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Doubly multiplicative error models with long- and short-run components

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ABSTRACT

We suggest the Doubly Multiplicative Error class of models (DMEM) for modeling and forecasting realized volatility, which combines two components accommodating *long*-run, respectively, *short*-run features in the data. Three such models are considered, the Spline-MEM which fits a spline to the slow-moving pattern of volatility, the Component-MEM, which uses daily data for both components, and the MEM-MIDAS, which exploits the logic of MIxed-DAta Sampling (MIDAS) methods. The parameters are estimated by the Generalized Method of Moments (GMM), for which we establish the theoretical properties and the equivalence with the Quasi Maximum Likelihood (QML) estimator under a Gamma assumption. The empirical application involves the S&P 500, NASDAQ, FTSE 100, DAX, Nikkei and Hang Seng indices: irrespective of the market, the DMEM's generally outperform the HAR and other relevant GARCH-type models.

1. Introduction

More than forty years have passed since Engle [2] pioneering work on modeling the conditional variance of financial returns as an autoregressive process of observable variables. GARCH-type models [3] play a significant role in the financial econometrics literature since such a class allows to reproduce several stylized facts, such as the persistence in the conditional second moments (volatility clustering) and the possibility of taking into account the slow-moving or state dependent average volatility level.

As a matter of fact, several suggestions in the GARCH literature assume that the dynamic evolution of the conditional variance of returns is driven by two components, a high- and a low-frequency one, which combine additively or multiplicatively [4, offer a comprehensive survey]. For instance, Hamilton and Susmel [5] and Dueker [6] consider a Markov Switching framework, Amado and Teräsvirta [7] a Smooth Transition context, Mazur and Pipień [8] and Engle and Rangel [9] introduce deterministic functions in order to accommodate a slow-moving behavior in the conditional variance. This latter contribution allows to establish a relationship between a time-varying average level of volatility and macroeconomic events related to the business cycle. Since the macro-variables are observed at a lower frequency than that of the asset returns, the MIxed-DAta Sampling (MIDAS) approach suggested by Ghysels et al. [10] was extended to allow the real economy to influence financial volatility [the GARCH-MIDAS model proposed by 11]. Some extensions are available, such as the Double Asymmetric GARCH-MIDAS (DAGM) introduced by Amendola et al. [12], where a variable available at a low-frequency drives the slow-moving level of volatility and is allowed to have differentiated effects according to its sign, determining a local time-varying trend around which a GJR-GARCH [13, GJR] describes the *short*-run dynamics.[2]

Volatility modeling has encountered a tremendous boost by the availability of ultra-high frequency data, and the ensuing stream of literature related to estimating volatility using tick-by-tick data, conveniently aggregated: following the pathbreaking paper by Andersen and Bollerslev [14], realized volatility measures [for a survey on these estimators in reference to forecasting, cf. 15] have become a suitable object to be modeled and an ideal target for evaluating volatility forecasting performances. Modeling realized volatility reproduces the dynamics of the conditional expectations of variances or volatility, or, yet, log-variances: in this respect, the variants of the Multiplicative Error Model [MEM, 16,17], the Heterogeneous Autoregressive Model (HAR) by Corsi [18], the Realized GARCH [RGARCH, 19], among others, have proven to be effective in translating the refinement of high-frequency volatility measurement into better out-of-sample model performances relative to GARCH. Previous contributions have considered the presence of a low-frequency component within the MEM context: regime-switching and smooth transition functions were introduced

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[1] The views expressed in the article are those of the authors and do not involve the responsibility of the Corte dei conti.
[2] A similar approach was independently developed by Pan and Liu [1].

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by Gallo and Otranto [20]; Brownlees and Gallo [21] consider splines; the semi-non-parametric vector MEM by Barigozzi et al. [22] delivers a low-frequency term affecting several assets non-parametrically.

In this paper we take an original stance by discussing the presence of *long*-run and *short*-run components of volatility, combining multiplicatively with one another within a unified original general MEM framework, which we label DMEM (Doubly Multiplicative Error Model): in it, the *short*-run component is seen as fluctuating around one and a function of past volatility or some predetermined variables, all observed at the same frequency. Next to a constant (giving back the base asymmetric MEM), the *long*-run component (which provides the time-varying average level of volatility) is here modeled as: i) a smooth function of time (here, a Spline-MEM); ii) a specification based on daily data which mirrors the structure of the *short*-run component with a higher persistence (a novel model, which we label Component-MEM); iii) the extension of the mixed data frequency approach to the MEM class, providing a tool in which weekly or monthly data for the *long*-run can be combined with daily data for the *short*-run (the novel MEM-MIDAS).

The theoretical discussion shows that both new models have desirable statistical properties for their estimators (either within a Maximum Likelihood or a Generalized Method of Moments framework).

From an empirical point of view, we estimate all the competing models for the realized volatility of a wide array of major market indices (the S&P 500, NASDAQ, FTSE 100, DAX, Nikkei, and Hang Seng). We compare the performance of these models against a few representative models in the GARCH class, in particular those based on a mixed data frequency approach on the one side and models for realized volatility keeping the MEM as a reference, together with (an asymmetric version of) the HAR, and the RGARCH, all characterized by the absence of such a low-frequency component.

We are in a position to address a few fundamental questions about the usefulness of our suggestion and the improvements we are able to achieve. To the question *Is a long-run component advisable?* we are able to answer yes: the models that do not use it are generally worse than the ones that do, both within the classes of models for realized volatility, on the one hand, and models for conditional variances of returns, on the other. To the question *Does modeling realized volatility perform better than a* GARCH*, even when the latter contains a long-term component?* our answer is still yes, pointing to the richness of intra-daily information over the consideration of just returns. To the question *Is there an improvement over existing models?* we show that our results favor the DMEM approach over the HAR in spite of the latter model's capability of mimicking long memory features in the data.

The rest of the paper is organized as follows. In Section 2 we suggest the rationale and the notation for the DMEM, introducing the new Component-MEM and MEM-MIDAS models. Section 3 focuses on the identification, stationarity conditions and h-steps ahead forecasts for the Component-MEM. Section 4 presents the theoretical results on the estimators' properties and statistical inference. Section 5 introduces the market indices used and presents the results in terms of in-sample estimation and out-of-sample comparison across the competing models. Section 6 concludes.

2. Multiplicative error models with components

Let $\{x_{i,t}\}$ be a time series coming from a non-negative discrete time process for the ith day ($i = 1, \dots, N_t$) of the period t (for example, a week, a month or a quarter; $t = 1, \dots, T$): this comprises most financial activity-related variables, such as realized volatility, high-low range, number of trades, volumes, durations, and so on.

Let $\mathcal{F}_{i,t}$ be the information set available at day i of period t. In its standard version [16], the MEM assumes that

$$x_{i,t} = \mu_{i,t}\epsilon_{i,t} = \tau\xi_{i,t}\epsilon_{i,t}, \tag{1}$$

where: τ is a constant; $\xi_{i,t}$ is a quantity that, conditionally on $\mathcal{F}_{i-1,t}$ and by means of a parameter vector $\boldsymbol{\theta}$, evolves deterministically; $\epsilon_{i,t}$ is an error term such that

$$\epsilon_{i,t}|\mathcal{F}_{i-1,t} \overset{iid}{\sim} D^{+}(1, \sigma^2), \tag{2}$$

meaning that it has a unit mean, unknown variance σ^2 and a probability density function defined over a non-negative support.[3]

Therefore, independently of the chosen distribution D^{+} and the function used to build the evolution of $\mu_{i,t}$, we have that

$$E(x_{i,t}|\mathcal{F}_{i-1,t}) = \tau\xi_{i,t}. \tag{3}$$

Evaluating expression (3) unconditionally, we can interpret τ to be the unconditional expectation of $x_{i,t}$ if we assume that $E(\xi_{i,t}) = 1$, so that $x_{i,t}$ moves around the constant term τ. Correspondingly, the conditional variance can be expressed as

$$Var(x_{i,t}|\mathcal{F}_{i-1,t}) = \sigma^2\tau^2\xi_{i,t}^2. \tag{4}$$

In this paper, we extend the specification for the conditional mean to have a multiplicative component structure, in which both factors of the conditional expectation are time-varying. We have

$$x_{i,t} = \mu_{i,t}\epsilon_{i,t} = \tau_{i,t}\xi_{i,t}\epsilon_{i,t}. \tag{5}$$

$\tau_{i,t}$ can be seen as a *slow*-moving component determining the average *level* of the conditional mean at any given time, or, which is the same, a *long*-run component. By the same token, since $\xi_{i,t}$ is a factor centered around one, it plays the role of dumping or amplifying $\tau_{i,t}$ depending on whether it is smaller or greater than one; for this reason, we label it as a *short*-run or *fast*-moving component. Eq. (5) with innovation (2) define a Doubly Multiplicative Error Model, or DMEM.[4]

Let us start by expressing the *short*-run component in general terms as the GARCH-type expression typical of a MEM, augmented by the contribution of a predetermined de-meaned (vector) variable $\boldsymbol{z}$ [DMEM − X to parallel the GARCH − X, cf. 24]:

$$\xi_{i,t} = \left(1 - \alpha_1 - \gamma_1/2 - \beta_1\right) + \alpha_1 x_{i-1,t}^{(\xi)} + \gamma_1 x_{i-1,t}^{(\xi-)} + \beta_1\xi_{i-1,t} + \boldsymbol{\delta}_1'\boldsymbol{z}_{i-1,t} \tag{6}$$

where

$$x_{i,t}^{(\xi)} \equiv \frac{x_{i,t}}{\tau_{i,t}} \qquad x_{i,t}^{(\xi-)} \equiv x_{i,t}^{(\xi)}\mathbb{1}_{i,t} \tag{7}$$

and $\mathbb{1}_{i,t} \equiv \mathbb{1}\left(r_{i,t} < 0\right)$ is an indicator variable. $x_{i,t}^{(\xi-)}$ is a variable derived from $x_{i,t}^{(\xi)}$ which takes a non-zero value only if it corresponds to a negative return (for asymmetric effects).

As far as the *long*-run is concerned, we consider here different alternatives, apart from it being constant (the resulting model would be the standard MEM).

- [Spline-MEM] We can specify $\tau_{i,t}$ by means of a spline function (for example a linear or a cubic spline)

  $$\tau_{i,t} = \exp(s(i,t))$$

  as a smoothing spline or a regression spline with a relatively low number of knots so as to guarantee the *slow*-moving feature. The resulting model is the so called Spline-MEM [the P-Spline MEM of21, corresponds to a specific choice of spline functions]. Spline-MEM is trend-stationary (stationary around the trend component represented by $\tau_{i,t}$). In practice, estimation follows the steps outlined in Appendix A.

[3] For ease of notation, we use the set $\mathcal{F}_{i-1,t}$ even when the first day of a new period, say $x_{1,t}$ depends on the information observed the last day of the period immediately preceding t, that is $\mathcal{F}_{N_{t-1},t-1}$

[4] The consideration of two multiplicative components in the univariate GARCH case is discussed by Conrad and Kleen [23].

- [Component-MEM] Another possibility is to specify $\tau_{i,t}$ in a way similar to $\xi_{i,t}$, namely

$$\tau_{i,t} = \omega^{(\tau)} + \alpha_1^{(\tau)} x_{i-1,t}^{(\tau)} + \gamma_1^{(\tau)} x_{i-1,t}^{(\tau-)} + \beta_1^{(\tau)} \tau_{i-1,t} \tag{8}$$

where

$$x_{i,t}^{(\tau)} \equiv \frac{x_{i,t}}{\xi_{i,t}} \qquad x_{i,t}^{(\tau-)} \equiv x_{i,t}^{(\tau)} \mathbb{1}_{i,t}. \tag{9}$$

The essential difference in comparison with $\xi_{i,t}$ is that $\tau_{i,t}$ is not constrained to move around a unit mean, although the persistence features of the components relative to one another characterize the fact that τ moves differently than ξ (see Section 3.1).

The model resulting from this specification of $\tau_{i,t}$, which we name Component-MEM, is similar to the model introduced by Brownlees et al. [25] who use, however, an additive (namely $\mu = \tau + \xi$) specification not examined here. Another specification which makes use of different multiplicative components is the Composite-MEM proposed by Brownlees et al. [26] to model intradaily volumes.

- [MEM-MIDAS] Yet another option is to allow $\tau_{i,t}$ to have a MIDAS-like structure, adapting the use of mixed frequency data models [11,23] to the multiplicative error model context. In its simplest form, for all days i of the same period t, $\tau_{i,t}$ can be expressed over a window of K periods as

$$\tau_{i,t} = \tau_t \equiv \exp\left\{ m + \zeta \sum_{k=1}^{K} \delta_k(\omega) X_{t-k} \right\}$$

where X_t indicates a variable available only at t times and $\delta_k(\omega)$ represents a weighting function. One possible choice is to adopt a Beta function for $\delta_k(\omega)$, that is:

$$\delta_k(\omega) = \frac{(k/K)^{\omega_1 - 1}(1 - k/K)^{\omega_2 - 1}}{\sum_{j=1}^{K} (j/K)^{\omega_1 - 1}(1 - j/K)^{\omega_2 - 1}}. \tag{10}$$

Assuming $\omega_1 = 1$ and $\omega_2 \geq 1$ in (10) identifies cases in which more emphasis is given to most recent observations. In this work, we only focus on the Beta function, but other choices like the Exponential Almon lag function [27] could be easily used.

A further refinement is inspired by the DAGM [1,12].

Regarding the choice of the MIDAS driver X, one could favor a variable X independent of ε as in Conrad and Kleen [23], as this simplifies the analysis, although it may be difficult to meet this condition in practice [as acknowledged by23, p.4].

3. Component-MEM: Properties

It is instructive to concentrate on some of the features of the novel Component-MEM: in this section, not to unnecessarily burden the notation, the double index i,t is replaced by t. We analyze, in turn, the identification, stationarity and forecasting properties of this model.

3.1. Component-MEM *identification*

The Component-MEM model as defined by Eqs. (5), (6) and (8) is not identified. Assuming mean-stationarity with the unconditional mean of τ_t denoted by τ, Eq. (8) can be rewritten as:

$$\mu_t = \tau_t \xi_t = \tau \tau_t^* \xi_t$$

where

$$\tau_t^* = \left[1 - \left(\beta_1^{(\tau)} + \alpha_1^{(\tau)} + \gamma_1^{(\tau)}/2\right)\right] + \left(\beta_1^{(\tau)} + \alpha_1^{(\tau)} \varepsilon_{t-1} + \gamma_1^{(\tau)} \mathbb{1}_{t-1} \varepsilon_{t-1}\right) \tau_{t-1}^*$$

$$\xi_t = \left[1 - \left(\beta_1 + \alpha_1 + \gamma_1/2\right)\right] + \left(\beta_1 + \alpha_1 \varepsilon_{t-1} + \gamma_1 \mathbb{1}_{t-1} \varepsilon_{t-1}\right) \xi_{t-1}.$$

It is thus clear that exchanging the β, α and γ parameters between τ_t^* and ξ_t (equivalently between τ_t and ξ_t, because the τ_t^* and the τ_t parameters are the same) provides exactly the same μ_t.[5]

To impose identification preserving the interpretation of the two components, we impose

$$\beta_1^{(\tau)} + \alpha_1^{(\tau)} + \gamma_1^{(\tau)}/2 > \beta_1 + \alpha_1 + \gamma_1/2, \tag{11}$$

namely that τ_t has a larger persistence than ξ_t (cf. Engle and Lee [28] for a similar strategy in a Component-GARCH framework).

3.2. Component-MEM *stationarity*

Stationarity properties the Component-MEM can be obtained by focusing on the dynamics of $\{\boldsymbol{v}_t = (\tau_t, \xi_t)'\}$. $\{\boldsymbol{v}_t\}$ is a generalized AR(1) process

$$\boldsymbol{v}_t = \boldsymbol{A}_t \boldsymbol{v}_{t-1} + \boldsymbol{b}_t \tag{12}$$

with coefficients

$$\boldsymbol{A}_t = \begin{pmatrix} \beta_1^{(\tau)} + \alpha_1^{(\tau)} \varepsilon_{t-1} + \gamma_1^{(\tau)} \mathbb{1}_{t-1} \varepsilon_{t-1} & 0 \\ 0 & \beta_1 + \alpha_1 \varepsilon_{t-1} + \gamma_1 \mathbb{1}_{t-1} \varepsilon_{t-1} \end{pmatrix}$$

$$\boldsymbol{b}_t = \boldsymbol{b} = \begin{pmatrix} \omega^{(\tau)} \\ \omega \end{pmatrix},$$

where $\omega = 1 - \beta_1 - \alpha_1 - \gamma_1/2$. Assuming $\omega^{(\tau)} > 0$, $\beta_1^{(\tau)}, \beta_1, \alpha_1^{(\tau)}, \alpha_1, \alpha_1^{(\tau)} + \gamma_1^{(\tau)}, \alpha_1 + \gamma_1 \geq 0$ and $\beta_1 + \alpha_1 + \gamma_1/2 < 1$ implies $\boldsymbol{b} > 0$ (meant element by element), while the $\boldsymbol{A}_t$ matrix random coefficients are non-negative and i. i. d.

The following result is a trivial transposition of Bougerol and Picard [29, Th. 1.3].

Theorem 1. *Under above parameter conditions, $\{\boldsymbol{v}_t\}$ has a strictly stationary and ergodic solution ⇔ the Lyapunov exponent[6] γ associated with the $\{\boldsymbol{A}_t\}$ coefficients is negative.*

Since all norms are equivalent in finite dimensional spaces, if we define the norm of a matrix $\boldsymbol{A}$ as[7]

$$\|\boldsymbol{A}\| \equiv \sqrt{\text{largest eigenvalue of}(\boldsymbol{A}'\boldsymbol{A})},$$

then the diagonal structure of $\boldsymbol{A}_t$ implies $\gamma = E\left(\ln \|\boldsymbol{A}_0\|\right)$. The condition $\gamma < 0$ by Theorem 1 is then tantamount to

$$E\left(\ln\left(\beta_1^{(\tau)} + \alpha_1^{(\tau)} \varepsilon_0 + \gamma_1^{(\tau)} \mathbb{1}_0 \varepsilon_0\right)\right) < 0, \tag{13}$$

$$E\left(\ln\left(\beta_1 + \alpha_1 \varepsilon_0 + \gamma_1 \mathbb{1}_0 \varepsilon_0\right)\right) < 0. \tag{14}$$

Some considerations are in order.

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[5] Still in case of mean stationarity of x_t, we notice that the τ constant cannot be set equal to the unconditional mean μ because τ_t and ξ_t are positively correlated due to the presence of the common ε_{t-1}. Using some algebra, we have

$$\tau = \mu \frac{1 - A^{(\tau,\xi)}}{1 - A^{(\tau)} A},$$

where A, $A^{(\tau)}$ and $A^{(\tau,\xi)}$ are defined in Eqs. (17) and (18).

[6] Under the condition

$$E\left[\max\left\{\ln \|\boldsymbol{A}_0\|, 0\right\}\right] < 0,$$

the (largest) Lyapunov exponent of a sequence of random matrices $\{\boldsymbol{A}_t\}$ is

$$\gamma = \lim_{T \to \infty} \frac{1}{T+1} \ln \|\boldsymbol{A}_0 \boldsymbol{A}_1 \dots \boldsymbol{A}_T\|,$$

which holds almost surely.

[7] Notice that, in case $\boldsymbol{A}$ is diagonal, then $\|\boldsymbol{A}\|$ is also equal to $\rho(\boldsymbol{A})$, the spectral radius of $\boldsymbol{A}$, which is defined as the absolute value of the largest eigenvalue of $\boldsymbol{A}$.

1. Jensen's inequality

$$E\left(\ln\left(\beta_1^{(\tau)}+\alpha_1^{(\tau)}\varepsilon_0+\gamma_1^{(\tau)}\mathbb{1}_0\varepsilon_0\right)\right)\le \ln E\left(\beta_1^{(\tau)}+\alpha_1^{(\tau)}\varepsilon_0+\gamma_1^{(\tau)}\mathbb{1}_0\varepsilon_0\right) = \ln\left(\beta_1^{(\tau)}+\alpha_1^{(\tau)}+\gamma_1^{(\tau)}/2\right), \quad (15)$$

implies that condition (13) is trivially satisfied in case the persistence $\beta_1^{(\tau)}+\alpha_1^{(\tau)}+\gamma_1^{(\tau)}/2$ of the τ_t component is smaller than one.
2. An inequality similar to (15) is valid for the ξ_t component. On the other hand, the identification condition in Eq. (11) implies that the persistence $\beta_1+\alpha_1+\gamma_1/2$ of ξ_t is always lower than that of τ_t, making condition (14) trivially satisfied and reducing the stationarity condition for $\{\boldsymbol{v}_t\}$ to (13).
3. The case $\beta_1^{(\tau)}+\alpha_1^{(\tau)}+\gamma_1^{(\tau)}/2 = 1$ can be managed similarly to Bougerol and Picard [29, Corollary 2.2]. In fact, we can easily check that, in this case $\rho(E(\boldsymbol{A}_0))$, the spectral radius of $E\left(\boldsymbol{A}_0\right)$, is one. Joining this with the facts that $\boldsymbol{A}_0$ has no zero rows or columns, it is not almost surely bounded, and $\boldsymbol{A}_1\boldsymbol{A}_0$ is positive, above result implies $\gamma<\ln\rho(E(\boldsymbol{A}_0))=\ln(1)=0$, which is the strong stationarity condition required by Theorem 1.

To summarize the previous points, $\beta_1^{(\tau)}+\alpha_1^{(\tau)}+\gamma_1^{(\tau)}/2\le 1$ is the condition needed to guarantee the strong stationarity of $\{\boldsymbol{v}_t\}$. That stated, the stationarity of $\{\mu_t\}$ comes from the fact that the mapping from (τ_t,ξ_t) to $(\mu_t,\nu_t)=(\tau_t\xi_t,\xi_t)$ is one-to-one.

3.3. Component-MEM *forecasts*

In a standard MEM based on only one lag, h-steps ahead forecasts can be derived easily by exploiting the recursion

$$\mu_{t+h}=\omega+A_{t+h-1}\mu_{t+h-1}$$

where

$$A_t=\beta_1+\alpha_1\varepsilon_t+\gamma_1\mathbb{1}_t\varepsilon_t. \quad (16)$$

Taking the expectation conditionally on $\mathcal{F}_t$ we have

$$\mu_{t+h|t}=\begin{cases}\omega+A_t\mu_t & \text{if } h=1\\ \omega+A\mu_{t+h-1|t} & \text{if } h>1\end{cases}$$

where $A=\beta_1+\alpha_1+\gamma_1/2$ is the persistence.

In the Component-MEM the derivation of h-steps ahead forecasts is more complex because of the presence of the same error term in both the contemporaneous τ and ξ components. In more detail

$$\mu_{t+h}=\tau_{t+h}\eta_{t+h}=\omega^{(\tau)}\omega+\omega^{(\tau)}A_{t+h-1}\xi_{t+h-1}+\omega A_{t+h-1}^{(\tau)}\tau_{t+h-1} + A_{t+h-1}^{(\tau)}A_{t+h-1}\mu_{t+h-1}$$

where

$$A_t^{(\tau)}=\beta_1^{(\tau)}+\alpha_1^{(\tau)}\varepsilon_t+\gamma_1^{(\tau)}\mathbb{1}_t\varepsilon_t$$

and A_t is defined in Eq. (16). Taking the expectation conditionally on $\mathcal{F}_t$ we obtain

$$\mu_{t+h|t}=\begin{cases}\left(\omega^{(\tau)}+A_t^{(\tau)}\eta_t\right)\left(\omega+A_t\xi_t\right) & \text{if } h=1\\ \omega^{(\tau)}\omega+\omega^{(\tau)}A\xi_{t+h-1|t}+\omega A^{(\tau)}\tau_{t+h-1|t}+A^{(\tau,\xi)}\mu_{t+h-1|t} & \text{if } h>1\end{cases}$$

where

$$B^{(\tau)}=\alpha_1^{(\tau)}+\gamma_1^{(\tau)}/2 \qquad B=\alpha_1+\gamma_1/2$$
$$A^{(\tau)}=\beta_1^{(\tau)}+B^{(\tau)} \qquad A=\beta_1+B \quad (17)$$
$$A^{(\tau,\xi)}=A^{(\tau)}A+\frac{\gamma_1^{(\tau)}}{2}\frac{\gamma_1}{2}+\sigma^2\left(B^{(\tau)}B+\frac{\gamma_1^{(\tau)}}{2}\frac{\gamma_1}{2}\right). \quad (18)$$

4. Inference

Inference on the model defined in Section 2 can be obtained by adapting the framework suggested in Brownlees et al. [25, Section 9.2.2]. Assuming that the conditional mean is correctly specified and indicating by $\boldsymbol{\theta}$ the vector of parameters entering it, two estimation strategies are illustrated in what follows: Generalized Method of Moments (GMM), and Quasi Maximum Likelihood (QML).

4.1. Generalized method of moments inference

GMM estimation can be conveniently used to estimate a DMEM without an explicit choice of the error term distribution and, importantly, does not suffer from the possible presence of zeros in the data.

Let

$$\varepsilon_{i,t}=\frac{x_{i,t}}{\tau_{i,t}\xi_{i,t}}. \quad (19)$$

Under model assumptions, $\varepsilon_{i,t}-1$ is a conditionally homoskedastic martingale difference, with conditional expectation zero and conditional variance σ^2. The *efficient* GMM estimator of $\boldsymbol{\theta}$, say $\widehat{\theta}_{GMM}$, solves the criterion equation

$$\sum_{t=1}^{T}\sum_{i=1}^{N_t}(\varepsilon_{i,t}-1)\boldsymbol{a}_{i,t}=\boldsymbol{0}, \quad (20)$$

and has asymptotic variance matrix

$$\mathrm{Avar}(\widehat{\theta}_{GMM})=\sigma^2\boldsymbol{A}^{-1}, \quad (21)$$

where

$$\boldsymbol{a}_{i,t}=\frac{1}{\mu_{i,t}}\nabla_\theta\mu_{i,t}=\frac{1}{\tau_{i,t}}\nabla_\theta\tau_{i,t}+\frac{1}{\xi_{i,t}}\nabla_\theta\xi_{i,t}, \quad (22)$$

$$\boldsymbol{A}=\lim_{N\to\infty}\left[N^{-1}\sum_{t=1}^{T}\sum_{i=1}^{N_t}E\left(\boldsymbol{a}_{i,t}\boldsymbol{a}_{i,t}'\right)\right],$$

and $N=\sum_{t=1}^{T}N_t$ is the number of observations. As a consequence, a consistent estimator of the asymptotic variance matrix is

$$\widehat{\mathrm{Avar}}(\widehat{\theta}_{GMM})=\widehat{\sigma}^2\widehat{\boldsymbol{A}}^{-1},$$

where

$$\widehat{\sigma}^2=N^{-1}\sum_{t=1}^{T}\sum_{i=1}^{N_t}\left(\widehat{\varepsilon}_{i,t}-1\right)^2$$

is the Method of Moments estimator of σ^2,

$$\widehat{\boldsymbol{A}}=N^{-1}\sum_{t=1}^{T}\sum_{i=1}^{N_t}\widehat{\boldsymbol{a}}_{i,t}\widehat{\boldsymbol{a}}_{i,t}',$$

$\widehat{\varepsilon}_{i,t}$ and $\widehat{\boldsymbol{a}}_{i,t}$ mean (19) and (22), respectively, evaluated at $\widehat{\theta}_{GMM}$.

4.2. Quasi maximum likelihood inference

Following Engle and Gallo [17], an alternative approach to DMEM inference is Quasi Maximum Likelihood (QML): this is obtained by assuming a Gamma distributed $\varepsilon_{i,t}$, and then estimating $\boldsymbol{\theta}$ by Maximum Likelihood.

Therefore, if (2) is specified as $\varepsilon_{i,t}|\mathcal{F}_{i,t-1}\sim Gamma(\phi,\phi)$ (so as to have $E\left(\varepsilon_{i,t}|\mathcal{F}_{i,t-1}\right)=1$ and $V\left(\varepsilon_{i,t}|\mathcal{F}_{i,t-1}\right)=\sigma^2=1/\phi$) the *log-likelihood* function is

$$l_N=\sum_{t=1}^{T}\sum_{i=1}^{N_t}\left[\phi\ln\phi-\ln\Gamma(\phi)+\phi\ln\varepsilon_{i,t}-\phi\varepsilon_{i,t}-\ln x_{i,t}\right].$$

Table 1
Model specifications.

Model	Functional form	Err. Distr.
MEM	$rvol_{i,t} \mid \mathcal{F}_{i-1,t} = \mu_{i,t}\epsilon_{i,t}$ $\mu_{i,t} = \alpha_0 + (\alpha_1 + \gamma_1 \mathbb{1}_{i-1,t}) rvol_{i-1,t} + \beta_1 \mu_{i-1,t}$ $\alpha_0 = (1 - \alpha_1 - \beta_1 - \gamma_1/2)\,\mu$, with $\mu = E\left[rvol_{i,t}\right]$	$\epsilon_{i,t} \overset{i.i.d}{\sim} D^+(1,\sigma^2)$
Spline-MEM	$rvol_{i,t} \mid \mathcal{F}_{i-1,t} = \tau_{i,t}\xi_{i,t}\epsilon_{i,t}$ $\xi_{i,t} = (1 - \alpha_1 - \gamma_1/2 - \beta_1) + \alpha_1 x^{(\xi)}_{i-1,t} + \gamma_1 x^{(\xi-)}_{i-1,t} + \beta_1 \xi_{i-1,t}$, with $x^{(\xi)}_{i,t} \equiv \frac{rvol_{i,t}}{\tau_{i,t}}$ and $x^{(\xi-)}_{i,t} \equiv x^{(\xi)}_{i,t} \mathbb{1}_{i,t}$ $\tau_{i,t} = \exp(s(i,t))$	$\epsilon_{i,t} \overset{i.i.d}{\sim} D^+(1,\sigma^2)$
Component-MEM	$rvol_{i,t} \mid \mathcal{F}_{i-1,t} = \tau_{i,t}\xi_{i,t}\varepsilon_{i,t}$ $\xi_{i,t} = (1 - \alpha_1 - \gamma_1/2 - \beta_1) + \alpha_1 x^{(\xi)}_{i-1,t} + \gamma_1 x^{(\xi-)}_{i-1,t} + \beta_1 \xi_{i-1,t}$, with $x^{(\xi)}_{i,t} \equiv \frac{rvol_{i,t}}{\tau_{i,t}}$ and $x^{(\xi-)}_{i,t} \equiv x^{(\xi)}_{i,t} \mathbb{1}_{i,t}$ $\tau_{i,t} = \omega^{(\tau)} + \alpha^{(\tau)}_1 x^{(\tau)}_{i-1,t} + \gamma^{(\tau)}_1 x^{(\tau-)}_{i-1,t} + \beta^{(\tau)}_1 \tau_{i-1,t}$, with $x^{(\tau)}_{i,t} \equiv \frac{rvol_{i,t}}{\xi_{i,t}}$ and $x^{(\tau-)}_{i,t} \equiv x^{(\tau)}_{i,t} \mathbb{1}_{i,t}$	$\epsilon_{i,t} \overset{i.i.d}{\sim} D^+(1,\sigma^2)$
MEM-MIDAS	$rvol_{i,t} \mid \mathcal{F}_{i-1,t} = \tau_t\xi_{i,t}\epsilon_{i,t}$ $\xi_{i,t} = (1 - \alpha_1 - \beta_1 - \gamma_1/2) + (\alpha_1 + \gamma_1 \cdot \mathbb{1}_{i-1,t}) \frac{rvol_{i-1,t}}{\tau_t} + \beta_1 \xi_{i-1,t}$ $\tau_t = \exp\left\{m + \zeta \sum_{k=1}^{K} \delta_k(\omega) X_{t-k}\right\}$	$\epsilon_{i,t} \overset{i.i.d}{\sim} D^+(1,\sigma^2)$
HAR	$rvol_{i,t} = c + (\beta_1 + \gamma_1 \mathbb{1}_{i-1,t}) rvol_{i-1,t} + \beta_5 \overline{rvol}_{(i-2):(i-5),t} + \beta_{22} \overline{rvol}_{(i-6):(i-22),t} + u_{i,t}$	$u_{i,t} \overset{i.i.d}{\sim} N(0,\sigma^2_u)$
GJR	$r_{i,t} \mid \mathcal{F}_{i-1} = \sqrt{h_{i,t}}\eta_{i,t}$ $h_{i,t} = const + (\alpha_1 + \gamma_1 \mathbb{1}_{i-1,t})\, r^2_{i-1,t} + \beta_1 h_{i-1,t}$	$\eta_{i,t} \overset{i.i.d}{\sim} N(0,1)$
GM	$r_{i,t} \mid \mathcal{F}_{i-1,t} = \sqrt{\tau_t \times \xi_{i,t}}\eta_{i,t}$ $\xi_{i,t} = (1 - \alpha_1 - \beta_1 - \gamma_1/2) + (\alpha_1 + \gamma_1 \cdot \mathbb{1}_{i-1,t}) \frac{r^2_{i-1,t}}{\tau_t} + \beta_1 \xi_{i-1,t}$ $\tau_t = \exp\left\{m + \zeta \sum_{k=1}^{K} \delta_k(\omega) X_{t-k}\right\}$	$\eta_{i,t} \overset{i.i.d}{\sim} N(0,1)$
DAGM	$r_{i,t} \mid \mathcal{F}_{i-1,t} = \sqrt{\tau_t \times \xi_{i,t}}\eta_{i,t}$ $\xi_{i,t} = (1 - \alpha_1 - \beta_1 - \gamma_1/2) + (\alpha_1 + \gamma_1 \cdot \mathbb{1}_{i-1,t}) \frac{r^2_{i-1,t}}{\tau_t} + \beta_1 \xi_{i-1,t}$ $\tau_t = \exp\left\{m + \zeta^+ \sum_{k=1}^{K} \delta_k(\omega)^+ X_{t-k} \mathbb{1}_{(X_{t-k} \geq 0)} + \zeta^- \sum_{k=1}^{K} \delta_k(\omega)^- X_{t-k} \mathbb{1}_{(X_{t-k} < 0)}\right\}$	$\eta_{i,t} \overset{i.i.d}{\sim} N(0,1)$
RGARCH	$r_{i,t} \mid \mathcal{F}_{i-1,t} = \sqrt{h_{i,t}}\eta_{i,t}$ $\log(h_{i,t}) = const + \beta_1 \log(h_{i-1,t}) + \alpha_1 \log(rvol_{i-1,t})$	$\eta_{i,t} \overset{i.i.d}{\sim} N(0,1)$

Notes: The table reports the functional forms for the Asymmetric MEM (MEM), Spline-MEM, Component-MEM, MEM-MIDAS, Asymmetric HAR (HAR), GJR, GARCH–MIDAS (GM), Double Asymmetric GARCH–MIDAS (DAGM), and Realized GARCH (RGARCH) specifications.

Table 2
Summary statistics.

	Obs.	Min.	Max.	Mean	SD	Skew.	Kurt.
Daily data							
S&P 500 log-returns	4859	−201.134	168.937	0.253	19.706	−0.382	11.517
S&P 500 Realized Kern. Vol.	4859	1.663	148.933	13.354	10.588	3.653	22.417
NASDAQ log-returns	4858	−208.605	210.785	0.423	23.565	−0.189	7.760
NASDAQ Realized Kern. Vol.	4858	2.979	166.377	17.654	12.981	3.322	19.512
FTSE 100 log-returns	4884	−160.912	150.568	−0.023	18.566	−0.324	7.857
FTSE 100 Realized Kern. Vol.	4884	2.353	149.440	14.751	10.534	3.911	27.770
DAX log-returns	4909	−188.321	190.920	0.157	23.344	−0.179	6.782
DAX Realized Kern. Vol.	4909	2.243	172.121	18.736	12.946	3.255	18.811
Nikkei log-returns	4726	−192.257	210.093	0.126	23.865	−0.389	6.443
Nikkei Realized Kern. Vol.	4726	2.071	145.944	17.780	10.783	3.368	24.030
Hang Seng log-returns	4742	−215.608	212.826	0.152	22.784	−0.054	8.508
Hang Seng Realized Kern. Vol.	4742	2.769	136.392	18.071	12.639	3.422	18.609
Monthly data							
NAI_t	233	−16.740	2.610	−0.330	1.361	−7.929	88.944
HSc_t	233	−92.585	91.317	0.712	31.404	0.052	0.142

Notes: The table reports the number of observations (Obs.), the minimum (Min.) and maximum (Max.), the mean, standard deviation (SD), Skewness (Skew.) and excess Kurtosis (Kurt.). The sample period is 2 January 2001–15 May 2020. The daily variables are the close–to–close log-returns and realized kernel volatility, both expressed in percentage annualized terms. The monthly variables are the first releases of the US National Activity Index (NAI_t) and Housing Starts (HS_t). Housing Starts are expressed as the annualized month-to-month percentage changes, that is $HSc_t = 12^{0.5} \cdot 100 \cdot ((HS_t/HS_{t-1}) - 1)$.

This equation shows that maximization with respect to θ can be achieved by maximizing the quasi-log-likelihood function

$$\sum_{t=1}^{T} \sum_{i=1}^{N_t} \left(\ln \varepsilon_{i,t} - \varepsilon_{i,t}\right),$$

which does not depend on ϕ. As a consequence, the first order condition for θ is given exactly by the GMM condition (20). Notice that Eq. (20) can be rewritten as

$$\sum_{t=1}^{T} \sum_{i=1}^{N_t} \frac{\partial \mu_{i,t}}{\partial \theta} \frac{x_{i,t} - \mu_{i,t}}{\mu^2_{i,t}} = \mathbf{0}, \tag{23}$$

which shows that, in case $\mu_{i,t}$ is correctly specified, the left-hand side has a zero expectation irrespective of whether $\varepsilon_{i,t}$ is Gamma-distributed.

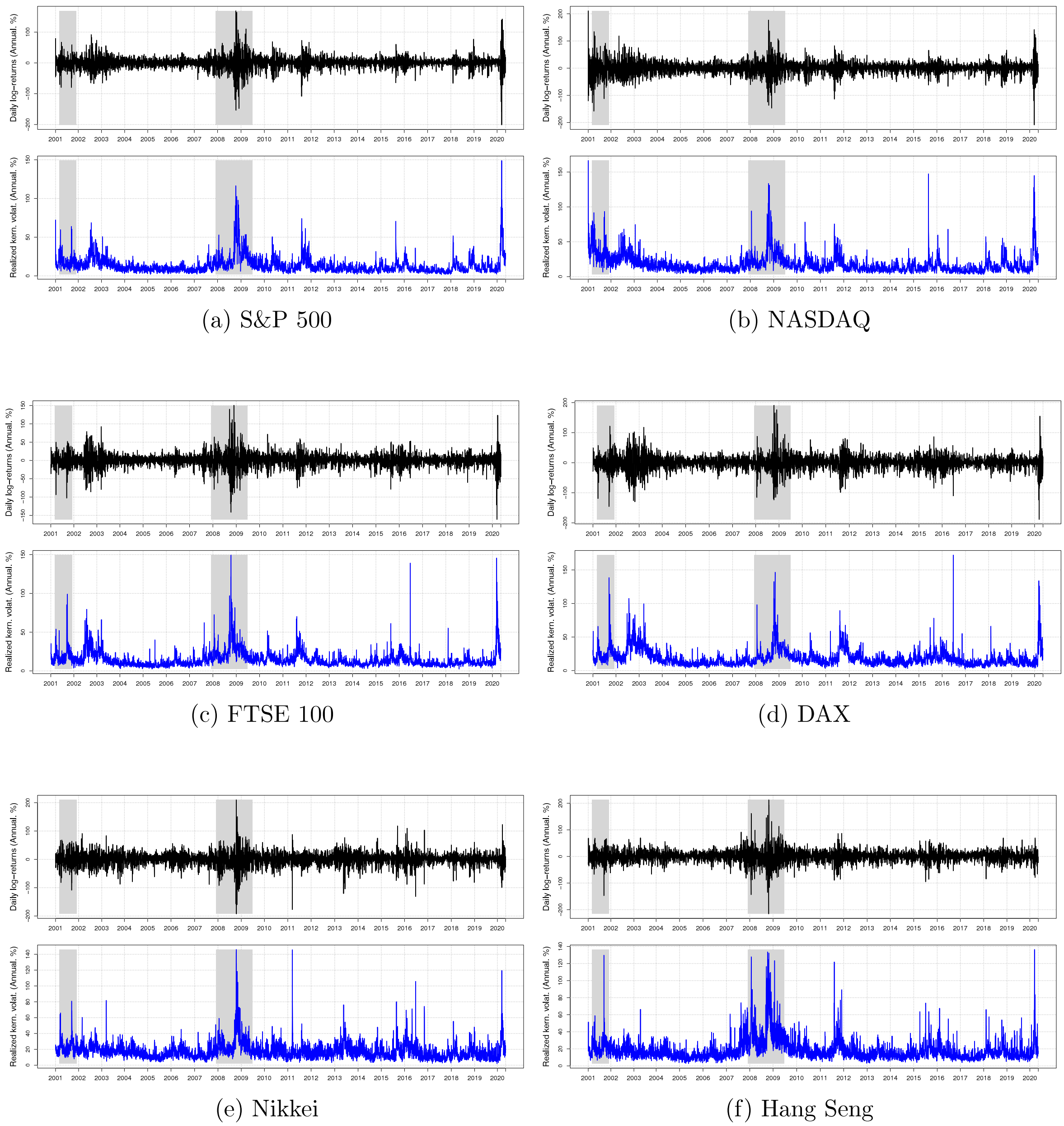


Fig. 1. Annualized daily log-returns and realized kernel volatility
Notes: Plots of close-to-close log-returns (top panels, black lines) and realized kernel volatilities (bottom panels, blue lines). Shaded areas represent US recession periods (NBER dating). Period: 2 January 2001–15 May 2020.

5. Empirical analysis

Volatility, our main object of interest, is expressed as the square root of the sum of the realized kernel variance [30,31] plus the overnight squared returns, expressed in percentage annualized terms.

We include in the set of competing models those having the realized volatility as the dependent variable, namely the multiplicative class (the MEM, plus the three proposed specifications Spline-MEM, Component-MEM, and MEM-MIDAS) and the asymmetric version of the HAR model, on the one side; and the GARCH class for the conditional variance of close-to-close returns, namely, GJR, GM, and the DAGM, on the other. To the latter, we add the RGARCH, which is still specified as a GARCH, but makes use of realized variance in its specification. All the functional forms are described in Table 1.

The testing ground for the models includes two different robust loss functions [LFs, 32]: QLIKE and MSE. All LFs have the realized kernel volatility as their target, and the GARCH models variance forecasts are modified to match that target. The evaluation uses the Model Confidence Set [MCS, 33], and the test statistic employed in the MCS procedure is the semi-quadratic T_{SQ}, as recently done by Cipollini et al. [34], for instance. The confidence level of the MCS procedure is 75%, as recently done by Catania [35], among others.

5.1. The data

Data on the S&P 500, NASDAQ, FTSE 100, DAX, Nikkei, and Hang Seng indices have been collected from the realized library of the Oxford-Man Institute [36], which allows us to derive close-to-close returns and their sign.

Within the possible choices of monthly macroeconomic variables, we favor the Chicago Fed NAI (National Activity Index), as it is an “index designed to gauge overall economic activity and related inflationary pressure” and is based on a variety of (85) US indicators related to industrial production in different sectors and personal income, to employment and unemployment, to personal consumption and housing, and to sales, orders and inventories. We feel that such a stationary index is a good candidate to represent how financial markets may react to underlying overall economic conditions represented by a US index. As an alternative, we include the US Housing Starts (HS) index as another MIDAS variable, expressed as the annualized month-to-month percentage change [as in 37]. Both the NAI and the HS indices are widely used in research focused on forecasting stock market or commodity volatility [see, for instance, 12,37,38, among others]; their data are taken as the first release in the Federal Reserve Economic Data database. The period under consideration for all the variables is from 2 January 2001 to 15 May 2020. All the MIDAS-based models were estimated using the `rumidas` package [39]. For reference purposes, some summary statistics (minimum, maximum, mean, standard deviation, skewness and kurtosis) for all variables considered are in Table 2.

Fig. 1 depicts the close-to-close log-returns (top panels, black lines) and realized kernel volatilities (bottom panels, blue lines) for the six indices considered over the full sample. We superimposed the US recession periods dated by the NBER in 2001 and then 2008–09, as a reference to periods of slowdown in economic activity. Although the scales are different, there are features in the dynamics of the series which are common to all six indices, notably the explosion of volatility around the Lehman Brothers demise in September 2008 and the COVID-19 outbreak in February 2020. There are also other episodes which are more idiosyncratic, although the surge in volatility at the end of 2002 is common to the US and UK indices, and the one in 2015 seems to have affected more the US markets and Hong Kong.

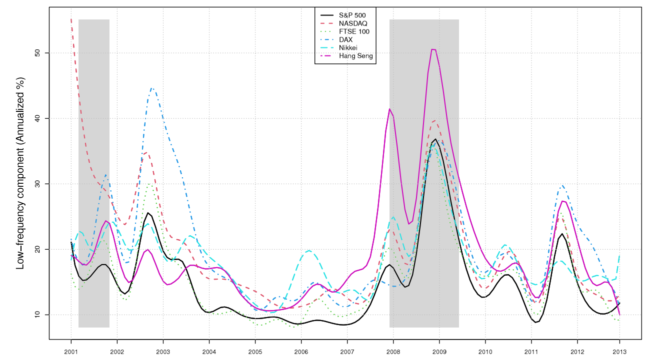
5.2. In-sample analysis

The in-sample period spans from January 2001 to December 2012. For the sake of simplicity, we report here only the in-sample estimates for the leading case of the S&P 500 (Table 3), while the in-sample estimates of all the other indices are in Appendix B (Tables from 6 to 8). In addition to the in-sample coefficient estimates, we also report the QLIKE, the MSE, as well as the Akaike and Bayesian Information Criteria (AIC and BIC, respectively). To be clear, the diagnostics results at the bottom of the Table differ by dependent variable and choice of the error distribution: for the first five columns (MEM's) the dependent variable is the realized volatility and the error distribution is Gamma; for the HAR the dependent variable is the realized volatility and the error distribution is Gaussian; for the remaining columns (GARCH) the dependent variable is the daily returns and the error distribution is Gaussian.

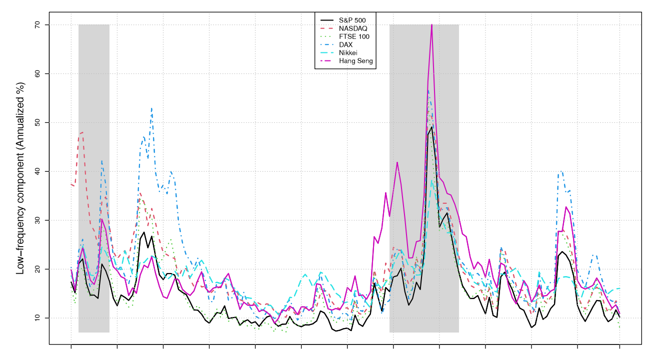
5.3. A graphical appraisal of the long-run

The three DMEM models produce an estimate of the *long*-run which is at a daily frequency for the Spline-MEM and Component-MEM, and at a monthly frequency for the MEM-MIDAS: in order for them to be compared, we choose to aggregate the first two at the monthly level by averaging to the same scale, with an obvious change of notation for the objects involved, by dropping the subscript i. In Fig. 2 we report the six τ_t components (for each index) estimated with the Spline-MEM (top panel), Component-MEM (middle panel), and MEM-MIDAS (bottom panel). It seems that the τ_t components have a similar pattern across all the indices, within the same specification. By and large, the commonality in the τ_t's for different indices is confirmed.[8]

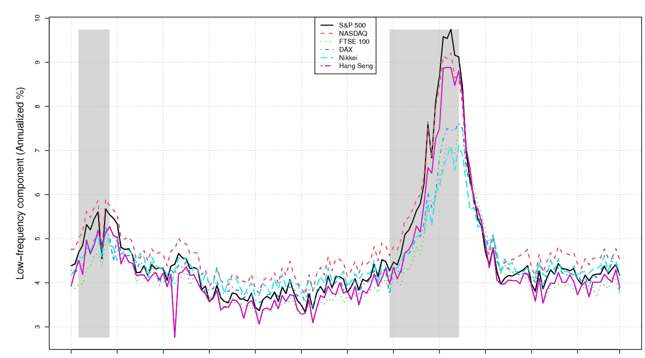
[8] It is not relevant, for the sake of our argument, to address the issue of the different opening schedules across time zones here.



(a) Spline-MEM τ_t



(b) Component-MEM τ_t



(c) MEM-MIDAS τ_t with National Activity Index as MIDAS variable

Fig. 2. Monthly τ_t term. Comparison among DMEM models
Notes: Plot of the Spline-MEM (top plot), Component-MEM (central plot) and MEM-MIDAS (bottom plot) τ_t terms. Shaded areas represent US recession periods (NBER dating). Period: 2 January 2001–28 December 2012.

5.4. Out-of-sample analysis

In the out-of-sample exercise, each model is estimated using a rolling window of twelve years (approximately, 3000 daily observations). The one-step-ahead forecasts are generated for the following two months, conditionally on the parameter estimates previously obtained. Then, the estimation window shifts forward by two months, new out-of-sample forecasts are similarly produced, and so forth until the end of the series. The first in-sample estimation period covers 2001–2012. Table 4 shows the out-of-sample (from 2 January 2013 to 15 May 2020) performance of each model for each market index volatility in terms of average loss functions (QLIKE and MSE). In Panel A of the table we report the benchmark results on a rolling window of twelve years with a two-month refitting frequency. The loss functions show similar values across classes of models: the lowest are fairly close to one another

Table 3
In-sample estimates. S&P 500.

MIDAS	MEM	Spline-MEM	Component-MEM	MEM-MIDAS NAI_t	MEM-MIDAS HSc_t	HAR	GJR	GM NAI_t	GM HSc_t	DAGM NAI_t	DAGM HSc_t	RGARCH
$const$	0.29					1.069**	4.014***					0.237***
						(0.459)	(1.119)					(0.077)
α_1	0.092***	0.052***	0.000	0.083***	0.092***		0.000	0.000	0.000	0.000	0.000	0.384***
	(0.011)	(0.013)	(0.021)	(0.011)	(0.011)		(0.015)	(0.017)	(0.015)	(0.02)	(0.031)	(0.036)
β_1	0.831***	0.798***	0.783***	0.824***	0.832***	0.155***	0.917***	0.898***	0.925***	0.898***	0.911***	0.784***
	(0.013)	(0.018)	(0.041)	(0.014)	(0.013)	(0.041)	(0.019)	(0.017)	(0.015)	(0.018)	(0.025)	(0.021)
β_5						0.525***						
						(0.06)						
β_{22}						0.161***						
						(0.054)						
γ_1	0.116***	0.138***	0.1***	0.124***	0.117***	0.175***	0.139***	0.154***	0.121***	0.156***	0.139*	
	(0.009)	(0.009)	(0.014)	(0.01)	(0.009)	(0.022)	(0.02)	(0.022)	(0.017)	(0.021)	(0.076)	
m				−0.134**	0.07			5.468***	5.507***	5.722***	5.597***	
				(0.061)	(0.123)			(0.145)	(0.164)	(0.219)	(1.457)	
ζ				−0.29***	−0.007**			−0.633***	−0.047			
				(0.038)	(0.003)			(0.091)	(0.029)			
ω_2				10.812***	4.669***			16.617*	1.002			
				(0.024)	(1.099)			(9.068)	(1.419)			
ζ^+										−1.761***	−0.075	
										(0.548)	(0.054)	
ω_2^+										4.415***	1.54	
										(0.215)	(5.439)	
ζ^-										−0.511***	−0.069	
										(0.103)	(0.116)	
ω_2^-										9.295***	1.001	
										(0.591)	(11.421)	
$\omega^{(\tau)}$			0.122***									
			(0.033)									
$\alpha_1^{(\tau)}$			0.072***									
			(0.015)									
$\beta_1^{(\tau)}$			0.894***									
			(0.015)									
$\gamma_1^{(\tau)}$			0.05***									
			(0.013)									
QLIKE	−3.781	−3.784	−3.782	−3.782	−3.781	−3.779	−3.761	−3.762	−3.763	−3.761	−3.763	−3.758
MSE	0.163	0.156	0.161	0.160	0.162	0.168	0.228	0.234	0.226	0.232	0.222	0.219
AIC	17 776.479	17 797.720	17 649.365	17 753.214	17 775.024	19 817.057	25 332.098	25 314.841	25 332.946	25 318.682	25 336.312	27 241.086
BIC	17 794.506	18 014.045	17 691.429	17 789.268	17 811.078	19 847.103	25 356.132	25 350.896	25 369.000	25 366.754	25 384.384	27 289.156

Notes: The table reports the estimated coefficients of the models in column. *, ** and *** represent the significance at levels 10%, 5%, 1%, respectively, associated to QML standard errors. The reported constant for the MEM model refers to α_0 parameter in Table 1. For ease of notation, the parameter α_1 referred to the RGARCH corresponds to the parameter labeled as γ in [19]. Moreover, the estimated parameters of the measurement equation of this latter model are not reported for space constraints.
Sample period: January 2001–December 2012. Daily observations: 3008. Number of lagged macro-economic variable realizations: $K = 36$. The AIC and BIC differ by dependent variable and choice of the error distribution: for the first five columns (MEM's) the dependent variable is the realized volatility and the error distribution is Gamma; for the HAR the dependent variable is the realized volatility and the error distribution is Gaussian; for the remaining columns (GARCH) the dependent variable is the daily returns and the error distribution is Gaussian.

within the class of MEM's with the Spline-MEM showing higher values, in spite of its good in-sample performance and the attention paid to the need for updating the τ_t component more frequently (Appendix A). We color in gray those models that enter the MCS at the significance level of $\alpha = 0.25$: MEM-based models are consistently entering the MCS, with a good performance of the base MEM and of the Component-MEM; the MEM-MIDAS have more mixed results (probably due to the fact that the US low-frequency variable is less sharp in capturing possible asymmetric shocks on other markets).

The overall message is reassuring in terms of modeling realized volatility directly, on the one hand, and, within that class, in terms of handling components multiplicatively, showing empirically that – at times – the *long*-run is better captured as a constant (e.g. the DAX). The specific message is that modeling conditional volatility through the conditional second moments of returns appears to be dominated according to both LF metrics.

Qualitatively, little changes when different choices are adopted about the rolling window and the frequency of the refitting period. Panel B of Table 4 reports results for a rolling window of twelve years with a lower six-month refitting frequency: there seems to be an inclusion of more models in the MCS (outside the MEM class, two cases for HAR, one for the GJR, GM and RGARCH – the DAX now favors also the Component-MEM and the MEM-MIDAS); Panel C reports results for a rolling window of six years with a two-month refitting frequency: the results are by and large similar to Panel A, with some more inclusions of models within the MEM class and generally lower values of the LFs.

To gain some further insights on the behavior of each model in relationship with the observed volatility pattern, we suggest a graphical comparison (Fig. 3) between the two proposed DMEM models introduced in this paper. To that end, we reproduce, for the out-of-sample period, the forecasts next to the realized kernel volatility. The same plots also report the volatility from the Spline-MEM.

6. Concluding remarks

Two different general approaches can be followed when forecasting asset return volatility: one is the GARCH approach, where the conditional variance is estimated from return data, the other is modeling the conditional expectation of volatility using ultra-high frequency realized measures of volatility. In the first approach, therefore, measurement and modeling are comprised within the same framework, while, in the second, the two aspects are decoupled. The merits of the GARCH model are testified by the hundreds of thousands of theoretical and empirical contributions since the seminal paper by Engle [2]. This type of approach has been enriched over the years by successive refinements, with the goal to capture some empirical regularities in the pattern of the observed time series. This is the case for the consideration of a time-varying local average in the conditional variance, a feature first addressed by Engle and Rangel [9], also in reference to its economic interpretation in terms of macroeconomic fluctuations. As a parallel approach, direct modeling of realized measures of volatility has the advantage of exploiting the better theoretical properties of these ultra-high frequency measures (less noisy than squared returns).

For either approach, the consideration of how complicated it is to collect the data and to fine-tune a model to derive the forecast has to be weighed against the actual reward in an improved forecasting performance. The availability of freely downloadable daily price data still ensures popularity with the GARCH approach (especially among practitioners), but it is also true that the number of high-frequency data

Table 4
Out-of-sample comparison.

MIDAS	MEM	Spline-MEM	Component-MEM	MEM-MIDAS NAI_t	MEM-MIDAS HSc_t	HAR	GJR	GM NAI_t	GM HSc_t	DAGM NAI_t	DAGM HSc_t	RGARCH
Panel A: twelve years as rolling window, two months as re-fitting frequency												
S&P 500												
QLIKE	−4.124	−4.115	−4.125	−4.121	−4.121	−4.115	−4.082	−4.067	−4.072	−4.065	−4.066	−4.107
MSE	0.105	0.133	0.11	0.12	0.106	0.123	0.18	0.24	0.189	0.293	0.197	0.127
NASDAQ												
QLIKE	−3.841	−3.832	−3.841	−3.838	−3.839	−3.832	−3.816	−3.793	−3.779	−3.78	−3.801	−3.821
MSE	0.207	0.253	0.214	0.26	0.208	0.233	0.262	0.378	0.382	0.519	0.304	0.252
FTSE 100												
QLIKE	−3.939	−3.934	−3.938	−3.937	−3.937	−3.931	−3.924	−3.905	−3.92	−3.912	−3.917	−3.934
MSE	0.147	0.163	0.158	0.15	0.148	0.164	0.169	0.216	0.173	0.201	0.174	0.16
DAX												
QLIKE	−3.696	−3.687	−3.694	−3.693	−3.695	−3.685	−3.682	−3.553	−3.655	−3.609	−3.653	−3.657
MSE	0.229	0.248	0.247	0.281	0.229	0.258	0.271	0.341	0.308	0.396	0.302	0.301
Nikkei												
QLIKE	−3.638	−3.625	−3.637	−3.637	−3.638	−3.63	−3.609	−3.589	−3.588	−3.579	−3.583	−3.612
MSE	0.263	0.299	0.268	0.281	0.264	0.279	0.356	0.434	0.363	0.438	0.364	0.364
Hang Seng												
QLIKE	−3.694	−3.685	−3.693	−3.69	−3.692	−3.688	−3.68	−3.668	−3.675	−3.665	−3.673	−3.686
MSE	0.218	0.231	0.217	0.23	0.219	0.23	0.255	0.385	0.263	0.398	0.266	0.238
Panel B: twelve years as rolling window, six months as re-fitting frequency												
S&P 500												
QLIKE	−4.123	−4.111	−4.125	−4.12	−4.122	−4.12	−4.081	−4.073	−4.086	−4.066	−4.086	−4.082
MSE	0.107	0.158	0.111	0.167	0.106	0.112	0.179	0.64	0.162	0.193	0.169	0.161
NASDAQ												
QLIKE	−3.84	−3.836	−3.841	−3.835	−3.839	−3.837	−3.816	−3.785	−3.8	−3.767	−3.809	−3.822
MSE	0.21	0.258	0.216	0.412	0.209	0.219	0.263	1.83	0.306	9.049	0.292	0.245
FTSE 100												
QLIKE	−3.938	−3.936	−3.938	−3.937	−3.937	−3.936	−3.924	−3.914	−3.924	−3.914	−3.914	−3.935
MSE	0.149	0.161	0.159	0.158	0.149	0.153	0.17	2.061	0.164	0.293	0.182	0.16
DAX												
QLIKE	−3.695	−3.69	−3.694	−3.691	−3.694	−3.688	−3.682	−3.676	−3.678	−3.671	−3.68	−3.63
MSE	0.231	0.246	0.247	0.345	0.231	0.25	0.271	0.451	0.298	0.501	0.27	0.463
Nikkei												
QLIKE	−3.637	−3.63	−3.637	−3.634	−3.637	−3.632	−3.609	−3.597	−3.612	−3.586	−3.613	−3.612
MSE	0.264	0.297	0.267	0.336	0.264	0.276	0.356	0.871	0.343	2.579	0.342	0.367
Hang Seng												
QLIKE	−3.693	−3.689	−3.693	−3.688	−3.692	−3.689	−3.679	−3.661	−3.674	−3.664	−3.678	−3.687
MSE	.22	0.229	0.217	0.328	0.22	0.229	0.256	5.844	0.26	0.271	0.255	0.238
Panel C: six years as rolling window, two months as re-fitting frequency												
S&P 500												
QLIKE	−4.125	−4.116	−4.126	−4.123	−4.123	−4.111	−4.085	−4.043	−4.068	−4.018	−4.065	−4.11
MSE	0.101	0.142	0.107	0.102	0.103	0.121	0.164	1.42	0.178	0.348	0.185	0.125
NASDAQ												
QLIKE	−3.842	−3.833	−3.841	−3.839	−3.839	−3.829	−3.822	−3.811	−3.816	−3.76	−3.812	−3.826
MSE	0.202	0.275	0.213	0.204	0.205	0.231	0.235	0.262	0.252	2.283	0.26	0.275
FTSE 100												
QLIKE	−3.939	−3.934	−3.938	−3.933	−3.936	−3.931	−3.926	−3.874	−3.916	−3.887	−3.914	−3.932
MSE	0.147	0.17	0.158	0.334	0.149	0.162	0.162	0.202	0.176	1.234	0.178	0.163
DAX												
QLIKE	−3.696	−3.687	−3.694	−3.695	−3.694	−3.686	−3.682	−3.642	−3.643	−3.65	−3.658	−3.675
MSE	0.228	0.26	0.247	0.23	0.231	0.255	0.264	0.391	0.333	0.719	0.312	0.285
Nikkei												
QLIKE	−3.639	−3.623	−3.636	−3.638	−3.638	−3.629	−3.609	−3.577	−3.591	−3.56	−3.589	−3.457
MSE	0.262	0.322	0.269	0.265	0.262	0.283	0.349	0.397	0.439	0.701	0.451	0.39
Hang Seng												
QLIKE	−3.694	−3.684	−3.693	−3.69	−3.688	−3.687	−3.681	−3.652	−3.649	−3.626	−3.663	−3.682
MSE	0.215	0.239	0.218	0.223	0.242	0.231	0.246	0.268	0.366	3.337	0.295	0.239

Notes: The table reports the averages of the QLIKE and MSE (multiplied by 10,000) loss functions for the out-of-sample period (from 2 January 2013 to 15 May 2020). Shades of gray denote inclusion in the MCS at significance level $\alpha = 0.25$.

vendors is expanding and that processing and storing tick-by-tick data (possibly by web scraping) is not a prohibitive task.

A comparison across models can be interpreted as an exercise that aims at assessing the capability of each model to reproduce empirical regularities in the data, but also at establishing how important those stylized facts are when taken on an out-of-sample terrain.

In this context, our paper has two clear outcomes: one is to suggest that modeling realized volatility delivers better results than going through a GARCH-type approach; the second is to show that incorporating the feature that average volatility by subperiod is time-varying provides an advantage in forecasting. For the first outcome, there are clear merits in using a model in which the errors enter multiplicatively,

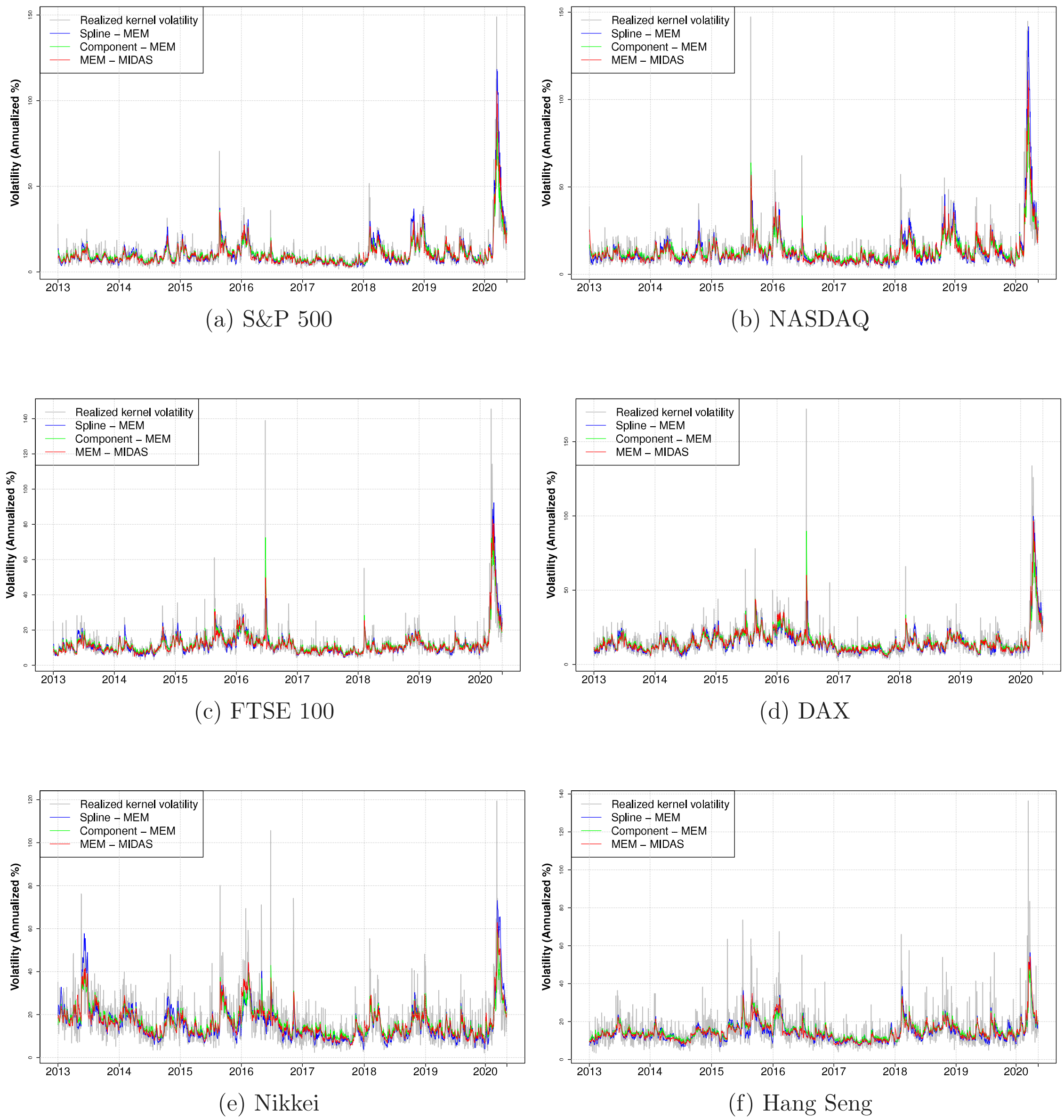


Fig. 3. DMEM out-of-sample volatilities
Notes: Plots of realized kernel and out-of-sample estimated volatilities from DMEM models. The MEM-MIDAS model reported here uses HSc_t as MIDAS variable. Period: 2 January 2013–15 May 2020.

as in the MEM: this mitigates the attenuation bias in realized volatility models as documented by Cipollini et al. [34], because it takes into explicit consideration the heteroskedastic nature of volatility measurement errors. For the second outcome, we suggest that doubling the multiplicative components incorporating a slow-moving and a *short*-run components of volatility dynamics delivers better results, at least for our six stock market indices. We considered three such models, differentiated by the type of information entering the low-frequency component: in the Spline-MEM we adopt a spline as a function of time; in the Component-MEM, we use the same daily data, but we allow for a more persistent dynamics; in the MEM-MIDAS, we can use one or more low-frequency variables to be filtered into a smooth component which exploits the mixed data sampling results by Ghysels et al. [40] and by Engle et al. [11]. The proposed DMEM's as well as the other MEM-based models generally perform better than the corresponding GARCH specifications, independently of the period or LFs considered.

The approach can be convenient in a context of policy analysis: the MEM-MIDAS lends itself to be used within a scenario-type approach for the low-frequency variable, by designing future prolonged periods of downturns in economic activity. The impact and aftermath of events on the financial market volatility (e.g. the COVID-19 health emergency, or the insurgence of a military conflict) may thus be studied in projecting this channel of transmission originating in the real economy to the medium term.

While refinements are still possible (e.g. the use of a second lag in making use of observed volatility values, or a DAGM extension within the MEM-MIDAS, or, again, several variables in the MIDAS filter), one indication that emerges from the empirical results (cf. Fig. 2) is that the components estimated by our models have some commonality that should be exploited – in a common factor sense – by a joint modeling of the series. The possibility that the average level of volatility depends upon sudden shifts is investigated in a Markov switching MEM-MIDAS by Scaffidi Domianello et al. [41] which extends and enriches the contributions by Pan et al. [42] and Segnon et al. [43].

This line of research has substantial bearings on risk assessment, given that volatility is the main ingredient to evaluate the tail risk of portfolios connected to negative downturns and the need for adequate capital requirements following the directives of the Basel agreements

Table 5
In-sample estimates. NASDAQ.

MIDAS	MEM	Spline-MEM	Component-MEM	MEM-MIDAS NAI_t	MEM-MIDAS HSc_t	HAR	GJR	GM NAI_t	GM HSc_t	DAGM NAI_t	DAGM HSc_t	RGARCH
const	0.388					1.459***	5.106***					0.142***
						(0.447)	(1.601)					(0.055)
α_1	0.128***	0.074***	0.000	0.123***	0.132***		0.000	0.000	0.000	0.000	0.000	0.439***
	(0.016)	(0.014)	(0.02)	(0.018)	(0.014)		(0.007)	(0.008)	(0.008)	(0.009)	(0.008)	(0.056)
β_1	0.809***	0.755***	0.698***	0.805***	0.825***	0.184***	0.93***	0.932***	0.934***	0.929***	0.932***	0.771***
	(0.02)	(0.022)	(0.051)	(0.021)	(0.016)	(0.035)	(0.015)	(0.011)	(0.011)	(0.011)	(0.012)	(0.027)
β_5						0.472***						
						(0.05)						
β_{22}						0.198***						
						(0.047)						
γ_1	0.086***	0.115***	0.104***	0.095***	0.08***	0.143***	0.119***	0.134***	0.129***	0.139***	0.132***	
	(0.009)	(0.009)	(0.013)	(0.011)	(0.009)	(0.022)	(0.022)	(0.022)	(0.022)	(0.023)	(0.022)	
m				−0.042	1.000***			8.078***	8.059***	8.246***	8.047***	
				(0.184)	(0.024)			(0.084)	(0.721)	(1.273)	(1.399)	
ζ				−0.238***	−0.01**			−0.284*	−0.045*			
				(0.044)	(0.005)			(0.158)	(0.025)			
ω_2				12.183	4.448***			6.678***	1.002			
				(29.322)	(0.742)			(1.424)	(1.09)			
ζ^+										−1.494	−0.037	
										(1.253)	(0.036)	
ω_2^+										7.351***	2.01	
										(1.407)	(1.681)	
ζ^-										−0.065	−0.01	
										(0.307)	(0.018)	
ω_2^-										1.44	6.89***	
										(1.382)	(0.069)	
$\omega^{(\tau)}$			0.154***									
			(0.045)									
$\alpha_1^{(\tau)}$			0.097***									
			(0.014)									
$\beta_1^{(\tau)}$			0.879***									
			(0.014)									
$\gamma_1^{(\tau)}$			0.031***									
			(0.009)									
QLIKE	−3.480	−3.484	−3.481	−3.481	−3.481	−3.479	−3.468	−3.462	−3.463	−3.463	−3.463	−3.469
MSE	0.281	0.259	0.275	0.278	0.276	0.286	0.339	0.398	0.393	0.391	0.389	0.340
AIC	19 272.781	19 269.782	19 065.970	19 251.292	19 233.530	21 406.096	26 829.445	26 821.594	26 820.060	26 822.706	26 826.124	28 365.334
BIC	19 290.808	19 486.096	19 108.031	19 287.344	19 269.582	21 436.139	26 853.478	26 857.646	26 856.112	26 870.775	26 874.194	28 413.401

Notes: The table reports the estimated coefficients of the models in column. *, ** and *** represent the significance at levels 10%, 5%, 1%, respectively, associated to QML standard errors. The reported constant for the MEM model refers to α_0 parameter in Table 1. For ease of notation, the parameter α_1 referred to the RGARCH corresponds to the parameter labeled as γ in [19]. Moreover, the estimated parameters of the measurement equation of this latter model are not reported for space constraints.
Sample period: January 2001–December 2012. Daily observations: 3007. Number of lagged macro-economic variable realizations: $K = 36$.

(see, for instance, Drumond [44] and Kinateder [45], among others). In this respect a natural extension of our model is to adapt it to focus on risk measures such as Value-at-Risk (VaR) and Expected Shortfall (ES). Including MIDAS components within a GARCH-type model for better forecasting daily VaR and ES measures has been recently explored by Candila et al. [46] in a quantile regression framework.

Declaration of competing interest

The authors declare that they have no competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Appendix A. Some details on Spline-MEM estimation and forecasting

The τ_t component of Spline-MEM is specified with regression splines using a natural cubic basis [47,48] corresponding to a given number of knots.

In our forecasting setup, for each in-sample period (say, between T_1 and T_2), we produce one-step ahead forecasts keeping coefficient estimates constant over the subsequent out-of-sample period T_2+1 to T_3. Keeping the τ_{T_2+h} component constant and equal to the latest in-sample estimate (up to T_2) is reasonable only for small h, and hence attention is needed in devising a suitable trade-off between the in-sample fitting performance and the use of splines in out-of-sample forecasting, which, as a matter of fact, is the sensitive portion of a Spline-MEM. The regression splines use a fixed number of equally spaced knots spread across the full January 4th, 2000–May 18th, 2020 period (5164 observations — with 50 knots we have about one knot every five months). For each in-sample period, we select only the relevant observations and each time an observation is added to the information set, the slow-moving component τ_t is then refitted: the corresponding estimate is obtained easily, since it is based on a GLM-Gamma regression of the x_t/ξ_t's on the observations from T_1 to $T_2 + h - 1$.

To complete the procedure, the update of the ξ_t component is carried out in the usual way, on the basis of the parameters estimated in-sample.

Appendix B. In-sample estimates for the remaining market indices

The parameter estimates for the other market indices are reported in Tables 5–9.

Table 6
In-sample estimates. FTSE 100.

	MEM	Spline-MEM	Component-MEM	MEM-MIDAS	MEM-MIDAS	HAR	GJR	GM	GM	DAGM	DAGM	RGARCH
MIDAS				NAI_t	HSc_t			NAI_t	HSc_t	NAI_t	HSc_t	
const	0.309					1.105***	4.034***					0.106**
						(0.409)	(0.849)					(0.042)
α_1	0.142***	0.108***	0.022	0.129***	0.134***		0.000	0.000	0.000	0.000	0.000	0.495***
	(0.016)	(0.015)	(0.023)	(0.018)	(0.017)		(0.01)	(0.018)	(0.012)	(0.019)	(0.019)	(0.034)
β_1	0.786***	0.746***	0.656***	0.782***	0.789***	0.225***	0.904***	0.885***	0.897***	0.873***	0.89***	0.744***
	(0.019)	(0.020)	(0.079)	(0.032)	(0.02)	(0.037)	(0.013)	(0.019)	(0.015)	(0.021)	(0.023)	(0.017)
β_5						0.447***						
						(0.058)						
β_{22}						0.18***						
						(0.041)						
γ_1	0.106***	0.117***	0.07***	0.111***	0.106***	0.152***	0.165***	0.178***	0.162***	0.186***	0.177***	
	(0.009)	(0.009)	(0.014)	(0.011)	(0.009)	(0.023)	(0.025)	(0.024)	(0.021)	(0.024)	(0.024)	
m				−0.208***	−0.101*			5.388***	5.513***	5.623***	5.628***	
				(0.079)	(0.06)			(0.174)	(0.15)	(0.185)	(1.007)	
ζ				−0.266***	−0.01			−0.598***	−0.071***			
				(0.047)	(0.006)			(0.084)	(0.012)			
ω_2				4.786	3.361***			4.71***	2.699***			
				(22.925)	(0.082)			(0.336)	(0.473)			
ζ^+										−2.061**	−0.096	
										(0.814)	(0.064)	
ω_2^+										2.86***	1.052***	
										(0.544)	(0.284)	
ζ^-										−0.489***	−0.088***	
										(0.1)	(0.028)	
ω_2^-										22.96***	1.448*	
										(0.103)	(0.757)	
$\omega^{(\tau)}$			0.185***									
			(0.041)									
$\alpha_1^{(\tau)}$			0.104***									
			(0.017)									
$\beta_1^{(\tau)}$			0.85***									
			(0.018)									
$\gamma_1^{(\tau)}$			0.065***									
			(0.012)									
QLIKE	−3.693	−3.695	−3.694	−3.693	−3.693	−3.691	−3.686	−3.689	−3.686	−3.689	−3.688	−3.687
MSE	0.174	0.168	0.171	0.173	0.173	0.180	0.192	0.184	0.188	0.181	0.186	0.198
AIC	17 672.860	17 716.050	17 564.921	17 667.643	17 679.661	20 100.962	25 203.312	25 181.486	25 194.911	25 177.649	25 185.885	26 169.644
BIC	17 690.901	17 932.530	17 607.014	17 703.723	17 715.741	20 131.029	25 227.364	25 217.566	25 230.991	25 225.756	25 233.992	26 217.748

Notes: The table reports the estimated coefficients of the models in column. *, ** and *** represent the significance at levels 10%, 5%, 1%, respectively, associated to QML standard errors. The reported constant for the MEM model refers to α_0 parameter in Table 1. For ease of notation, the parameter α_1 referred to the RGARCH corresponds to the parameter labeled as γ in [19]. Moreover, the estimated parameters of the measurement equation of this latter model are not reported for space constraints.
Sample period: January 2001–December 2012. Daily observations: 3021. Number of lagged macro-economic variable realizations: $K = 36$.

Table 7
In-sample estimates. DAX.

	MEM	Spline-MEM	Component-MEM	MEM-MIDAS	MEM-MIDAS	HAR	GJR	GM	GM	DAGM	DAGM	RGARCH
MIDAS				NAI_t	HSc_t			NAI_t	HSc_t	NAI_t	HSc_t	
const	0.398					1.165**	6.832***					0.211***
						(0.49)	(1.409)					(0.071)
α_1	0.163***	0.135***	0.000	0.158***	0.161***		0.000	0.000	0.000	0.000	0.000	0.507***
	(0.016)	(0.015)	(0.023)	(0.019)	(0.017)		(0.009)	(0.007)	(0.009)	(0.011)	(0.01)	(0.057)
β_1	0.769***	0.723***	0.613***	0.764***	0.769***	0.289***	0.907***	0.925***	0.909***	0.896***	0.907***	0.724***
	(0.02)	(0.019)	(0.072)	(0.023)	(0.02)	(0.05)	(0.012)	(0.009)	(0.012)	(0.014)	(0.012)	(0.029)
β_5						0.403***						
						(0.066)						
β_{22}						0.18***						
						(0.037)						
γ_1	0.098***	0.112***	0.086***	0.1***	0.098***	0.14***	0.16***	0.13***	0.153***	0.164***	0.157***	
	(0.008)	(0.008)	(0.012)	(0.008)	(0.008)	(0.022)	(0.023)	(0.015)	(0.022)	(0.023)	(0.022)	
m				−0.127*	−0.022			6.039***	6.21***	6.26***	5.66***	
				(0.065)	(0.136)			(0.18)	(0.182)	(0.257)	(0.768)	
ζ				−0.108***	−0.014			0.333**	−0.032			
				(0.019)	(0.01)			(0.137)	(0.022)			
ω_2				5.513	1.001			1.863**	1.002***			
				(6.835)	(1.376)			(0.887)	(0.352)			
ζ^+										−0.606	0.000	
										(0.374)	(0.023)	
ω_2^+										1.993***	2.852**	
										(0.597)	(1.135)	
ζ^-										−0.159**	−0.036	
										(0.069)	(0.029)	
ω_2^-										7.39***	1.006	
										(0.727)	(1.167)	
$\omega^{(\tau)}$			0.22***									
			(0.049)									
$\alpha_1^{(\tau)}$			0.132***									
			(0.016)									
$\beta_1^{(\tau)}$			0.829***									
			(0.017)									
$\gamma_1^{(\tau)}$			0.054***									

(*continued on next page*)

Table 7 (*continued*).

	MEM	Spline-MEM	Component-MEM	MEM-MIDAS NAI_t	MEM-MIDAS HSc_t	HAR	GJR	GM NAI_t	GM HSc_t	DAGM NAI_t	DAGM HSc_t	RGARCH
			(0.01)									
QLIKE	−3.461	−3.463	−3.462	−3.461	−3.461	−3.460	−3.451	−3.451	−3.452	−3.453	−3.452	−3.453
MSE	0.235	0.229	0.232	0.234	0.234	0.241	0.270	0.268	0.268	0.265	0.268	0.269
AIC	18 940.303	18 954.830	18 842.642	18 935.215	18 945.045	21 186.169	27 015.552	27 046.827	27 015.640	27 009.534	27 019.666	27 831.898
BIC	18 958.371	19 171.654	18 884.802	18 971.352	18 981.183	21 216.283	27 039.642	27 082.964	27 051.778	27 057.717	27 067.849	27 880.079

Notes: The table reports the estimated coefficients of the models in column. *, ** and *** represent the significance at levels 10%, 5%, 1%, respectively, associated to QML standard errors. The reported constant for the MEM model refers to α_0 parameter in Table 1. For ease of notation, the parameter α_1 referred to the RGARCH corresponds to the parameter labeled as γ in [19]. Moreover, the estimated parameters of the measurement equation of this latter model are not reported for space constraints.
Sample period: January 2001–December 2012. Daily observations: 3050. Number of lagged macro-economic variable realizations: $K = 36$.

Table 8
In-sample estimates. Nikkei.

MIDAS	MEM	Spline-MEM	Component-MEM	MEM-MIDAS NAI_t	MEM-MIDAS HSc_t	HAR	GJR	GM NAI_t	GM HSc_t	DAGM NAI_t	DAGM HSc_t	RGARCH
const	0.554					2.217***	12.355***					0.182**
						(0.672)	(3.332)					(0.08)
α_1	0.129***	0.094***	0.053**	0.125***	0.127***		0.032***	0.025**	0.033***	0.06	0.033***	0.562***
	(0.014)	(0.014)	(0.024)	(0.015)	(0.014)		(0.011)	(0.012)	(0.011)	(0.098)	(0.011)	(0.117)
β_1	0.803***	0.757***	0.811***	0.794***	0.801***	0.174***	0.883***	0.872***	0.881***	0.904***	0.882***	0.714***
	(0.02)	(0.023)	(0.028)	(0.021)	(0.02)	(0.047)	(0.015)	(0.000)	(0.014)	(0.011)	(0.01)	(0.059)
β_5						0.437***						
						(0.062)						
β_{22}						0.206***						
						(0.051)						
γ_1	0.076***	0.096***	0.082***	0.08***	0.077***	0.127***	0.126***	0.132***	0.121***	0.07	0.121***	
	(0.01)	(0.010)	(0.024)	(0.073)	(0.024)	(0.028)	(0.026)	(0.018)	(0.014)	(0.07)	(0.014)	
m				−0.073*	0.008			6.078***	6.286***	0.952	6.206***	
				(0.04)	(0.044)			(0.136)	(0.152)	(39.566)	(0.558)	
ζ				−0.086***	−0.023***			−0.194**	−0.035*			
				(0.017)	(0.008)			(0.091)	(0.02)			
ω_2				5.498***	1.255***			7.07	1.268***			
				(0.051)	(0.158)			(36.408)	(0.339)			
ζ^+										1.22	−0.035	
										(77.43)	(0.026)	
ω_2^+										1.01**	2.42	
										(0.476)	(1.551)	
ζ^-										0.14*	−0.04	
										(0.083)	(0.024)	
ω_2^-										1.121	1.001**	
										(13.673)	(0.417)	
ω^τ			0.138**									
			(0.069)									
$\alpha_1^{(\tau)}$			0.066***									
			(0.019)									
$\beta_1^{(\tau)}$			0.922***									
			(0.023)									
$\gamma_1^{(\tau)}$			0.009									
			(0.012)									
QLIKE	−3.500	−3.502	−3.500	−3.500	−3.500	−3.498	−3.480	−3.481	−3.480	−3.478	−3.480	−3.477
MSE	0.257	0.251	0.256	0.255	0.256	0.262	0.382	0.367	0.376	0.402	0.377	0.378
AIC	18 956.826	19 000.835	18 870.974	18 947.707	18 951.157	20 613.152	26 288.914	26 277.818	26 289.147	26 428.170	26 291.916	27 830.410
BIC	18 974.775	19 216.226	18 912.856	18 983.606	18 987.055	20 643.067	26 312.845	26 313.716	26 325.046	26 476.035	26 339.781	27 878.272

Notes: The table reports the estimated coefficients of the models in column. *, ** and *** represent the significance at levels 10%, 5%, 1%, respectively, associated to QML standard errors. The reported constant for the MEM model refers to α_0 parameter in Table 1. For ease of notation, the parameter α_1 referred to the RGARCH corresponds to the parameter labeled as γ in [19]. Moreover, the estimated parameters of the measurement equation of this latter model are not reported for space constraints.
Sample period: January 2001–December 2012. Daily observations: 2931. Number of lagged macro-economic variable realizations: $K = 36$.

Table 9
In-sample estimates. Hang Seng.

MIDAS	MEM	Spline-MEM	Component-MEM	MEM-MIDAS NAI_t	MEM-MIDAS HSc_t	HAR	GJR	GM NAI_t	GM HSc_t	DAGM NAI_t	DAGM HSc_t	RGARCH
const	0.235					1.429**	6.208***					0.082***
						(0.614)	(1.701)					(0.031)
α_1	0.095***	0.058***	0.000	0.09***	0.091***		0.023***	0.012	0.017**	0.01	0.017**	0.266***
	(0.01)	(0.012)	(0.02)	(0.01)	(0.01)		(0.008)	(0.008)	(0.008)	(0.009)	(0.008)	(0.011)
β_1	0.865***	0.831***	0.813***	0.856***	0.865***	0.107***	0.921***	0.905***	0.916***	0.901***	0.914***	0.862***
	(0.013)	(0.022)	(0.076)	(0.014)	(0.013)	(0.04)	(0.01)	(0.014)	(0.012)	(0.015)	(0.012)	(0.003)
β_5						0.476***						
						(0.068)						
β_{22}						0.291***						
						(0.068)						
γ_1	0.057***	0.071***	0.047***	0.061***	0.058***	0.106***	0.087***	0.103***	0.095***	0.106***	0.097***	
	(0.008)	(0.009)	(0.013)	(0.009)	(0.008)	(0.026)	(0.021)	(0.022)	(0.021)	(0.024)	(0.022)	
m				−0.177***	−0.058			5.87***	6.13***	6.033***	6.43***	
				(0.055)	(0.071)			(0.182)	(0.165)	(0.241)	(0.477)	
ζ				−0.294***	−0.011**			−0.627***	−0.066***			
				(0.049)	(0.005)			(0.098)	(0.025)			
ω_2				8.405***	4.858***			9.487	1.787***			
				(0.06)	(0.028)			(23.296)	(0.517)			
ζ^+										−1.55***	−0.092**	
										(0.344)	(0.045)	
ω_2^+										5.087	1.317***	

(*continued on next page*)

Table 9 (*continued*).

	MEM	Spline-MEM	Component-MEM	MEM-MIDAS	MEM-MIDAS	HAR	GJR	GM	GM	DAGM	DAGM	RGARCH
MIDAS				NAI_t	HSc_t			NAI_t	HSc_t	NAI_t	HSc_t	
										(11.88)	(0.358)	
ζ^-										−0.522***	−0.061	
										(0.14)	(0.043)	
ω_2^-										13.268	2.093	
										(14.082)	(4.184)	
$\omega^{(\tau)}$			0.155***									
			(0.051)									
$\alpha_1^{(\tau)}$			0.085***									
			(0.014)									
$\beta_1^{(\tau)}$			0.892***									
			(0.015)									
$\gamma_1^{(\tau)}$			0.03***									
			(0.012)									
QLIKE	−3.472	−3.474	−3.472	−3.472	−3.472	−3.470	−3.462	−3.463	−3.462	−3.463	−3.462	−3.463
MSE	0.373	0.364	0.373	0.372	0.374	0.386	0.408	0.396	0.403	0.392	0.401	0.423
AIC	19 518.325	19 573.539	19 424.533	19 513.157	19 520.088	21 795.234	26 095.508	26 075.379	26 090.602	26 075.748	26 092.904	27 741.046
BIC	19 536.281	19 789.016	19 466.432	19 549.070	19 556.001	21 825.161	26 119.448	26 111.292	26 126.515	26 123.632	26 140.787	27 788.927

Notes: The table reports the estimated coefficients of the models in column. *, ** and *** represent the significance at levels 10%, 5%, 1%, respectively, associated to QML standard errors. The reported constant for the MEM model refers to α_0 parameter in Table 1. For ease of notation, the parameter α_1 referred to the RGARCH corresponds to the parameter labeled as γ in [19]. Moreover, the estimated parameters of the measurement equation of this latter model are not reported for space constraints.
Sample period: January 2001–December 2012. Daily observations: 2938. Number of lagged macro-economic variable realizations: $K = 36$.

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