Case Studies in Thermal Engineering 42 (2023) 102693

Contents lists available at ScienceDirect

Case Studies in Thermal Engineering

journal homepage: www.elsevier.com/locate/csite

An approximate analytical solution for dynamic heat transfer of building walls

C. Concilio, P. Di Luccia, G. Cuccurullo *

Dipartimento di Ingegneria Industriale, Università degli Studi di Salerno, Via Giovanni Paolo II, 132, 84084, Fisciano (SA), Italy

ARTICLE INFO

Handling Editor: Huihe Qiu

ABSTRACT

The present work deals with an approximate analytical solution for the dynamic temperature field due to the coupled effect of building walls to the adjacent environment. The solution is derived within the framework of weighted residuals and can be used as a handy tool to realize a reasonable estimation of the building thermal performance and to elucidate the dependence on the room descriptive parameters. Results from the present model are compared with predictions arising from both a commercial FEM code and the analytical solution from the standard EN 13786, showing satisfactory results. This conclusion is not surprising since it is demonstrable in view of the characteristic-time responses of the climatic forcing and the building walls. Finally, as an example of application of this approach, the model was focused on describing the thermal response of an open-loop regulated indoor environment.

1. Introduction

Rating issues related to building walls design and thermal comfort analysis require adequate characterisation of unsteady thermal performances. Problems of considerable technical importance fall within this definition, e.g. problems such as optimal insulation conditions for wall structures, design of the building fabric, design of passive and active systems necessary to realize the required thermal conditions in the respect of energy saving or the need to assess the impacts of various climatic situations on building energy requirements.

The question is not obvious since several parameters must be considered to describe the connections among the involved sub-systems, namely, building walls and adjacent room, HVAC plants and the related temperature control strategies, external state environment or inner sources. For achieving such a task, several techniques have been developed: semi empirical models [1–7] or transfer functions-based methods [8–10] which solve numerically complex PDE systems can be easily found in literature. A broad spectrum of software that rely on numerical processing for simulating the unsteady thermal performance of buildings has been well established for many decades (DOE, TRNSYS, BLAST, EnergyPlus, Genopt, MatLab/Simulink, SUNCODE, COMSOL), see for instance Refs. [11–20]. Moreover, mathematical models based on RC electric circuit analogy, adequate for high mass building, can also be found [21–24]. A more conventional and widespread analysis, based simply on the periodic steady-state wall behaviour, is proposed by the "ISO 13786-2017 - periodic thermal transmittance". The latter approach involves periodic II and III type boundary conditions for the external environment, whereas a fixed internal temperature is considered [25–28]. This overly straightforward model exhibits the attractive feature of being easy to understand and manipulate, therefore is diffusely used to describe wall thermal response, but it is irrespective of the features of the interconnected sub-systems. The analysis introduced in the present paper aims to overcome this

* Corresponding author.
E-mail address: gcuccurullo@unisa.it (G. Cuccurullo).

https://doi.org/10.1016/j.csite.2022.102693
Received 21 October 2022; Received in revised form 17 December 2022; Accepted 31 December 2022
Available online 2 January 2023

problem whilst ensuring easy implementation of the model and satisfactory accuracy. This objective is pursued by using the integral and the collocation methods for satisfying the energy balance equations related to the composite wall; the latter is coupled to the inner ambient described by a lumped approach. Following such a scheme, the problem is expressed in terms of an easy-to-handle system of ordinary differential equations. As usual, when using weighted residuals methods, a proper selection of basic profiles is essential to obtain accurate results.

2. Basic equations

A three-layers composite plane wall is considered which is subjected to 1D steady periodic regime due to the outer environment temperature oscillations. A sinusoidal temperature – time dependence for outer temperature is sought as: $T_o(t) = T_{o,a} - \Delta T_o \cos(\Omega\, t)$, where $T_{o,a}$, ΔT_o and Ω are the outer fluid average temperature, its amplitude and angular velocity, respectively, Fig. 1. Two layers, made of solid and perforated bricks respectively, are separated by an interposed insulating layer. The wall-building made up as described is assumed to represent the envelope enclosing the ambient to control. A lumped approach is used to follow with the inner ambient temperature, therefore the ambient thermal capacity C_i, encompassing the effect of air, furniture and inner walls are assumed to be known. Boundary conditions on both wall surfaces provide heat transfer linearly depending on wall temperature excess with constant heat transfer coefficients, h_i and h_o for inner and outer surfaces. The initial temperature T_{in} is supposed to be uniform across the wall and the indoor environment.

Since commonly used insulation materials include rock wool, mineral wool, polystyrene, and polyurethane which exhibit very little heat storage capacity, the insulating layer thermal inertia is neglected and it is reviewed merely as a thermal resistance, R_{in}. This assumption makes it possible to replace the insulating layer by introducing a finite temperature gap at the interface between the two residual layers, Fig. 1. The remaining two layers (thicknesses L_1 and L_2) are assumed to be isotropic homogeneous solids, with thermal conductivities (λ_1 and λ_2) and diffusivities (α_1 and α_2) independent of temperature.

Referring to the coordinate system shown in Fig. 1, the energy balance equations and the related boundary conditions for the equivalent thermal problem can be written as

$$\frac{\partial T_1}{\partial t} = \alpha_1 \frac{\partial^2 T_1}{\partial x^2} \tag{1}$$

$$\frac{\partial T_2}{\partial t} = \alpha_2 \frac{\partial^2 T_2}{\partial x^2} \tag{2}$$

$$\lambda_1 \left.\frac{\partial T_1}{\partial x}\right|_{0,t} = h_i [T_1(0,t) - T_i(t)] \tag{3}$$

$$-\lambda_2 \left.\frac{\partial T_2}{\partial x}\right|_{L_1+L_2,t} = h_o [T_2(L_1+L_2,t) - T_o(t)] \tag{4}$$

$$\lambda_1 \left.\frac{\partial T_1}{\partial x}\right|_{L_1,t} = \lambda_2 \left.\frac{\partial T_2}{\partial x}\right|_{L_1,t} \tag{5}$$

$$T_2(L_1,t) - T_1(L_1,t) = R_{in} \lambda_1 \left.\frac{\partial T_1}{\partial x}\right|_{L_1,t} \tag{6}$$

while $T(x,0) = T_i(0) = T_{in}$ for each point in the domain. The above equations can be arranged in nondimensional form to narrow-down the independent parameters involved in the description:

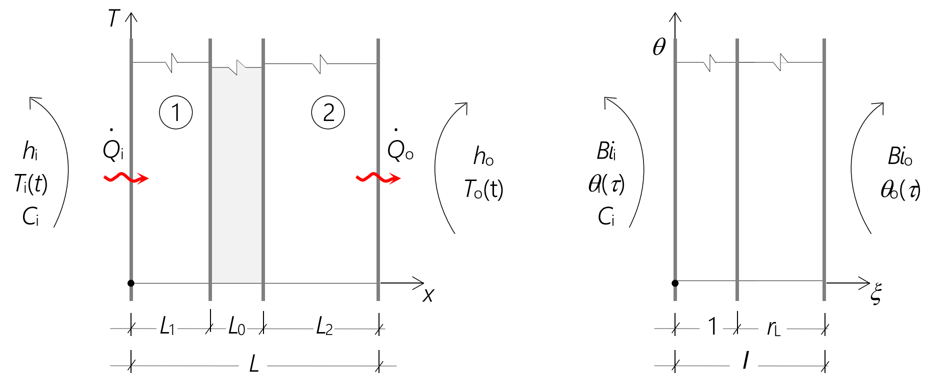

Fig. 1. Sketches of the original and equivalent-dimensionless problems.

$$\frac{\partial\theta_1}{\partial\tau}=\frac{\partial^2\theta_1}{\partial\xi^2} \tag{1}$$

$$\frac{\partial\vartheta_2}{\partial\tau}=r_\alpha\frac{\partial^2\vartheta_2}{\partial\xi^2} \tag{2}$$

$$\left.\frac{\partial\theta_1}{\partial\xi}\right|_{0,\tau}=Bi_{\mathrm{i}}[\theta_1(0,\tau)-\theta_{\mathrm{i}}(\tau)] \tag{3}$$

$$\left.\frac{\partial\theta_2}{\partial\xi}\right|_{l,\tau}=-\frac{Bi_{\mathrm{o}}}{r_L}[\theta_2(l,\tau)-\theta_0(\tau)] \tag{4}$$

$$\left.\frac{\partial\theta_1}{\partial\xi}\right|_{1,\tau}=r_{\mathrm{k}}\left.\frac{\partial\theta_2}{\partial\xi}\right|_{1,\tau} \tag{5}$$

$$\theta_2(1,\tau)+\theta_{\mathrm{c}}-\theta_1(1,\tau)=\frac{R_{\mathrm{in}}}{L_1/\lambda_1}\left.\frac{\partial\theta_1}{\partial\xi}\right|_{1,\tau} \tag{6}$$

while the initial condition requires: $\theta_1(\xi,0)=0$; $\theta_2(\xi,0)=-\theta_{\mathrm{c}}$; $\theta_{\mathrm{i}}(0)=0$; $\theta_{\mathrm{o}}(0)=-1$.

The non-dimensional variables are defined as

$$\theta_1=\frac{T_1-T_{\mathrm{in}}}{\Delta T_{\mathrm{o}}};\theta_2=\frac{T_2-T_{\mathrm{o,a}}}{\Delta T_{\mathrm{o}}};\theta_{\mathrm{i}}=\frac{T_{\mathrm{i}}-T_{\mathrm{in}}}{\Delta T_{\mathrm{o}}};\xi=\frac{x}{L_1};\tau=\frac{t}{L_1^2/\alpha_1};\theta_{\mathrm{o}}=\frac{T_{\mathrm{o}}-T_{\mathrm{o,a}}}{\Delta T_{\mathrm{o}}}=-\cos(\omega\tau)$$

while, in the course of normalizing the problem, the following dimensionless groups appeared.

$$Bi_{\mathrm{i}}=\frac{h_{\mathrm{i}}}{\lambda_1}L_1;Bi_{\mathrm{o}}=\frac{h_{\mathrm{o}}}{\lambda_2}L_2;\theta_{\mathrm{c}}=\frac{T_{\mathrm{o,a}}-T_{\mathrm{in}}}{\Delta T_{\mathrm{o}}};r_{\mathrm{L}}=L_2\Big/L_1;r_{\mathrm{k}}=k_2\Big/k_1;r_\alpha=\alpha_2\Big/\alpha_1$$

The coupling of the wall with the inner environment is accounted on by following a lumped approach; the related inner-ambient energy balance equation involves the storage, the convective and two diffusive terms, Fig. 2:

$$C_{\mathrm{i}}\frac{\partial T_i}{\partial t}=\dot{Q}_{\mathrm{co}}-\dot{Q}_{\mathrm{tr}}+\dot{m}_{\mathrm{ve}}c_{\mathrm{p,a}}[T_{\mathrm{o}}(t)-T_{\mathrm{i}}(t)] \tag{7}$$

The convective term refers to the ventilation due to an external air mass flow, $\dot{m}_{\mathrm{ve}}$, $c_{\mathrm{p,a}}$ being the air specific heat. The first diffusive term is the heating power delivered by the coil, $\dot{Q}_{\mathrm{co}}$, the latter is due to the transmission through the building envelope, $\dot{Q}_{\mathrm{tr}}$.

The heat through the wall is expressed with reference to the convective heat flow at the inner surface:

$$\dot{Q}_{\mathrm{tr}}=h_{\mathrm{i}}A_{\mathrm{i}}[T_{\mathrm{i}}(t)-T_1(0,t)] \tag{8}$$

The fan coil unit (FCU) is supposed a water-to-air heat exchanger, therefore, the heating capacity delivered by the water (hot fluid, subscript “w”) to the air (cold fluid) can be cast in the form [29–31],

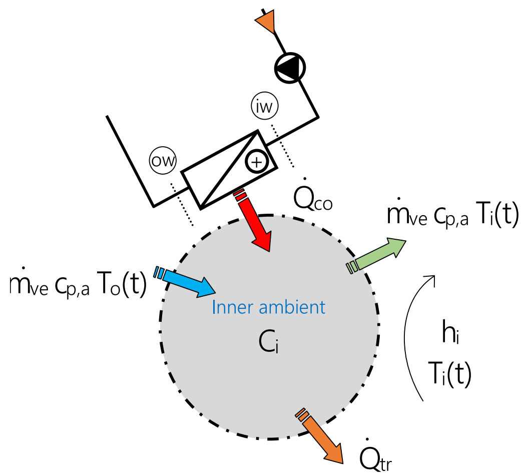

Fig. 2. Lumped energy balance for the inner-ambient.

$$\dot{Q}_{\mathrm{co}} = U\left[T_{\mathrm{w,i}}(t) - T_{\mathrm{i}}(t)\right]; \quad U = \dot{Q}^{*}_{\mathrm{co}} \Big/ \left[T^{*}_{\mathrm{w,i}} - T^{*}_{\mathrm{i}}\right] \tag{9}$$

where the specific-heating-capacity U refers to the heating capacity delivered by the FCU as driven by unitary maximum temperature difference over the device under design conditions (superscript "*"). The latter expression points-out the maximum temperature difference across the heat exchanger, related to water and air inlets, and it is applicable if the changes in temperature from the design conditions are moderate. The use of this parameter is convenient for rating the performance of the heat exchanger because it depends exclusively on air and water mass flow rates. The heating regulation strategy is supposed to vary the water temperature supplied to the FCU in dependence on the external one while maintaining a constant flow rate through it. Therefore, it is driven by an open-loop controller according to the following linear relationship

$$T_{\mathrm{w,i}}(t) = \frac{T^{*}_{\mathrm{w,i}} - T^{*}_{\mathrm{i}}}{T^{*}_{\mathrm{o}} - T^{*}_{\mathrm{i}}}\left(T_{\mathrm{o}}(t) - T^{*}_{\mathrm{i}}\right) + T^{*}_{\mathrm{i}} \tag{10}$$

Considering equations 8–10, the energy balance (7) can be written in dimensionless form as

$$\frac{\partial\theta_{\mathrm{i}}}{\partial\tau} = \frac{1}{\tau_{\mathrm{co}}}\left[\left(\theta^{*}_{\mathrm{i}} + \frac{T^{*}_{\mathrm{i,w}} - T^{*}_{\mathrm{i}}}{T^{*}_{\mathrm{o}} - T^{*}_{\mathrm{i}}}\left(\theta_{\mathrm{o}} - \theta^{*}_{\mathrm{i}} + \theta_{\mathrm{c}}\right)\right) - \theta_{\mathrm{i}}\right] - \frac{1}{\tau_{\mathrm{i}}}\left[\theta_{\mathrm{i}} - \theta_{1}(0,\tau)\right] + \frac{1}{\tau_{\mathrm{ve}}}\left(\theta_{\mathrm{o}} + \theta_{\mathrm{c}} - \theta_{\mathrm{i}}\right) \tag{7'}$$

where the characteristic dimensionless times of internal environment, battery and ventilation are introduced:

$$\tau_{\mathrm{i}} = \frac{C_{\mathrm{i}}/h_{\mathrm{i}}A_{\mathrm{i}}}{t_{\mathrm{rif}}}; \; \tau_{\mathrm{co}} = \frac{C_{\mathrm{i}}/U}{t_{\mathrm{rif}}}; \; \tau_{\mathrm{ve}} = \frac{C_{\mathrm{i}}/\dot{m}_{\mathrm{ve}}c_{\mathrm{p,a}}}{t_{\mathrm{rif}}} \tag{11}$$

3. Method of solution

In the present study, the unsteady temperature field is solved as an approximate analytical solution following the weighted residuals approach, for instance see Refs. [32–34]. For the problem at hand, the integral energy balance is imposed on both layers. Therefore, information concerning the integration variable must be explicitly provided by a proper choice of the basic profiles. It's a good rule to select such profiles to be as closely as possible related to the problem under consideration; for instance, they can be chosen in such a way that they meet the boundary conditions while exhibiting the appearance of simpler limiting cases for which the solution is known. For the case under consideration, the semi-infinite body model is helpful. It can be estimated that the thermal wavelength β inside each layer turns out to be $\beta = 2\,\pi\,\delta$, where $\delta = (2\alpha/\Omega)^{1/2}$ is the thermal penetration depth, [33,35–37]. Consequently, it's reasonable to conclude that, for the usual thickness of the layers of the building walls, the ratio of the layer thickness to the corresponding wavelength is less than unit, thus suggesting that a simple polynomial shape can be suitably sought for drawing temperature profiles. Third-degree polynomials are selected to characterize the approximate solutions within the two remaining layers:

$$\widetilde{\vartheta}_{\mathrm{j}}(\xi,\tau) = \sum_{k=0}^{3} a_{\mathrm{j,k}}(\tau)\xi^{k}\;; \quad j = 1,2 \tag{12}$$

The search for the 8 unknown trial functions, $a_{1,1}$, $a_{1,2}$ … $a_{2,3}$, plus the unknown inner temperature $\theta_{\mathrm{i}}(\tau)$ is triggered by invoking equation (3')-(7') and, in addition, the integral form of equation (1') and (2') in the corresponding domains

$$\int_{0}^{1}\frac{\partial\widetilde{\vartheta}_{1}}{\partial\tau}\mathrm{d}\xi = \int_{0}^{1}\frac{\partial^{2}\widetilde{\vartheta}_{1}}{\partial\xi^{2}}\mathrm{d}\xi \qquad \int_{1}^{1+r_{L}}\frac{\partial\widetilde{\vartheta}_{2}}{\partial\tau}\mathrm{d}\xi = \int_{1}^{1+r_{L}}\frac{\partial^{2}\widetilde{\vartheta}_{2}}{\partial\xi^{2}}\mathrm{d}\xi \tag{13}$$

Two further equations are determined by imposing that energy balances, equation (1') and (2'), are satisfied at two given points within the range [0,l]; for this purpose, the internal and external building wall surfaces identified @ $\xi = 0$ and $\xi = l$ are selected, then one can write

$$\left.\frac{\partial\widetilde{\vartheta}_{1}}{\partial\tau}\right|_{0,\tau} = \left.\frac{\partial^{2}\widetilde{\vartheta}_{1}}{\partial\xi^{2}}\right|_{0,\tau} \qquad \left.\frac{\partial\widetilde{\vartheta}_{2}}{\partial\tau}\right|_{1+r_{L},\tau} = r_{\alpha}\left.\frac{\partial^{2}\widetilde{\vartheta}_{2}}{\partial\xi^{2}}\right|_{1,\tau} \tag{14}$$

The convenience of using the approximate approach lies in the ability to represent the addressed set of non-dimensional equations in a concise and effective way; in fact, the system of ODEs can be set into the following matrix form,

$$\underline{\underline{\mathrm{C}}}\dot{\underline{\mathrm{x}}} + \underline{\underline{\mathrm{K}}}\underline{\mathrm{x}} = \underline{\mathrm{F}}_{0} \tag{15}$$

where $\underline{\mathrm{x}}(\tau) = \{a_{1,0}(\tau), a_{1,1}(\tau), a_{1,2}(\tau), a_{1,3}(\tau), a_{2,0}(\tau), \ldots, \theta_{\mathrm{i}}(\tau)\}$ is the array exposing the unknown trial functions, $\underline{\underline{\mathrm{C}}}$ is the "thermal capacity matrix", $\underline{\underline{\mathrm{K}}}$ is the "thermal conductance matrix" and $\underline{\mathrm{F}}_{0}(\tau)$ is the driving term collecting all the forcing functions:

$$\underline{\mathrm{F}}_{0}(\tau) = \left\{0,0,0,-\frac{Bi_{\mathrm{o}}}{r_{\mathrm{L}}}\cos(\omega\tau),0,\theta_{\mathrm{c}},\frac{1}{\tau_{\mathrm{co}}}\left(\theta^{*}_{\mathrm{i}} + \frac{\theta^{*}_{\mathrm{i,w}} - \theta^{*}_{\mathrm{i}}}{\theta^{*}_{\mathrm{o}} - \theta^{*}_{\mathrm{i}}}\left(\theta_{\mathrm{o}} - \theta^{*}_{\mathrm{i}} + \theta_{\mathrm{c}}\right)\right) + \frac{1}{\tau_{\mathrm{ve}}}\left(\theta_{\mathrm{c}} - \theta_{\mathrm{o}}\right),0,0\right\} \tag{16}$$

Finally, the initial condition gives: $\underline{\mathrm{x}}(0) = \{0, 0, 0, 0, -1, 0, 0, 0, 0\}$.

4. The infinite-capacity ambient

A first practical use of the model was to review the case of an ambient behaving as a thermal reservoir with constant and uniform temperature (infinite capacity ambient case). Assuming an infinite inertia for the inner ambient, its temperature will be constant over time and fixed to the initial value, which is taken to be $T_i^* = 20\ ^\circ C$, that is equal to the design inner temperature. Provided periodic steady-state conditions are attained, the configuration given by the standard EN ISO 13786 (Thermal performance of building components - Dynamic thermal characteristics) is recovered. Therefore, the EN-standard serves as a baseline against which to validate the solution obtained with the proposed model. In addition, a commercial FEM code is used to evaluate temperature profiles in the building-wall as a function of time; this is because the standard makes it possible to calculate only the temperature of the interfaces.

Numerical results are evaluated with reference to a typical test-case featured as described below. Outer and inner boundary conditions are for: $T_{o,a} = 0\ ^\circ C$; $\Delta T_o = 5\ ^\circ C$; $h_o = 25\ W/(m^2\ K)$; $h_i = 7.7\ W/(m^2\ K)$, while the wall structure is sought as shown in Table 1.

The curves of Fig. 3, quantitatively displayed in Table 2 for selected points, show that the approximate solution closely matches the numerical ones. The maximum relative error between the approximate and the numerical solution is generally contained within the 1.60%.

The dynamic building wall behaviour is evaluated in terms of dynamic transmittance and time shift between the inner and outer temperatures. Results exposed in Table 3 compare the approximate, the EN-standard and the numerical solution: again, the ability of the approximate model to follow-up the dynamic behaviour of the wall seems clear.

It almost goes without saying that the infinite-capacity ambient may be considered as the result of the action of an ideal control system, able to maintain the inner temperature fixed at the design value, irrespective of both the fancoil and the control strategy and in use. In the following paragraph, the constraint of infinite capacity is removed and the above-mentioned effects can be taken into consideration.

5. The finite-capacity ambient

The configuration for infinite ambient capacity is assumed as a baseline for studying a further case including the effect of the ambient inertia. Consequently, the influence of air exchange and the heating coil features on the wall building profiles are included in the analysis. In turn, the impact of the open-loop strategy for the temperature control is implemented. Numerical results are still evaluated based on the wall structure and boundary conditions described in the preceding paragraph. Additionally, the inner ambient is featured as follows: plan dimensions are of 15 × 10 m, its height is of 3 m, the frontal area of surfaces exposed to the external environment is 25 m^2, furniture correspond to 600 kg of wood ($\rho = 450\ kg/m^3$, $c = 1380\ J/(kg\ K)$). Natural ventilation is supposed to realize a fresh air flow equal to half a volume in an hour.

The nominal size of the FCU must be such as to meet the design ambient load. It follows that the specific heating capacity turns out to be $U^* = 60\ W/K$ assuming inlet temperatures equal to $T_{w,i}{}^* = 50\ ^\circ C$ and $T_i{}^* = 20\ ^\circ C$, respectively for water and air. The latter value uniquely identifies the water flow rate to the heat exchanger for a selected fan velocity.

Finally, according to equation (15), the system of equations is written as follows.

In order to display results, the specific heating capacity is assumed to be an independent variable, ranging around the nominal value, because of the limited availability of commercial sizes. As expected, increasing average temperatures are reached in the indoor environment as the specific heating capacity increases, see Fig. 4; in particular, the target indoor temperature is attained when the specific heating capacity attains the nominal value U^*. The open-loop control system fails to comply with the design ambient temperature depending on the FCU features, therefore a suitable correction of relationship between the water supply temperature and the external one should be adopted.

Fluctuation of the inner temperature around the average value turns out to be negligible, see for example Fig. 5 where curves are evaluated in relation to the nominal specific heating capacity. This occurrence confirms that the hypothesis of indoor temperature independent of time, as suggested by the standard EN 13786, is typically suitable. Curves in Fig. 5 also exhibit that the temperatures of the inner and outer surfaces of the wall are shifted with respect to the corresponding air temperatures.

Dynamic features of the wall buildings are given in terms of time lag between indoor and outdoor temperatures, Fig. 6. It is clear that the dynamic thermal response of the wall is influenced by both the finite capacity environment and the chosen FCU. Large deviations from the results related to the infinite-capacity ambient appear. As expected, since the inner ambient capacity helps both to shift the peaks and to reduce the severity of external fluctuations, best performance is achieved with increasing ambient capacity; all curves are monotonically decreasing with increasing the FCU size because of higher heat transfer rates through the wall.

6. Conclusions

In this work an approximate analytical model has been proposed which gives out the unsteady thermal field in a composite wall due to a periodic oscillating external ambient temperature while the inner boundary condition is coupled with the inner ambient regarded

Table 1
Building Wall structure.

LAYER	λ [W m^{-1} K^{-1}]	ρ [kg m^{-3}]	c [J kg^{-1} K^{-1}]	L [cm]
Brick 1	0.777	1800	920	8
Polyurethane panel	0.045	335	850	5
Brick 2	0.400	1800	920	14

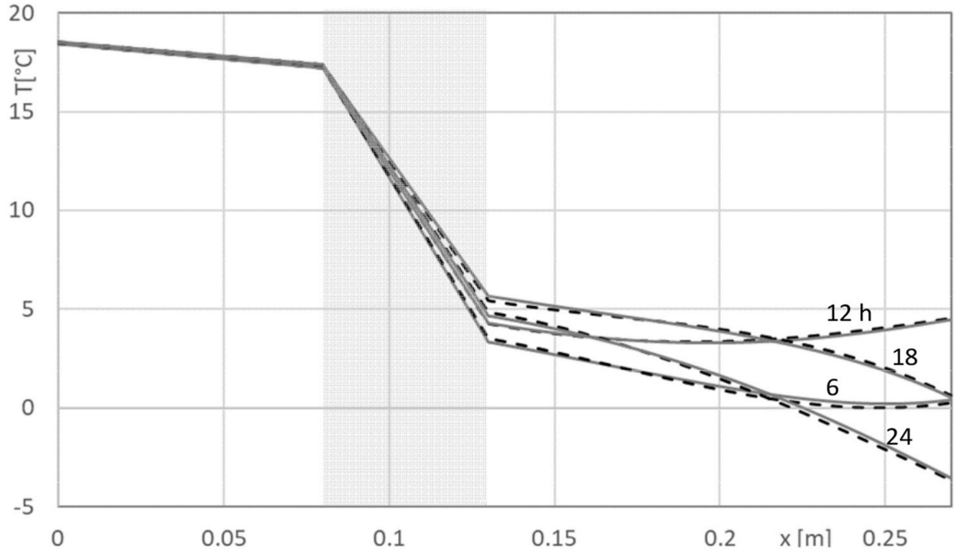

Fig. 3. Numerical (continuous) vs approximate (dashed) temperature profiles for the infinity capacity ambient.

Table 2
relative error for the infinite ambient capacity.

time [h]	position				
	0 [%]	L_1 [%]	L_1+ L_0 [%]	L_1+L_0+L_2 [%]	Max [%]
6	0.04	0.10	1.06	0.57	1.28
12	0.04	0.10	1.07	0.64	1.24
18	0.07	0.09	0.20	0.31	0.66
24	0.01	0.01	0.82	0.59	1.60

Table 3
dynamic building wall behaviour.

solution	parameter	
	Dynamic transm. [10^{-3} W/(m^2 K)]	time lag [h]
numeric	90	11.04
approx.	89	11.15
standard	87	11.46

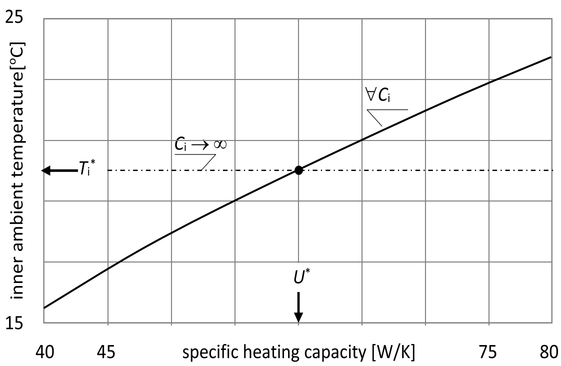

Fig. 4. Inner ambient temperature.

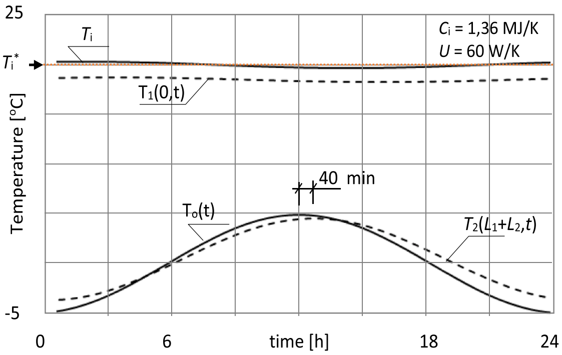

Fig. 5. temperature-time dependence.

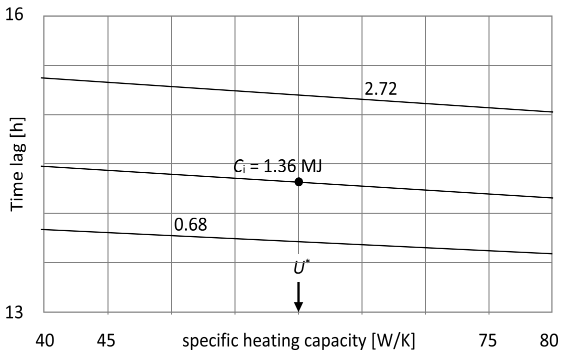

Fig. 6. Time lag.

as a lumped system. Contributions due to both air renewal and to the control strategy of the FCU heat output are accounted for. The approximate solution fairly agrees with numerical check-runs arising from both the standard EN ISO 13786 and the FEM calculation, however, it is highly simplified and convenient to use. The proposed method makes thermal profiles available within the wall-building, a feature not allowed by the standard.

The availability of a simple model for studying the transient thermal behaviour of buildings with a satisfactory level of accuracy suggested to start a study of the wall envelope performance encompassing the ability to account for the influence of the internal environment and of the FCU feature, as well. The proposed method has been applied to account for the effect of the heating coil with a climatic (open loop) regulation.

With reference to the configuration at hand, the functional dependence on the main parameters was brought to light and meaningful quantitative differences were pointed out with respect to the reference case encompassing an inner ambient with infinite thermal capacity as suggested by the EN-standard. It was than proven that the method allows a simple and quick way for studying the transient thermal behaviour of buildings. Further developments of the present work will include the implementation of thermostatic environmental regulations to clarify the effect of the indoor features ignored by EN ISO 13786.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper

Data availability

No data was used for the research described in the article.

References

[1] N.W. Wilson, W.G. Colborne, R. Ganesh, Determination of thermal parameters of an occupied house, Build. Eng. 90 (1984) 39–50.
[2] R.R. Crawford, J.E. Woods, A method for deriving a dynamic system model from actual building performance data, Build. Eng. 91 (1985) 1859–1874.
[3] X. Fang, T. Yang, Regression methodology for sensitivity analysis of solar heating walls -, Appl. Therm. Eng. 28 (17–18) (2008) 2289–2294.
[4] Biao Yan, Xi Meng, Jinlong Ouyang, Enshen Long - Impact of Occupant Behavior on Thermal Performance of the Typical-Composite Walls of a Building.

[5] P. Monzó, P. Mogensen, J. Acuña, F. Ruiz-Calvo, C.- Montagud, A novel numerical approach for imposing a temperature boundary condition at the borehole wall in borehole fields, Geothermics 56 (2015) 35–44.
[6] M. Arfan, I. Mahariq, K. Shah, T. Abdeljawad, G. Laouini, P.O. Mohammed, Numerical computations and theoretical investigations of a dynamical system with fractional order derivative, Alex. Eng. J. 61 (3) (2022) 1982–1994. Results in Physics 36 (2022): 105431.
[7] B. Xu, H. Ye, C.- Zhang, Numerical simulation on the dynamic air-conditioning load of the top floor building with thermal insulation coatings, Taiyangneng Xuebao/Acta Energiae Solaris Sinica 29 (7) (2008) 862–870.
[8] G. Beccali, M. Cellura, V.L. Brano, A. Orioli, Is the transfer function method reliable in a European building context? A theoretical analysis and a case study in the south of Italy, Appl. Therm. Eng. 25 (2–3) (2005) 341–357.
[9] E. Sassine, Y. Cherif, E. Antczak, J. Dgheim, An efficient and simple method for investigating dynamic heat transfer in multilayered building components, Int. J. Magn. Reson. Imag. 7 (3) (2022) 281–309.
[10] American Society of Heating, Refrigerating and AirConditioning Engineers, ASHRAE Fundamentals, Atlanta, GA, 1993.
[11] J. Savoyat, K. Johannes, J. Virgone, Integration of thick wall in trnsys simulation, in: 30th ISES Biennial Solar World Congress 2011 vol. 3, SWC, 2011, pp. 1836–1847, 2011.
[12] D.G. Stephenson, G.P. Mitalas, Calculation of heat conduction transfer functions for multi-layer slabs, Build. Eng. 77 (Pt 2) (1971) 117–126.
[13] Y. Wang, L. Wang, W. Hu, P.- Sun, Influence of the heat transfer coefficient of the outer-surface on the natural convective heat transfer characteristics in a heating room, Harbin Gongcheng Daxue Xuebao/J. Harbin Eng. Univ. 36 (9) (2015) 1206–1211.
[14] Suliman S. Alfarawi, Hossin Omar, Azeldin El-Sawi, Ayad M. Al Jubori, Thermal performance assessment of external wall construction for energy-efficient buildings, Eur. J. Sustain. Dev. Res. 6 (3) (2022), em0189.
[15] P. Bahramnia, S.M.H. Rostami, J. Wang, G.-J.- Kim, Modeling and controlling of temperature and humidity in building heating, ventilating, and air conditioning system using model predictive control, Energies 12 (24) (2019) art. no. 4805.
[16] D. Kim, W. Zuo, J.E. Braun, M. Wetter, Comparisons of building system modeling approaches for control system design, in: Proceedings of BS 2013: 13th Conference of the International Building Performance Simulation Association, 2013, pp. 3267–3274.
[17] V. Pukhkal, Numerical modeling of energy consumption in residential buildings, IOP Conf. Ser. Mater. Sci. Eng. 1079 (2021), 052025.
[18] A. Mahdavi, C. Pröglhöf, A model-based method for the integration of natural ventilation in indoor climate systems operation -, in: IBPSA 2005 - International Building Performance Simulation Association, 2005, pp. 685–692, 2005.
[19] Ioannis Alexopoulos, Christos Tzivanidis, Evangelos Bellos, Georgios Mitsopoulos, Dimitrios Korres, Thermal behavior of a building simulation using COMSOL, in: Conference: 7th IC-SCCE at, vol. 2, 2016, pp. 409–413. Athens.
[20] L. De Santoli, et al., A general approach for retrofit of existing buildings towards NZEB: the windows retrofit effects on indoor air quality and the use of low temperature district heating, in: 2018 IEEE International Conference on Environment and Electrical Engineering and 2018 IEEE Industrial and Commercial Power Systems Europe, EEEIC/I&CPS Europe, 2018, pp. 1–6, https://doi.org/10.1109/EEEIC.2018.8493730.
[21] K.A. Antonopoulos, E. Koronaki, On the heat capacity of Greek buildings, in: *Proc. 1st Int. Conf. On Energy And the Environment*, vol. 2, 1997, pp. 463–470. Limassol, Cyprus.
[22] K.A. Antonopoulos, E. Koronaki, Apparent and effective thermal capacitance of buildings, Energy Int. J. 23 (1998) 183–192, 24.
[23] M.E. Hoffman, M. Feldman, Calculation of the thermal response of buildings by total thermal time constant method, Build. Environ. 16 (1981) 71–85.
[24] J. Campbell, Calculation of heat requirements with intermittent heating, Build. Eng. 96 (1) (1990), 120123.
[25] G. Cuccurullo, G. Sorrentino, An analytical solution for evaluating wall thermal-insulating performances, in: Atti XXII Congr. Naz. U.I.T. sulla Trasmissione del Calore, 2004. Genova.
[26] E.-H. Wang, W.H. Liu, Unsteady heat transfers through a multi-layer wall, Appl. Energy 31 (2) (1988) 83–99.
[27] H. Asan, Effects of wall's insulations thickness and position on time lag and decrement factor, Energy Build. 28 (1998) 299–305.
[28] K. Hulgen, Experimental and theoretical investigation of effects of wall's themophysical properties on time lag and decrement factor, Energy Build. 34 (2002) 273–278.
[29] G. Cuccurullo, Elementi di termodinamica e trasmissione del calore, Maggioli editore, 2020, 8891645289.
[30] T. Kusuda, Effectiveness method for predicting the performance of finned-tube coils, Heat and mass transfer to extended surfaces, in: ASHRAE Symposium, 1969. Jan. 27-30, 1969, Chicago.
[31] Technical Journal from Caleffi Hydronic Solutions, Hydronic Fundamentals, n.13, 2013 (Milwaukee, Wisconsin).
[32] P.G. Berardi, G. Cuccurullo, An approximate solution for velocity and temperature developing in pipe flow, Int. J. Heat. Technol. 16 (1998) 51.
[33] P.G. Berardi, G. Cuccurullo, Simultaneously developing velocity and temperature profiles in pipe flow, in: Atti XV Congr. Naz. U.I.T., 1997. Torino, 1997.
[34] G. Cuccurullo, P.G. Berardi, Developing of velocity and temperature in entrance pipe flow for power law fluids, 11th int. Heat transfer conference, Kjongju-Corea 3 (1998) 15.
[35] R.J. Duffin, A passive wall design to minimize building temperature swings, Sol. Energy 33 (3/4) (1984), 337342.
[36] T.R. Knowles, Proportioning composites for efficient thermal storage walls, Sol. Energy 31 (3) (1983) 319–326.
[37] R.J. Duffin, G. Knowles, Use of layered walls to reduce building temperature swings, Sol. Energy 33 (6) (1984) 543–549.