419	0.3089	0.3098	0.4114	0.4289
1.10	0.0172	0.0172	0.1453	0.1453	0.2522	0.2559
1.15	0.0010	0.0010	0.0625	0.0625	0.1745	0.1757
1.20	3.08E-05	3.08E-05	0.0229	0.0229	0.1029	0.1032
1.25	3.89E-07	3.89E-07	0.0068	0.0068	0.0556	0.0556
1.30	3.44E-09	3.44E-09	0.0019	0.0019	0.0294	0.0294

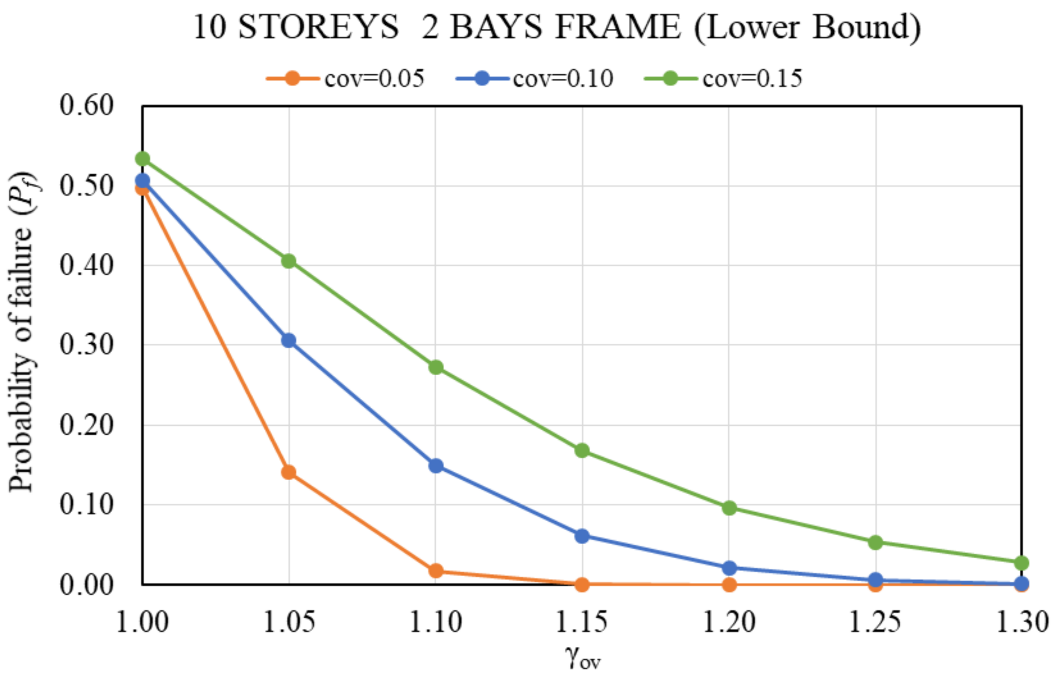

Fig. 8. Lower bound of the probability of failure as affected by γ_{ov} and cov for the 10 storey-2 bay frame

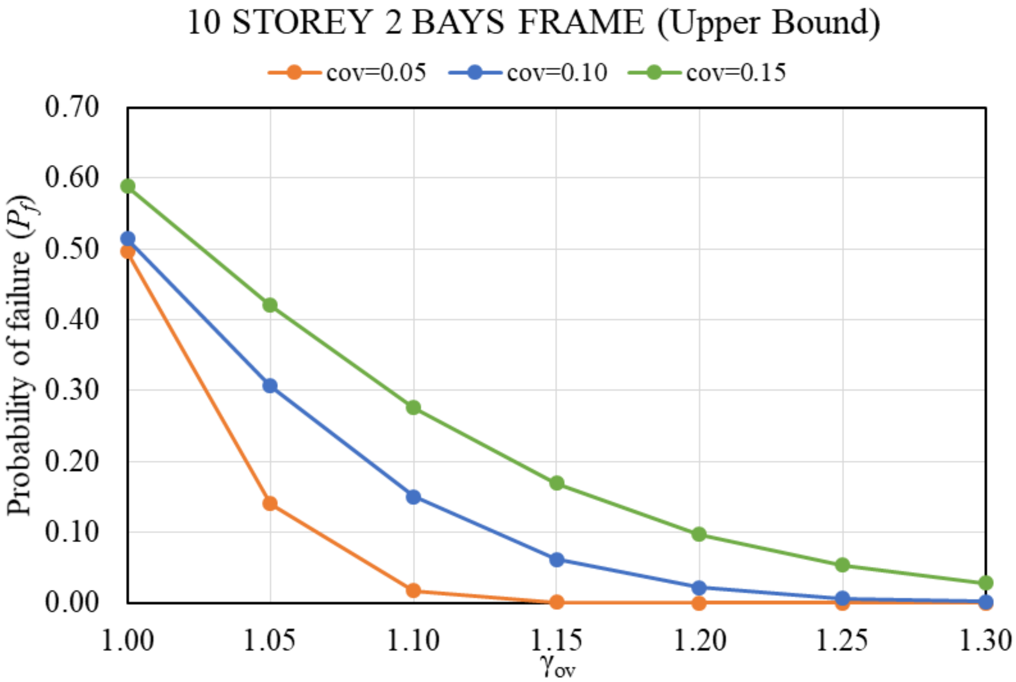

Fig. 9. Upper bound of the probability of failure as affected by γ_{ov} and cov for the 10 storey-2 bay frame

As an example, the results of the analysis described above are depicted in Fig. 8 and Fig. 9 with reference to the 10 storey-2 bay frame presented in Fig. 4. It is possible to observe that:

a) As expected, both the upper bound and the lower bound of the probability of failure increase as the coefficient of variation of the yield strength cov increases, independently of the overstrength factor adopted in the design.
b) As expected, independently of the coefficient of variation, both the upper bound and the lower bound of the probability of failure decrease as the overstrength factor adopted in the design is increased.
c) By properly comparing the upper bound values of the probability of failure with the corresponding lower bounds, for a given coefficient of variation, it is possible to note that the narrowness of the Ditlevsen bounds improves as the overstrength factor of the beams adopted in the design increases.

From the analysis presented above, it is also evident that as soon as the design goal is established in probabilistic terms, it is possible to calibrate the overstrength factor to be adopted in the deterministic design approach in order to properly account for the influence of material uncertainty. As an example, if the design goal is the occurrence of a probability of failure P_f equal to 0.05 (i.e. a probability equal to 0.95 of developing a collapse mechanism of global type), given the coefficient of variation (as an example $cov = 0.10$) it is possible to compute the overstrength factor γ_{ov} needed to assure the design goal with the predefined target probability.
The results of the parametric analysis carried out by varying the number of storeys and the number of bays are reported in Table 3 where reference is made to a value of the coefficient of variation $cov = 0.10$ and to a probability of failure P_f equal to 0.05. Therefore, the resulting γ_{ov} values assure the development of a collapse mechanism of global type with a target probability equal to 95%.

Table 3. Values of $\boldsymbol{\gamma_{ov}}$ obtained by means of the parametric analysis carried out by P-TPMC for P_f=0.05 and cov=0.10

Storeys n_s	**Number of bays** n_b				
	2	3	4	5	6
4	1.170	1.140	1.123	1.108	1.090
6	1.167	1.138	1.122	1.108	1.090
8	1.165	1.137	1.121	1.108	1.090
10	1.163	1.134	1.121	1.108	1.090
12	1.161	1.134	1.120	1.108	1.090

The results presented in Table 3 show that, given a coefficient of variation of the yield strength equal to 0.10, the values of the overstrength factor needed to account for the influence of the material uncertainty vary from a minimum value equal to 1.090 to a maximum value equal to 1.17. Despite of the range of variation of the overstrength factor with the number of bays and the number of storeys is quite narrow, it is possible to derive a more accurate relationship directly accounting for the influence of the frame geometrical configuration (n_s and n_b). To this scope, a regression analysis has been carried out by means of the least square method. The resulting relationship proposed is:

$$\gamma_{ov} = (C_1 \cdot n_s + C_2) \cdot n_b^{(C_3 \cdot e^{C_4 \cdot n_s})} \tag{51}$$

where the coefficient are reported in Table 4.

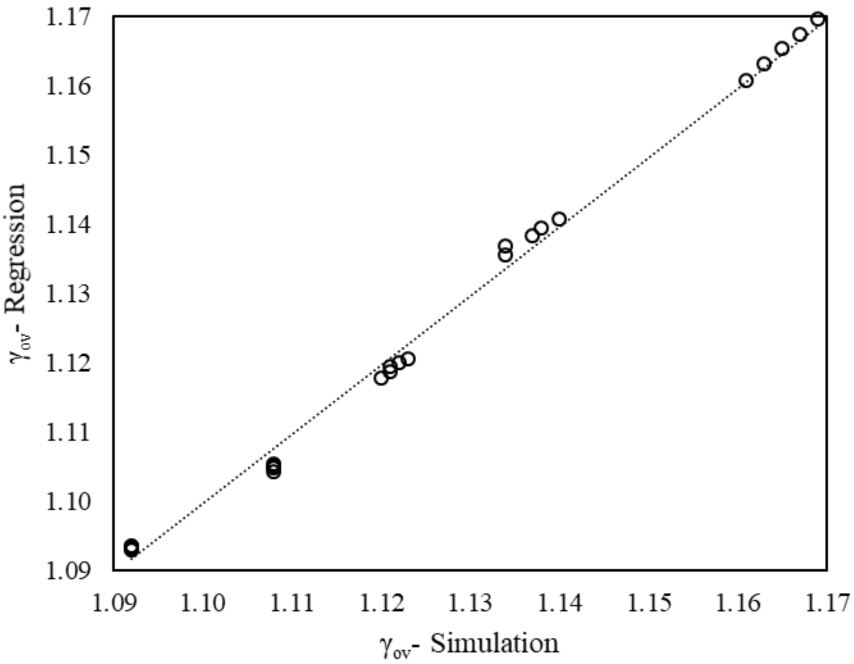

Fig. 10. Accuracy of the relationship proposed for evaluating the overstrength factor accounting also for the frame geometrical configuration

Table 4. Regression coefficients

$\boldsymbol{C_1}$	$\boldsymbol{C_2}$	$\boldsymbol{C_3}$	$\boldsymbol{C_4}$
$-1.90 \cdot 10^{-3}$	1.23	$-6.50 \cdot 10^{-2}$	$-1.60 \cdot 10^{-2}$

The coefficient of determination is $R^2 = 0.9945$, and the standard deviation is equal to 0.0019 showing the accuracy of the proposed relationship as further testified by the graphical representation given in Fig. 10.

9 CONCLUSIONS

The Theory of Plastic Mechanism Control has been extended to the case of stochastic MRFs where the influence of material uncertainty on the collapse mechanism exhibited under seismic horizontal forces has to be considered. Probabilistic Theory of Plastic Mechanism Control, derives from the combination of TPMC and FORM. Specifically, FORM is used to compute the probability of developing undesired collapse mechanisms because of the harmful influence of material uncertainty. The combined use of TPMC and FORM allows the evaluation of the over-strength factor to be considered, in the computation of the maximum bending moment that beams are able to transmit to the columns, to guarantee the desired reliability level, i.e. a given target probability. The comparison between the values of the failure probability obtained by the Probabilistic Theory of Plastic Mechanism Control (P-TPMC) and those obtained by means of Monte Carlo simulations has shown the accuracy of the approach proposed for the failure mode control for MRFs. The results obtained have shown that, as expected, both the upper bound and the lower bound of the probability of failure increase as the coefficient of variation of the yield strength increases, independently of the overstrength factor adopted in the design. Moreover, given the coefficient of variation, both the upper bound and the lower bound of the failure probability decrease as the overstrength factor adopted in the design increases. The narrowness of Ditlevsen bounds improves as the overstrength factor of the beams adopted in the design increases. Finally, a simple relationship for evaluating the value of γ_{ov} to assure a given reliability in the achievement of the design goal, i.e. the development of a collapse mechanism of global type, has been proposed. Its accuracy is testified by a coefficient of determination very close to 1.0.

Nomenclature

$\alpha_0^{(g)}$	*First order collapse mechanism multiplier of horizontal forces for the global mechanism*	$\mathrm{G}(\mathbf{x})$	*Function of the random vector* $\mathbf{x}$
$\alpha_{0.i_b.i_t}^{(t)}$	*First order collapse mechanism multiplier of horizontal forces for the generic mechanism*	h_k	*Storey height of the k-th storey*
$\boldsymbol{\beta}_\mathbf{C}$	*Vector of reliability (or Cornell) indexes*	h_{i_b}, h_{i_t}	*Storey height of the* i_b*-th storey or* i_t*-th storey*
$\gamma^{(g)}$	*The slope of the mechanism equilibrium curves for the global mechanism*	h_{n_s}	*Sum of the interstorey heights of the storeys involved by the generic mechanism*
$\gamma_{i_b.i_t}^{(t)}$	*The slope of the mechanism equilibrium curves for the generic mechanism*	i	*Column index*
γ_{ov}	*Overstrength factor*	i_b, i_t	*Mechanism index*
$\boldsymbol{\Gamma}^{(sb)}$	*Vector of coefficients accounting for second order effects in case of global mechanism and shear band mechanisms*	j	*Bay index*
$\boldsymbol{\Gamma}^{(up)}$	*Vector of coefficients accounting for second order effects in case of global mechanism and upper partial mechanisms*	k	*Storey index*
δ	*Top sway displacement*	$M_{b.j.k}$	*The plastic moment of the beam of j-th bay at k-th storey*
δ_u	*The design displacement compatible with the ductility supply of the structure*	$M_{c.i.k}$	*the plastic moment of i-th column of k-th storey*
μ_G	*Mean value of* G	n	*Size of the frame sample in MC simulation*
$\boldsymbol{\mu}_\mathbf{x}$	*Vector of means of* $\mathbf{x}$	n_0	*Number of the failure cases in MC simulation*
ρ	*Correlation coefficient between* y_1 *and* y_2	n_b	*Number of bays for each storey*
ρ_{ij}	*Correlation coefficient between i-th and j-th events*	n_c	*Number of columns for each storey*
$\boldsymbol{\sigma}_\mathrm{G}$	*Standard deviation of* G	n_s	*Number of storeys*
$\boldsymbol{\sigma}_\mathrm{G}^2$	*Covariance matrix of* G	N_{tot}	*Total number of possible mechanisms*
Φ	*Standard Gaussian CDF*	$\boldsymbol{p}_f$	*Vector of probability of the failure events*
a_0	*Vector the known quantities*	P_f	*Probability of failure*
$\mathbf{b}_i, \boldsymbol{b}_j$	*i-th and j-th row of* **B** *matrix*	$P_{f.i}$	*Probability of failure of i-th event*
$\mathbf{B}$	*Matrix of deterministic coefficients*	$P_{f.ij}$	*Joint probabilities between i-th and j-th events*
$\boldsymbol{B}_b^{(sb)}$	*Submatrix of coefficients of sum of the plastic moments of beams at each storey with reference to events related to the "shear band" mechanisms*	R	*Reliability factor*
$\boldsymbol{B}_c^{(sb)}$	*Submatrix of coefficients of sum of the plastic moments of columns at each storey with reference to events related to the "shear band" mechanisms*	sb	*Shear band mechanisms*
$\boldsymbol{B}_b^{(up)}$	*Submatrix of coefficients of sum of the plastic moments of beams at each storey with reference to events related to the "upper partial" mechanisms*	t	*Mechanism index*
$\boldsymbol{B}_c^{(up)}$	*Submatrix of coefficients of sum of the plastic moments of columns at each storey with reference to events related to the "upper partial" mechanisms*	up	*Upper partial mechanisms*
cov	*Coefficient of variation*	V_k	*The total gravity load applied at k-th storey*
$\mathbf{C}_\mathbf{x}$	*Vector of covariance of* $\mathbf{x}$	$\mathbf{x}$	*Vector random variables*
$E[\,]$	*Linear operator (expected value)*	$\boldsymbol{x}_b$	*Vector collecting the sum of the plastic moments of beams at each storey*
$E_{i_b.i_t}^{(t)}$	*Failure event*	$\boldsymbol{x}_c$	*Vector collecting the sum of the plastic moments of columns at each storey*
F_k	*The seismic force applied at k-th storey*	y_1, y_2	*Independent standard Gaussian variables*

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