

# An approach to calibrate the unsaturated hydraulic properties of a soil through numerical modelling of a small-scale slope model exposed to rainfall

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## Abstract

Numerous studies have looked at rainfall infiltration as a triggering factor for rainfall-induced landslides. In addition to rainfall characteristics, soil hydraulic properties are recognized to play a crucial role in instability mechanisms. This study proposes a methodology for assessing the soil water characteristic curve and hydraulic conductivity function through finite-element seepage modelling of physical model tests. The approach is applied to an instrumented, small-scale slope model built with uniformly graded sand with a 30 degree inclination, exposed to homogeneous rainfall until the occurrence of failure. In a first stage, a sensitivity analysis is carried out to assess the influence of the Mualem-van Genuchten model parameters on the slope response. For this purpose, six physically based indicators are utilized to compare the numerical modelling results with the experimentally obtained data regarding the hydraulic response of the slope model. In the subsequent stage, a trial-and-error calibration stage is conducted to determine the set of parameters for which the difference between the numerically and experimentally acquired data is minimized. Ultimately, the suggested methodology facilitates the assessment of the optimal set of hydraulic parameters, to which both the sensitivity analysis and the trial-and-error calibration phase are anchored. The approach has demonstrated its effectiveness in calibrating the unsaturated hydraulic properties of the considered soil, as it properly addresses the physical mechanisms associated with rainfall infiltration in a slope.

**Keywords:** Rainfall infiltration, Landslide, Wetting front, Soil Water Characteristic Curve, Numerical modelling, Physical modelling

## Introduction

As widely recognised, rainfall is one of the main triggering factors of both slow-moving and flow-like landslides [2, 47, 16, 38, 11, 10, 49, 44, 41, 61]. Indeed, the rainfall infiltration process can trigger a series of phenomena on a slope, potentially causing instability [69, 6, 45, 58], such as: i) decreasing matric suction; ii) increasing porewater pressures and rising of the groundwater level; iii) generation of destabilizing seepage forces; iv) increase in unit weight of soil due to increasing of water content. Many studies have been conducted to investigate the hydrological processes and the initiation

46 mechanisms of landslides resulting from rainfall infiltration. Both field monitoring and experimental tests have shown  
47 that the unsaturated hydraulic properties of the soil layers very often play a fundamental role in these processes, clearly  
48 in combination with other important factors of the analysed boundary value problem, such as stratigraphy, soil types,  
49 slope inclination, and boundary conditions [73, 50, 17, 34, 53, 71, 68, 4, 9, 13, 46, 58].

50 Direct measurements of the unsaturated hydraulic parameters using traditional methods can be complex, time-consuming  
51 and not always effective, particularly when only data from laboratory tests are used. The Soil Water Characteristic Curve  
52 (SWCC), which relates soil suction to soil moisture content, is one of the most important features in unsaturated soil  
53 mechanics. Typically, a SWCC is highly nonlinear and relatively difficult to measure accurately [72, 54, 18]. While a  
54 wide range of soil suction, which fine-grained soils typically exhibit, requires the combination of different measurement  
55 techniques and devices [e.g. 59, 45, 48], measurements in uniformly graded coarse-grained soils, such as the material  
56 considered in this study, are often challenging due to a very steep and narrow SWCC where saturated and residual soil  
57 moisture conditions are typically separated by only a few kPa of matric suction [29, 57]. Depending on the measurement  
58 technique, equilibration times during suction measurements can vary from a few minutes to several months [63].  
59 Moreover, not all measurement techniques and devices can provide a characterisation of the soil that is relevant for the  
60 process under consideration [28]. Hydraulic hysteresis or non-uniqueness in the functional relationship between soil water  
61 potential and water content is a well-known phenomenon that is characteristic of SWCC and has direct implications for  
62 hydraulic characterisation and hydraulic response modelling of unsaturated soils [72, 76]. The latter is reflected in the  
63 different pathways that the unsaturated soil undergoes depending on whether it is subjected to a drying or a wetting  
64 process. In general, for a given soil water potential, the soil tends to retain more water in a drying process than in a wetting  
65 process. To account for this, depending on the problem being considered, the main drying and wetting branches of a  
66 SWCC are usually determined in two different ways: i) the initially saturated soil is subjected to a drying process and  
67 measurements are conducted to define the main drying branch, or ii) measurements are made on an initially dry soil, i.e.  
68 with a moisture content close to residual conditions, which is subjected to a wetting process to obtain the main wetting  
69 branch [35, 28]. The Hydraulic Conductivity Function (HCF) provides a functional relationship between hydraulic  
70 conductivity and soil suction and, together with the SWCC, controls the seepage process in unsaturated soils. However,  
71 uncertainties and limitations in terms of prolonged testing times, expensive equipment, and limitations in measuring range  
72 make the determination of HCF through direct measurements very rare [e.g. 39, 19, 44]. To overcome these difficulties,  
73 many researchers have proposed different estimation methods or indirect methods to obtain the HCF of the soil [e.g. 21,  
74 62, 35, 63, 70, 50].

75 Herein, a new methodology is defined aiming at calibrating the unsaturated hydraulic parameters of the soil in a slope  
76 subjected to rainfall, when monitoring data of soil water content are available. The procedure is applied and tested using  
77 a small-scale sandy slope exposed to a constant rainfall intensity and equipped with porewater pressure and soil moisture  
78 sensors at various locations. The proposed methodology and the obtained results are also used to investigate how the  
79 variation of each parameter defining the SWCC and the HCF of the soil affect the hydraulic response of a considered  
80 slope model. While previous studies have investigated how various factors affect the hydraulic response and stability  
81 conditions of slopes under rainfall infiltration conditions [e.g. 69, 59, 22], the results of the parametric analyses conducted  
82 in this study aim to guide and facilitate the calibration process. Although the test procedure could be modified to generalise  
83 its application, the methodology proposed here provides a hydraulic characterisation of soil undergoing wetting processes  
84 and is particularly well suited for the use with laboratory flume test results.

## 85 **Rainfall infiltration in unsaturated soils**

86 To understand the hydraulic mechanisms related to rainfall infiltration in soils, numerous approaches based on the  
87 advancement of the “wetting front” have been developed. The first model was proposed by Green-and Ampt (1911) [33],  
88 which describes the infiltration process through partially saturated soil. The model is based on Darcy’s law and makes  
89 the following simplifying assumptions: i) ponding condition at ground level; ii) homogeneous and isotropic soil; iii) sharp  
90 wetting front with fully saturated conditions above it and initial degree of saturation below it; iv) constant hydraulic  
91 conductivity of the soil, neglecting variations due to changes in moisture content. According to these hypotheses, the time  
92 necessary to saturate the soil,  $t_{sat}$ , to a depth  $z_{sat}$  is:

$$t_{sat} = \frac{S_{sat} - S_{in}}{k_{sat}} \left[ z_{sat} - (h_p + C_c) \ln \left( \frac{h_p + C_c + z_{sat}}{h_p + C_c} \right) \right] \quad (1)$$

93 where  $S_{in}$  and  $S_{sat}$  are respectively the degree of saturation below and above the wetting front,  $k_{sat}$  is the saturated  
94 hydraulic conductivity of the soil,  $C_c$  is the capillarity coefficient, and  $h_p$  is the ponding head.

95 Several studies have been carried out to remove some of the limitations of the Green-Ampt model, considering layered  
96 soil stratigraphy [14, 15, 20] and time-dependent infiltration rate [27, 1]. However, most of the contributions keep the  
97 ponding condition at ground level, which may correspond to unrealistic precipitation rates for some soils. Very few studies  
98 have been carried out considering an infiltration rate lower than the saturated hydraulic conductivity of the soil, such as  
99 the one by Sun et al. (1998)[67], who identified the depth of the wetting front as:

$$z_w = \frac{(q_1 - q_0)}{n_s (S_{q_1} - S_{q_0})} d \quad (2)$$

100 where  $d$  is the rainfall duration,  $q_0$  is the surface flux at steady state condition (initial condition),  $q_1$  is the water flux  
 101 under transient conditions,  $n_s$  is the soil porosity,  $S_{q_0}$  is the initial degree of saturation in steady state conditions in  
 102 equilibrium with the water flux  $q_0$ , and  $S_{q_1}$  is the degree of saturation corresponding to the depth  $z_w$  of the wetting front.

103 Considering the simplifying assumptions required to obtain an analytical solution, the conceptual wetting front model is  
 104 not easily applicable to real cases. Instead, numerical methods enable the solution of the differential equation describing  
 105 a transient groundwater flow (Richards, 1931 [62]) by properly schematizing any boundary value problem, thus  
 106 considering, for instance, soil stratigraphy, heterogeneous initial conditions, time-dependent boundary conditions, and,  
 107 most importantly, the non-linear hydraulic conductivity within partially saturated soils.

108 Indeed, the hydraulic soil behaviour in unsaturated conditions is highly dependent on matric suction, volumetric water  
 109 content, and their variation over time. Several studies have been carried out to define the SWCC and the HCF  
 110 characterising unsaturated soils, respectively differing for the number of parameters required and their applicability to  
 111 different soil types [5, 15, 37, 16, among others]. One of the most frequently SWCC adopted in the literature is the van  
 112 Genuchten model [30], which can be expressed in terms of the degree of saturation ( $S$ ) as follows:

$$S = S_r + \frac{(S_{sat} - S_r)}{\left[1 + \left(\frac{h}{a}\right)^n\right]^m} \quad (3)$$

113 where  $h$  is the soil water potential,  $S_r$  is the residual degree of saturation,  $S_{sat}$  is the degree of saturation at fully saturated  
 114 conditions,  $a$  and  $n$  are the fitting curve parameters related to an air-entry value and to the slope of the transition zone,  
 115 respectively, with  $m = 1 - 1/n$ , as proposed by van Genuchten [51].

116 The dependence of the unsaturated hydraulic conductivity on the soil water content can be expressed as (van Genuchten,  
 117 [51]):

$$k(S_e) = k_{sat} S_e^l \left[1 - \left(1 - S_e^{\frac{1}{m}}\right)^m\right]^2 \quad \text{with} \quad S_e = (S - S_r)/(S_{sat} - S_r) \quad (4)$$

118 where  $S_e$  is the effective degree of saturation, and  $l$  is an empirical constant.

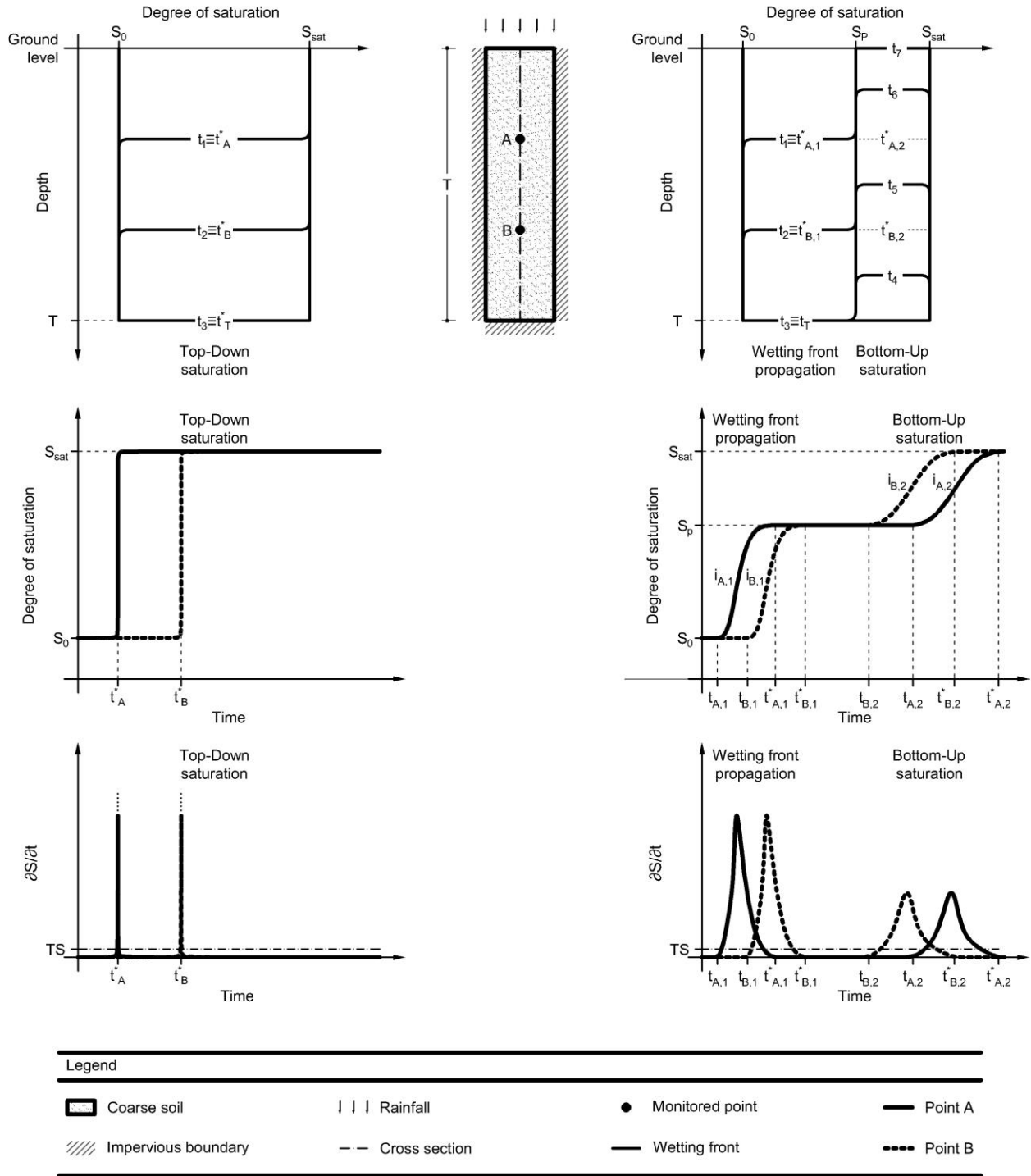
119 The factor  $S_e^l$  was introduced by Burdine [8] and Fatt and Dykstra [24] to account for pore tortuosity and connectivity  
 120 (Schaap and Leij [64]). Mualem [51] found out that  $l$  can assume both positive and negative value.

## 121 **Theoretical framework for the model calibration using six control** 122 **parameters**

123 As already stated, the wetting front concept does not allow to consider many realistic stratigraphical settings, even  
 124 relatively simple ones, such as the presence of an impermeable layer underlying a coarse-grained soil. Indeed, in this  
 125 common stratigraphical condition for shallow rainfall-induced landslides, also a low intensity rainfall can initiate the  
 126 saturation of the coarse soil layer, starting from the bottom boundary after the wetting front has reached it. To illustrate  
 127 the hydraulic mechanisms that may occur with this stratigraphical setting yet neglecting, for explanatory reasons, the role  
 128 of the slope angle, we can consider a soil column in one dimensional condition with a thickness  $T$  and an impervious  
 129 boundary at the bottom (Fig. 1). Once the infiltration capacity drops to a constant value, it can be assumed that the amount  
 130 of rainfall that can infiltrate the soil can be approximated by the saturated hydraulic conductivity ( $k_{sat}$ ). In this sense, the  
 131 value of the ratio between the rainfall intensity ( $q$ ) and the saturated hydraulic conductivity ( $k_{sat}$ ) allows us to distinguish  
 132 between 2 types of saturation mechanisms: top-down saturation, for  $q/k_{sat} \geq 1$ ; and bottom-up saturation preceded by a  
 133 downward wetting front propagation in unsaturated conditions, for  $q/k_{sat} < 1$  and the boundary conditions considered in  
 134 the scheme.

135 In the first case (left side of Fig. 1), the rainfall produces a saturation mechanism from the ground level to the inner part  
 136 of the soil column (Top-Down mechanism), which is in line with Green-Ampt conceptual model and can be expressed  
 137 according to Eq. (1). The sharp wetting front fully saturates the soil, starting from the ground level, as time progresses  
 138 (from  $t_1$  to  $t_3$ ). Tracing the degree of saturation in two points within the soil (A and B in Fig. 1), once the saturation  
 139 conditions are reached, they remain unchanged until the end of the process, i.e., the complete saturation of the soil column.  
 140 In terms of first derivative of the degree of saturation ( $S'$  hereafter), at any time the function assumes a null value until  
 141 the wetting front reaches the point, then assumes an infinitive-value due to the "instantaneous" variation of the saturation  
 142 condition, and then returns to a null value once a new equilibrium state is achieved. In the second case (right side of Fig.  
 143 1), when the rainfall intensity is lower than the saturated hydraulic conductivity of the soil, two phases are observable: a  
 144 first one for which the increase of the degree of saturation, albeit not reaching fully saturated conditions, propagates

145 downwards from the ground level to the inner zones of the soil, essentially following the behaviour described by the  
146 Green-Ampt model; a second Bottom-Up saturation mechanism that can be assimilated to a rapid upwards advancement  
147 of another wetting front, moving from the bottom boundary until the soil column is fully saturated. At any given point,  
148 right side of Fig. 1 shows that the initial conditions  $S_0$  are kept until  $t_1$ , which is the time necessary for the rainwater to  
149 reach the point depth. Then the degree of saturation increases until time  $t_1^*$ , where a new constant “plateau” value ( $S_p$ ) is  
150 reached and kept. Afterward the degree of saturation starts increasing again at time  $t_2$ , which corresponds to the arrival  
151 of the Bottom-Up mechanism, and then reaches its final value  $S_{sat}$  at time  $t_2^*$ , when fully saturated conditions are  
152 established at that depth. It is worth highlighting that in this scheme with an impervious bottom layer, the degree of  
153 saturation at the plateau  $S_p$  represents an apparent steady state condition, since it is kept constant until the upward  
154 saturation mechanism reaches the point of interest. The apparent steady state condition corresponds to a steady state  
155 condition in equilibrium with the rainfall applied only in the case of indefinite thickness of the permeable soil. In terms  
156 of  $S'$ , the two phases are easily distinguishable considering the two peak values assumed by the rate of change of the  
157 degree of saturation. Of course, the time separating the two phases is dependent on the depth considered, and it decreases  
158 as the distance from the impervious layer decreases. The porewater pressures will behave in a similar way, keeping  
159 negative values during the first phase and then reaching positive values as soon as the second saturation mechanism  
160 reaches the point of interest.



161

162 **Fig. 1** Scheme of the saturation degree evolution in one-dimensional conditions at fixed times (first row in the figure) and over time  
 163 at fixed depths (second and third row in the figure) for a coarse soil with a thickness of  $T$  over an impervious layer for: a)  $q \geq k_{sat}$ ,  
 164 i.e. top-down saturation (left side of the figure); b) and  $q < k_{sat}$ , i.e. bottom-up saturation preceded by a downward propagation of the  
 165 wetting front in unsaturated conditions (right side of the figure)

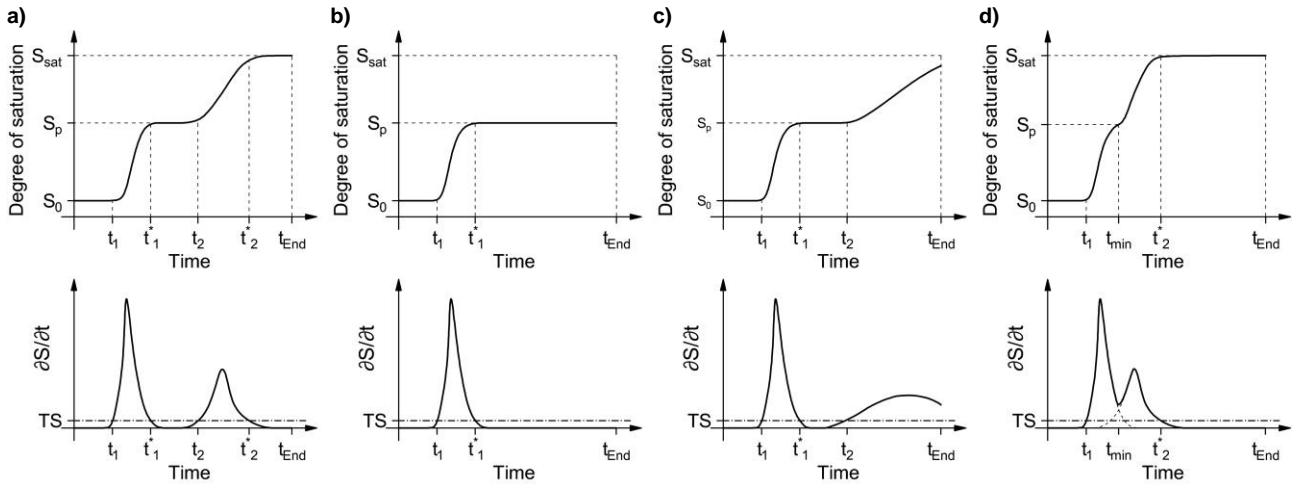
166 Considering the two phases previously described for  $q/k_{sat} < 1$ , a downward propagation of the wetting front in  
 167 unsaturated conditions followed by bottom-up saturation, it is possible to define some reference values for the time-  
 168 dependent changes in the saturation degree at a given depth, which can be used as control parameters to evaluate the fit  
 169 between the monitoring data, i.e. observations, and the results of a numerical model, i.e. predictions. At this scope, the  
 170 following six parameters are considered: i) the initial degree of saturation,  $S_0$ ; ii) the degree of saturation for the apparent  
 171 steady state conditions,  $S_p$ ; iii) the time of the initial change in the saturation degree,  $t_1$ ; iv) the time to reach the fully  
 172 saturated conditions,  $t_2^*$ ; v) the average rate of change of the saturation degree for the wetting front advancement,  $i_1$ ; vi)  
 173 the average rate of change of the saturation degree for the bottom-up mechanism,  $i_2$ . The time derivative of the saturation  
 174 degree is herein used to objectively define, by means of some threshold values (TS), the times at which the initial wetting

175 front propagation and the bottom-up saturation start,  $t_1$  and  $t_2$ , respectively, and come to an end,  $t_1^*$  and  $t_2^*$ , respectively,  
 176 at the considered location. The case for  $q/k_{sat} \geq 1$  can be considered a special case of the one described, for which only  
 177 the following two control parameters can be defined: the initial degree of saturation,  $S_0$ , and the time at which the  
 178 saturation degree abruptly changes from  $S_0$  to  $S_{sat}$ ,  $t^*$ .

179 In real cases, considering the many possible stratigraphical settings, slope geometries, rainfall characteristics and  
 180 boundary conditions of a monitored site [12, 40, 25], it may not be straightforward to identify the above parameters, as  
 181 the two phases may not fully develop where the monitoring points are located (e.g., shallow points at the top of a slope),  
 182 or alternatively they may interact (e.g., points at the toe of a slope and close to the bottom boundary). Fig. 2 and Table 1  
 183 show how to correctly define the six control parameters in such cases, by depicting and considering four types of  
 184 schematic trends (ST), defined as follows:

- 185 – ST0, when the Bottom-Up saturation mechanism fully develops;
- 186 – ST1, when only the wetting front propagation is present (which corresponds to the ideal case of infinite thickness  
 187 of the soil);
- 188 – ST2, when the Bottom-Up saturation starts but does not fully saturate the soil (e.g., change in rainfall conditions);
- 189 – ST3, when the distinction between the first and the second phases is not clear (e.g., proximity of the monitored  
 190 point to the lower boundary) and consequently the Bottom-Up mechanism shows its effect before the wetting  
 191 front propagation phase is over.

192 While the scheme considered for the one-dimensional soil column can generally be considered representative and valid  
 193 for analogous 2D or 3D slope problems, lateral seepage and water flow along the sloping bottom of the domain give rise  
 194 to somewhat different conditions in the degree of saturation in different parts of the model. For example, the highest part  
 195 of the slope may experience only the ST1 case due to lateral flow and gravity-induced seepage to lower parts of the  
 196 domain.



197  
 198 **Fig. 2** Bottom-up saturation mechanism: a) ST0, mechanism completely developed (ideal situation); ST1, wetting front propagation  
 199 only; c) ST2, bottom-up saturation initiated but not fully developed; d) ST3, wetting front propagation interacting with bottom-up  
 200 saturation

201 **Table 1** Computation of the six control parameters needed to calibrate a rainfall infiltration model for the four schematic trends  
 202 identified in Fig. 2

Control parameter	Schematic Trend (ST)			
	0	1	2	3
Initial degree of saturation, $S_0$	$S_0$	$S_0$	$S_0$	$S_0$
Degree of saturation for the apparent steady state conditions, $S_p$	$\frac{1}{N} \sum_{i=t_1^*}^{t_2} S_i$	$\frac{1}{N} \sum_{i=t_1^*}^{t_{End}} S_i$	$\frac{1}{N} \sum_{i=t_1^*}^{t_2} S_i$	$S_{t_{min}}$
Time of initial change in saturation degree, $t_1$	$t_1$	$t_1$	$t_1$	$t_1$
Time of fully saturated conditions, $t_2^*$	$t_2^*$	-	-	$t_2^*$

Rate of change of saturation degree for downward wetting front propagation, $i_1$	$\frac{S_{t_1^*} - S_{t_1}}{t_1^* - t_1}$	$\frac{S_{t_1^*} - S_{t_1}}{t_1^* - t_1}$	$\frac{S_{t_1^*} - S_{t_1}}{t_1^* - t_1}$	$\frac{S_{t_{min}} - S_{t_1}}{t_{min} - t_1}$
Rate of change of saturation degree for bottom-up saturation, $i_2$	$\frac{S_{t_2^*} - S_{t_2}}{t_2^* - t_2}$	-	$\frac{S_{t_{End}} - S_{t_2}}{t_{End} - t_2}$	$\frac{S_{t_2^*} - S_{t_{min}}}{t_2^* - t_{min}}$

## Case study: thoroughly monitored slope model built with uniformly graded sand

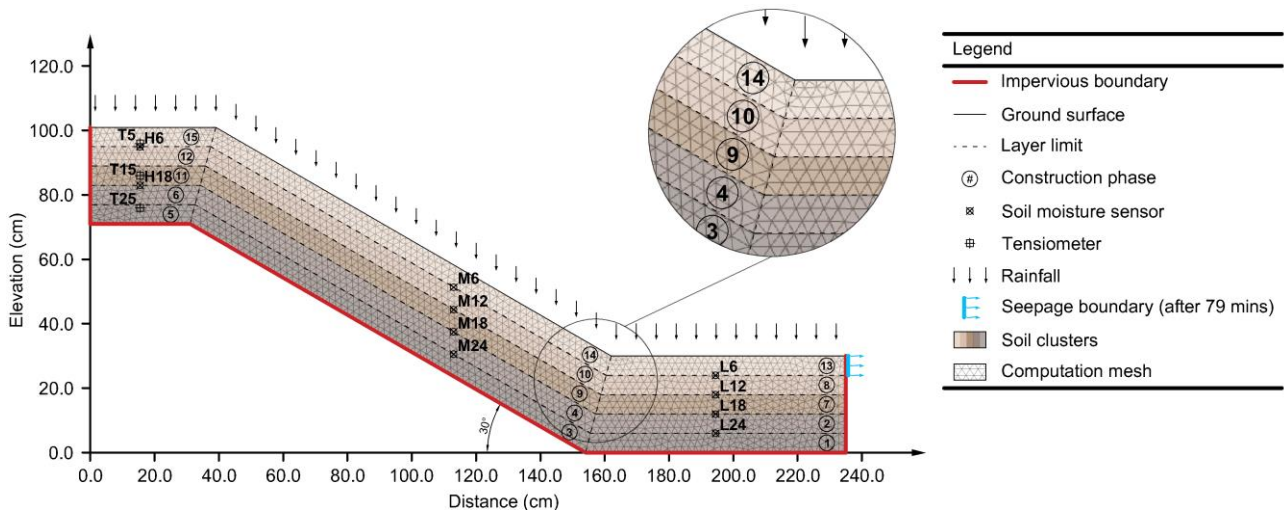
The small-scale slope model considered as a test case has been developed within the four-year research project “Physical modelling of landslide remediation constructions’ behaviour under static and seismic actions” (2018-2022) at the Faculty of Civil Engineering, University of Rijeka, Croatia. The platform for testing slope models exposed to various rainfall conditions under 1 g loading conditions is 1 m wide and consists of 3 main parts: the lower (L), the middle (M), and the upper part (H), which are 0.81, 1.43 and 0.31 m long, respectively. While the lower and upper parts were horizontal, the middle section was inclined at 30°. The slope was built from uniformly graded fine sand in strata of 6 cm with a total of 5 layers and compacted according to Ladd’s under-compaction method [39] to obtain a slope with a relative density of 50% and an initial gravimetric moisture content of 2%. The motivation for using clean sand in the study was to obtain a relatively uniform material for the laboratory tests, which would allow repeatability of the tests through the desired initial conditions in terms of target density or soil moisture content. Another consideration was that the soil with the basic properties reported in Table 2 undergoes negligible volumetric deformation when wetted, while its relatively high hydraulic conductivity ensures that all simulated rainfall infiltrates into the soil without causing surface runoff. These two aspects greatly simplify the interpretation of the results. During the model construction, i.e. after the compaction of each soil layer was completed, the soil moisture sensors TEROS 10 and TEROS 12 (METER Group AG, Munich, Germany), among others, were placed at the predefined locations and connected to data loggers, with the measuring interval set to 1 min. Once the entire model was completed and all sensors and monitoring equipment were installed, the slope model was subjected to a constant rainfall intensity of 79 mm/h. The rainfall intensity used in the study did not consider any specific rainfall event but resulted from the following requirements: no surface runoff should occur during the rainfall simulation, the working pressures in the rainfall simulator should ensure uniform coverage of the entire model, and the impact velocities of the raindrops should not cause surface erosion. These conditions were achieved using 3 axial flow full-cone spray nozzles at a working pressure of 2 bar, mounted 80 cm above the ground level and aligned to ensure uniform rainfall conditions in the model. The platform is also equipped with three drainage lines located on the front Plexiglas sidewall, allowing control of the water level in the lower part of the model. All drainage lines were kept closed until the groundwater level reached the surface of the model at the bottom of the slope, 79 min after the rainfall simulation began. Then the upper drainage line was opened to prevent the water level from rising further and to keep the groundwater in the lowest part of the model at the level of the model surface until the end of the test. Continuously collected data on changes in soil moisture, interpreted using the manufacturer’s generic calibration equation, and occasional readings from analogue tensiometers installed in the upper part of the model provided information on the hydraulic response of the model, while the optical 3D measurement system ARAMIS (GOM mbH; Braunschweig, Germany) and MEMS accelerometers monitored any surface or movements within the model. More details on the flume, model build-up procedure, the rainfall simulator and monitoring sensor network can be found in [55] and [57].

**Table 2** Property of the soil used for the small-scale physical model

Parameter	Value
Specific gravity	$G_s$ (-) 2.7
Particle size	$D_{10}$ (mm) 0.19
Particle size	$D_{60}$ (mm) 0.37
Uniformity coefficient	$CU$ (-) 1.95
Minimum void ratio	$e_{min}$ (-) 0.64
Maximum void ratio	$e_{max}$ (-) 0.91
Saturated hydraulic conductivity	$k_{sat}$ (m/s) 1.6e-4
Initial porosity	$n_{s,i}$ (-) 0.44
Initial relative density	$D_{r,i}$ (%) 50
Initial gravimetric water content	$w_i$ (%) 2

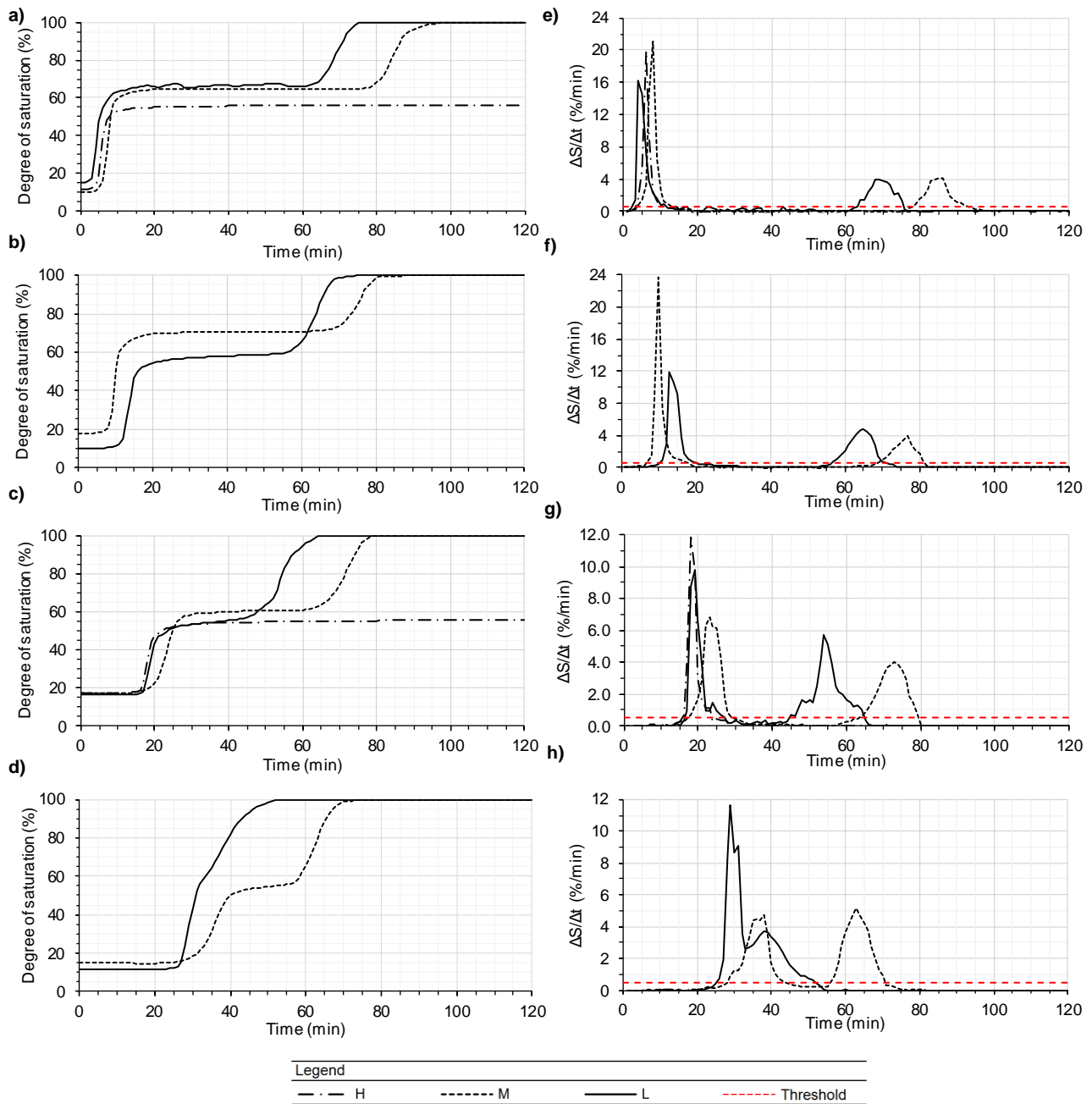
Fig. 3 shows the main characteristics of the physical and numerical models, including the construction stages of the slope, the monitored points equipped with soil moisture (H6, H18, M6, M12, M18, M24, L6, L12, L18, L24) and porewater pressure (T5, T15, T25) sensors, the simulated rainfall conditions and the boundary conditions. Based on the negligible

241 volumetric deformations observed during the 120-minute rainfall simulation, the numerical uncoupled single-phase  
 242 groundwater flow analyses have been implemented adopting the commercial finite element software Plaxis2D [5]. The  
 243 numerical model consists of 3649 15-noded triangular elements and 29767 nodes. The size of the mesh elements varies  
 244 between  $0.3704 \times 10^{-3} \text{ m}^2$  and  $0.047 \times 10^{-3} \text{ m}^2$ , while the hydraulic properties of the soil were described with the model of van  
 245 Genuchten (1980) [30] and an isotropic hydraulic conductivity was assumed. All boundaries except for the ground level  
 246 are set as impervious for a duration of 79 mins. For the later period of the numerical simulation, the first 6 cm of the  
 247 bottom vertical boundary is set as a seepage area (Fig. 3) to allow water to drain from the model, which is consistent with  
 248 the previously described opening of the upper drainage line to prevent the water table in the lower part of the physical  
 249 model from rising above the ground level. A homogeneous rainfall with an intensity of 79 mm/h (i.e.,  $2.2 \times 10^{-5} \text{ m/s}$ ) was  
 250 applied along the ground level of the domain for the entire duration of the analysis (120 min). The rainfall boundary  
 251 condition is applied as a normal component of intensity in relation to the ground surface.  
 252



253  
 254 **Fig. 3** Main characteristics of the physical and numerical models: stratigraphical and stage construction settings, monitored points,  
 255 and boundary conditions assumed for the numerical analysis

256 The results of the physical model in terms of  $S$  are reported in the Fig. 4a,b,c,d. In addition, Fig. 4e,f,g,h show the first  
 257 derivative of  $S$ ,  $S'$ , over the time where the Top-Down wetting front and Bottom-Up saturation mechanisms are clearly  
 258 distinguishable. Generally, the first peak is higher than the second one for all the points monitored, indicating that the  
 259 Top-Down wetting front is usually sharper than the Bottom-Up saturation. Moreover, the amplitude of the first peak is  
 260 smaller than that of the second one, implying that the wetting front advancement is exhausted faster than the saturation  
 261 one. This evidence is also visible in  $S$  curves, even if it is not easy to assess. Furthermore, for point L24 in Fig. 4d, is  
 262 noticeable that the Top-Down wetting front is not totally exhausted when the upward saturation mechanism initiates and  
 263 reaches the L24 point (ST3 saturation mechanism in Fig. 2d). In terms of  $S'$ , once a threshold value has been assigned,  
 264 the times  $t_1$ ,  $t_1^*$ ,  $t_2$ , and  $t_2^*$  can be easily computed. The value of the threshold to be considered depends on the data  
 265 available, soil properties, and boundary conditions. In this study, a threshold value of 0.5 has been considered adequate.  
 266 Once the timing has been assessed, the control parameters are evaluated as reported in Table 1 for each ST.



267

268 **Fig. 4** Measurements of the physical model at the monitored points in the lower part (L, in continuous line), in the middle part (M, in dashed line) and in the upper part (H, in dot-dashed line) in terms of: saturation degree at depths 6 cm (a), 12 cm (b), 18 cm (c), 24 cm (d) from ground level, and corresponding prime derivatives at depths 6 cm (e), 12 cm (f), 18 cm (g), 24 cm (h) from ground level with threshold used to identify control parameters  $t_1$ ,  $t_1^*$ ,  $t_2$ , and  $t_2^*$  (see also Fig. 2)

272

## Results and discussions

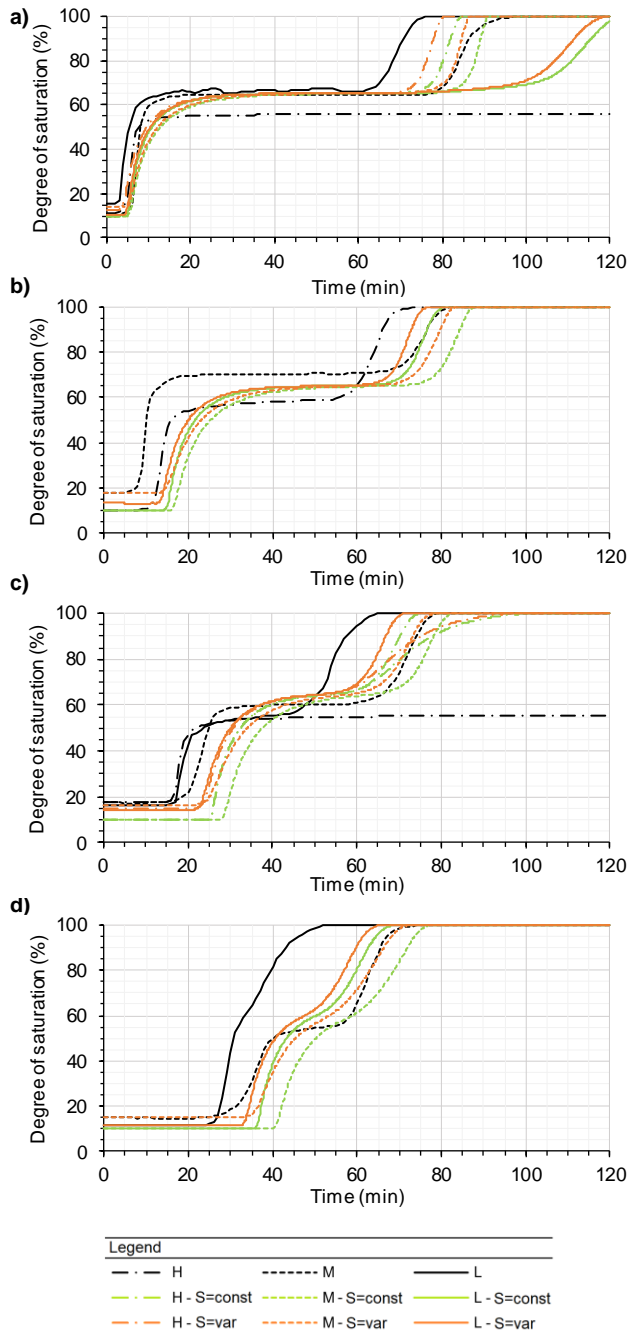
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### Initialisation of the numerical model

274 In the first numerical simulation, a homogeneous slope in terms of soil properties and initial degree of saturation has been considered. The hydraulic properties of the soil used for this analysis are those given as 'base case' in Table 3. In this case, as shown in Fig. 5, the degree of saturation is constant and equal to 10% for all the points monitored (green lines), but slightly distant from the actual conditions observed in the physical model (black lines). Therefore, a calibration procedure was applied in a second numerical simulation to obtain more realistic initial conditions for the moisture distribution in the numerical model (orange lines), i.e. conditions similar to those observed in the experiment. Indeed, knowing the volumetric water content in correspondence of each sensor and the measured values of volumetric water content under saturated conditions, it is possible to define an initial degree of saturation for each layer of the slope. The final result is a heterogeneous slope in terms of initial soil moisture distribution, but a homogeneous one in terms of porosity. Fig. 5 shows the effect of the initial soil moisture distribution on the numerical response in terms of the degree of saturation evolution over time. A higher initial degree of saturation speeds up both the initial downward wetting front propagation

284

285 and the Bottom-Up saturation due to: i) higher values of unsaturated hydraulic conductivity; ii) a reduced rainwater  
 286 storage capacity of the soil. However, the  $S_p$  value remained unaltered, indicating that these conditions are only correlated  
 287 with the  $q/k_{sat}$  ratio and soil properties. This evidence is in line with Sun et al. [67] and highlights the importance of a  
 288 correct assumption for the initial conditions. Indeed, it is clear from Eq. (2) that the initial degree of saturation can speed  
 289 up the wetting front advancement and consequently the general response of the slope.



290  
 291 **Fig. 5** Effect of the initial saturation degree for the monitored points in the lower part (L, in continuous line), in the middle part (M,  
 292 in dashed line), and in the upper part (H, in dot-dashed line) in terms of the saturation degree at depths 6 cm (a), 12 cm (b), 18 cm (c),  
 293 24 cm (d) from ground level

## 294 Parametric analysis

295 To evaluate the sensitivity of the results to changes in the unsaturated hydraulic properties of the sand used in the small-  
 296 scale slope, a parametric analysis has been carried out considering 4 parameters:  $n$ ,  $a$ ,  $l$ , and  $k_{sat}$  (Table 3). The base case  
 297 (NM\_BC) is characterized by a set of parameters usually valid for this kind of soil [31, 37] whereas the saturated hydraulic  
 298 conductivity was determined from the constant head laboratory test results. In the other cases (NM\_xx) the four  
 299 parameters were varied once at the time, while keeping the other three constant to the preassigned basic values. The ranges  
 300 of values of the unsaturated hydraulic parameters adopted in this parametric analysis are the following:

- 301 -  $n$  from 2.1 to 4.8;
- 302 -  $a$  from 1.96 kPa to 0.49 kPa (closely approximating the range of air-entry value [42, 26] );
- 303 -  $l$  from  $-1.0$  to  $0.5$ , where the latter value is usually adopted without any distinction between the different soils
- 304 and unsaturated hydraulic properties [51];
- 305 -  $k_{sat}$  from  $5.0 \cdot 10^{-5} m/s$  to  $3.3 \cdot 10^{-4} m/s$  (equivalent to  $180 mm/h$  to  $1188 mm/h$ , respectively).

306 In particular, the upper limit of  $a$  and  $n$  parameters of the SWCC were estimated using the pedotransfer function based  
 307 on the  $D_{60}$  and  $CU$  values of the sand used to build the physical slope model according to Benson et al [3]. The upper  
 308 limit of the  $k_{sat}$  was derived by the  $D_{10}$  value according to Hazen [26].

309 **Table 3** Values of the unsaturated hydraulic properties used in the different cases (NM\_xx) of the parametric analysis

		Base case	Changing a			Changing n			Changing l			Changing k		
ID: NM_		BC	a2.0	a0.7	a0.5	n2.1	n3.9	n4.8	l-1.0	l-0.5	l0.5	k5.0	k10.0	k32.5
Parameters	$S_r$ (%)	5	5	5	5	5	5	5	5	5	5	5	5	5
	$S_{sat}$ (%)	100	100	100	100	100	100	100	100	100	100	100	100	100
	$a$ (kPa)	0.98	1.96 <sup>#</sup>	0.65	0.49	10	10	10	10	10	10	10	10	10
	$n$ (-)	3.0	3.0	3.0	3.0	2.1	3.9	4.8 <sup>#</sup>	3.0	3.0	3.0	3.0	3.0	3.0
	$l$ (-)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-1.0	-0.5	0.5 <sup>§</sup>	0.0	0.0	0.0
	$k_{sat}$ ( $\times 10^{-5}$ m/s)	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	5.0	10.0	32.5 <sup>*</sup>

<sup>#</sup>Benson et al. [3], <sup>§</sup> Mualem [51], <sup>\*</sup> Hazen [32]

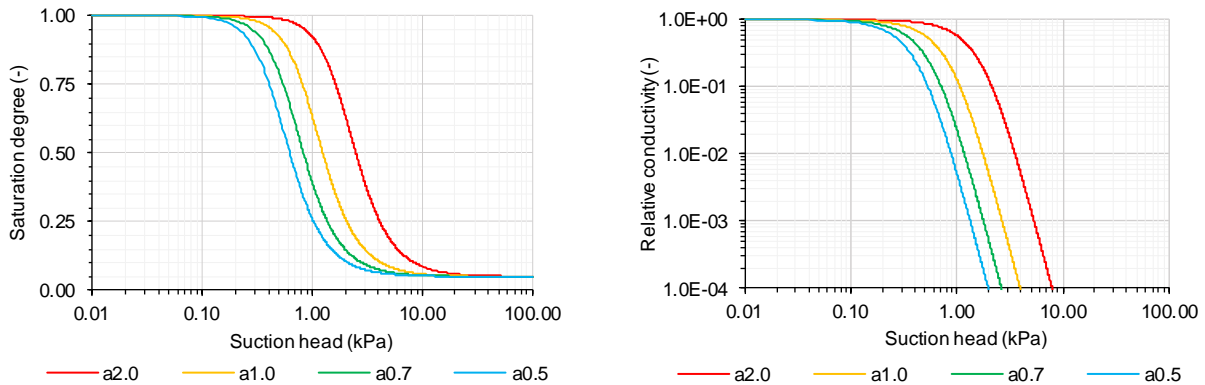
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### 311 Effect of parameter “a”

312 Fig. 6 shows the SWCC and the HCF for the cases examined, indicating that decreasing of the air-entry value ( $a$   
 313 parameter) leads to a decrease of saturation degree at assigned suction and a corresponding decrease of the relative  
 314 conductivity. Fig. 7 shows the response of the numerical model when the  $a$  parameter is varied. For increasing values of  
 315  $a$ , there is a faster downward propagation of the wetting front ( $t_1$  decrease) and a slower Bottom-Up saturation ( $t_2^*$   
 316 increase). As the depth increases, these two phases tend to be closer to each other, and for the highest value of  $a$  sometimes  
 317 they are indistinguishable (i.e., middle and lower parts for  $a = 1.96$  kPa). Moreover, the steeper change in  $S$  for lower  
 318 values of  $a$  suggests a wetting front advancement that, albeit slower, lasts a shorter time (greater  $t_1$  with smaller  $t_1^*$ ). The  
 319 apparent steady state degree of saturation is not significantly affected by this parameter, as all the saturation time series  
 320 tend towards to the same  $S_p$  value.

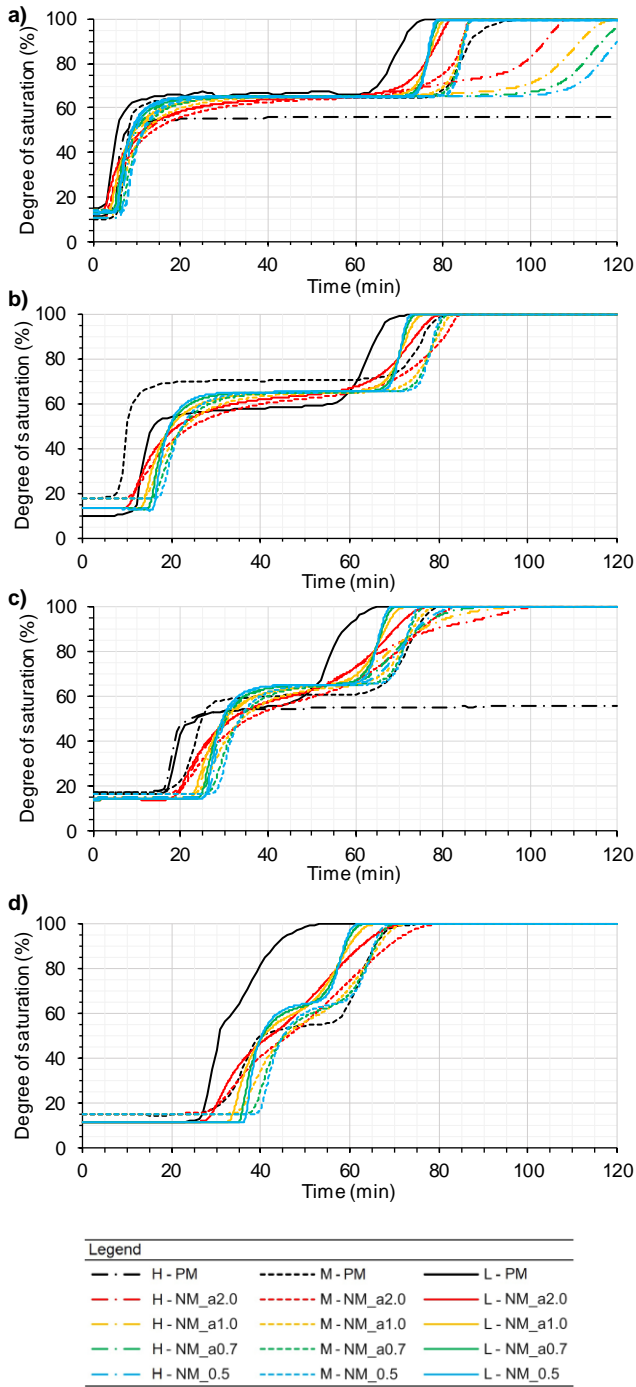
321 With respect to the initial porewater pressure, the strong dependence on  $a$  parameter is evident from Fig. 6, as the change  
 322 in  $a$  causes the shift in SWCC. Consequently, assigning an initial degree of saturation corresponds to assigning an initial  
 323 porewater pressure that increases as  $a$  decreases.

324



325

326 **Fig. 6** SWCC (left) and HCF (right) for the values of the “a” parameter adopted in the parametric analysis



327

328 **Fig. 7** Effect of the variation of the “ $a$ ” parameter for the monitored points in the lower part (L, in continuous line), in the middle  
 329 part (M, in dashed line), and in the upper part (H, in dot-dashed line) in terms of saturation degree at depths 6 cm (a), 12 cm (b), 18 cm  
 330 (c), 24 cm (d) from ground level. [Legend: PM=Physical model, NM\_xx=Numerical model case xx (see Table 3)]

### 331 Effect of parameter “ $n$ ”

332 The  $n$  parameter represents the slope of the SWCC. In this study, four values for  $n$  have been considered, ranging from  
 333 2.1 to 4.8, with a base value of 3.0. This parameter has two different effects in relation to the value of suction (Fig. 8). At  
 334 very low suction, an increase in  $n$  leads to higher values of saturation degree and relative conductivity in the SWCC and  
 335 HCF respectively. In contrast, for higher suction values, the degree of saturation decreases with increasing of  $n$  and,  
 336 consequently, the relative conductivity also decreases.

337 As for the model response, different values of  $n$  have the following effects (Fig. 9):

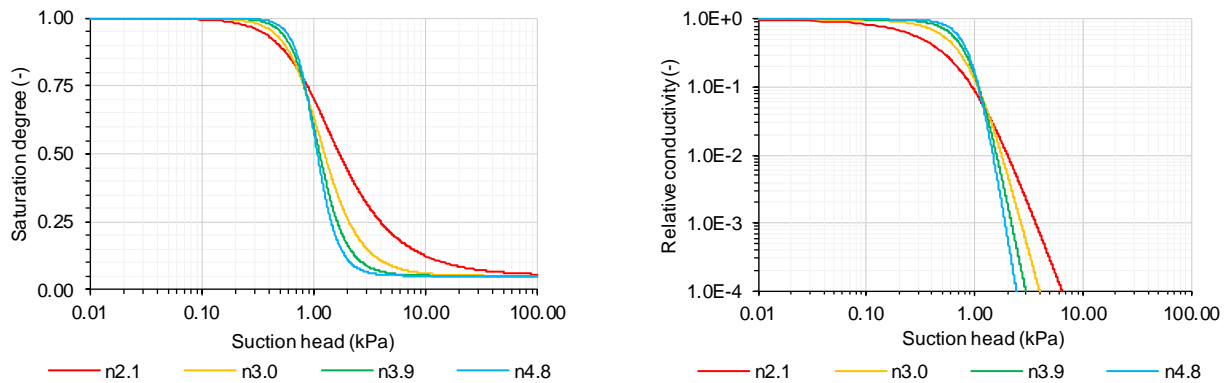
- 338 – Decreasing values of  $n$  produce a delay in the downward wetting front propagation at all depths, although more
- 339 noticeable for deeper points. Similar statements are valid also for the Bottom-Up saturation.
- 340 – For decreasing values of  $n$ , higher  $S_p$  values are obtained. This can be explained by looking at the HCF for
- 341 relatively low suction values (i.e. comprises between 0.0 to 1.0 kPa, in the range of values present in the slope
- 342 after the wetting front passes and before the Bottom-Up saturation reaches the observed point), corresponding
- 343 to lower relative conductivity and thus to a higher water accumulation during the wetting front propagation.
- 344 – In terms of the degree of saturation curve's gradient, lower values of  $n$  lead to smaller values of both  $i_1$  and  $i_2$ ,
- 345 with the Bottom-Up saturation more significantly affected.

346 With regard to the initial porewater pressure, a similar statement can be made for the parameter  $n$  as for the parameter  $a$ .

347 In this case, a decrease in the slope of the SWCC for decreasing values of  $n$  can be observed in Fig. 8, which corresponds

348 to an increase in the initial porewater pressure for a fixed initial degree of saturation (values around 0.15).

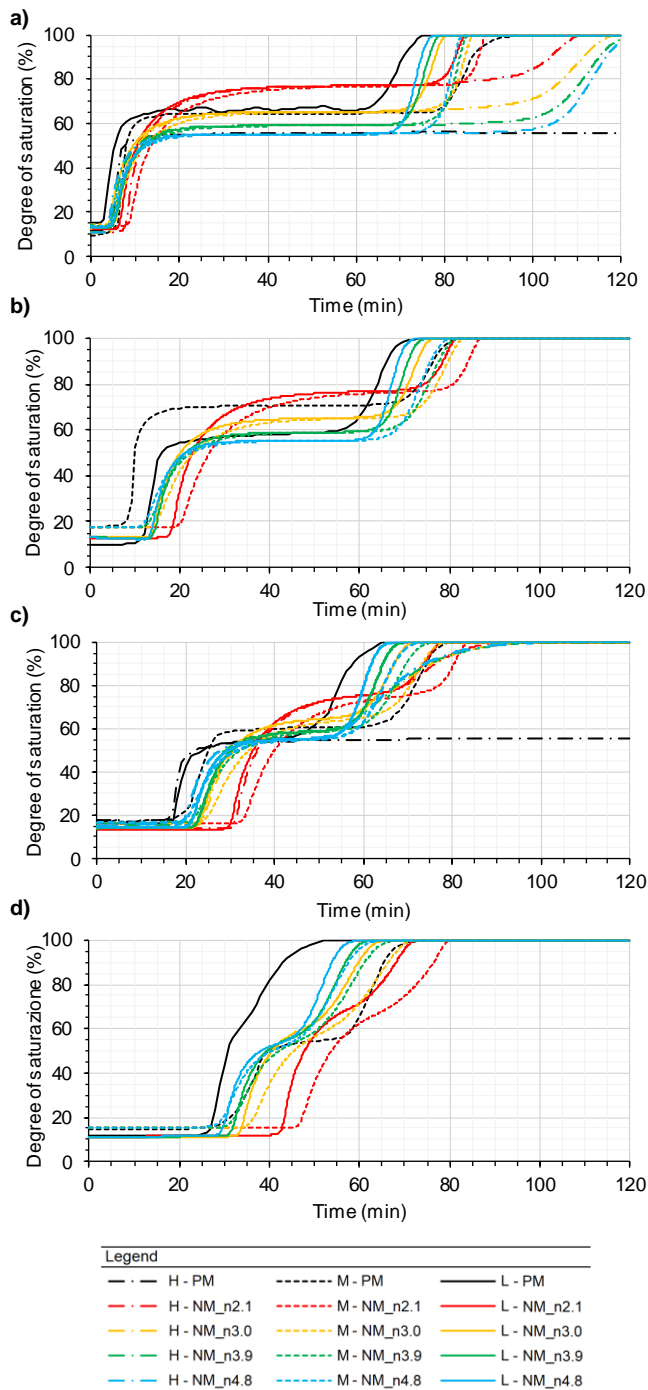
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**Fig. 8** SWCC (left) and HCF (right) for the values of the  $n$  parameter adopted in the parametric analysis



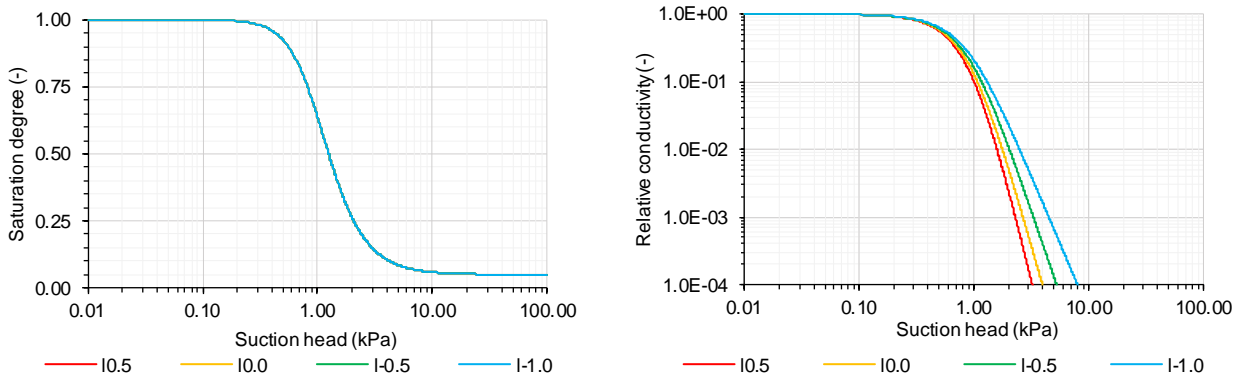
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353 **Fig. 9** Effect of the variation of the  $n$  parameter for the monitored points in the lower part (L, in continuous line), in the medium  
 354 part (M, in dashed line), and in the higher part (H, in dot-dashed line) in terms of saturation degree at depths 6 cm (a), 12 cm (b), 18  
 355 cm (c), 24 cm (d) from ground level. [Legend: PM=Physical model, NM\_xx=Numerical model case xx (see Table 3)]

### 356 Effect of parameter “ $l$ ”

357 According to Eq. (3) and Eq. (4), the  $l$  parameter does not influence the SWCC, but only the HCF. Fig. 10 shows the  
 358 effect of this parameter on the hydraulic conductivity, revealing that decreasing values of  $l$  imply an increase of the  
 359 relative conductivity.

360



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**Fig. 10** SWCC (left) and HCF (right) for the values of the  $l$  parameter adopted in the parametric analysis

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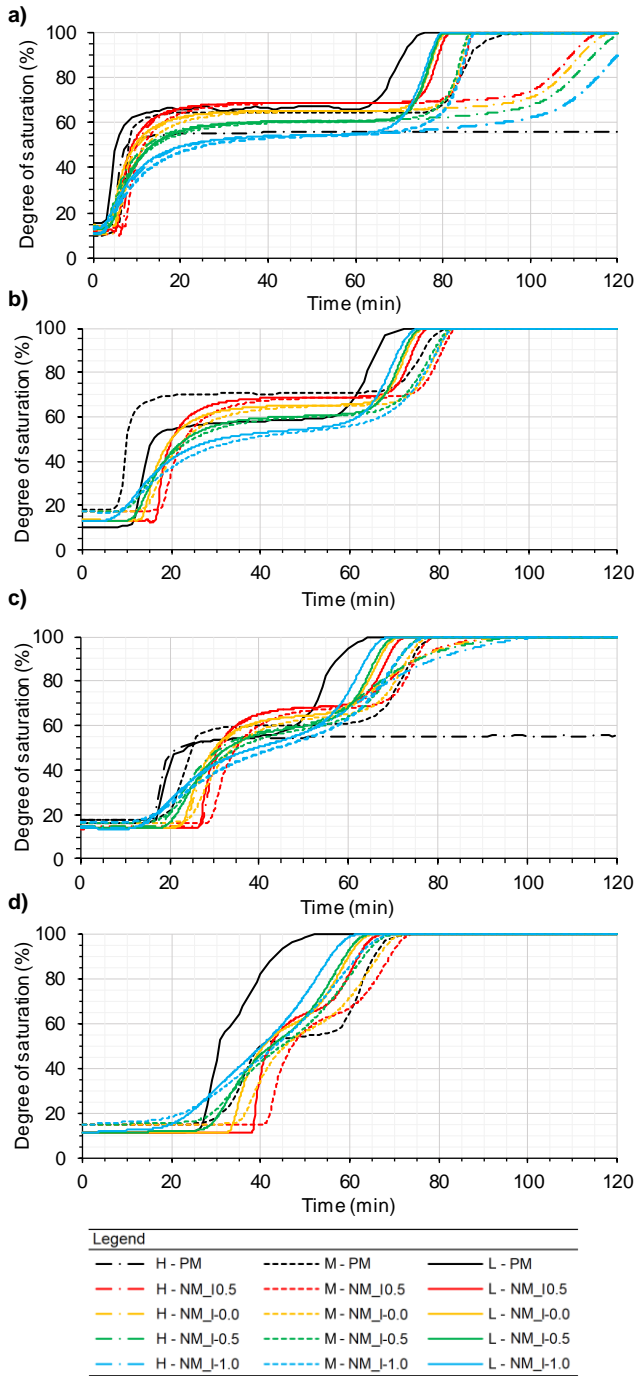
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This parameter does not significantly affect the initial and final state but acts on the timing for reaching the apparent steady state conditions (Fig. 11). Specifically, for increasing values of  $l$ , there is a delay in the changes of the saturation degree due to a lower unsaturated hydraulic conductivity, which decreases the velocity of the propagation of the wetting front, leaving the soil at greater depths unaffected by the infiltrating rainfall for a longer time. This implies higher  $t_1$  values, while  $t_2^*$  is not significantly affected by these changes. Furthermore, the lower conductivity associated to higher value of  $l$ , means that the accumulation of water content during the wetting front advancement is higher, and that translates into a higher degree of saturation of the apparent steady state conditions,  $S_p$ . The rate of change of the saturation degree for the downward wetting front propagation,  $i_1$ , is also strongly dependent on the  $l$  value: increasing values of  $l$  imply increasing values of  $i_1$  for all the monitored points. In contrast,  $i_2$  is not notably influenced by this parameter. Thus, the  $l$  parameter mainly influences the downwards advancement of the wetting front, whereas the Bottom-Up saturation is not clearly impacted by its changes.



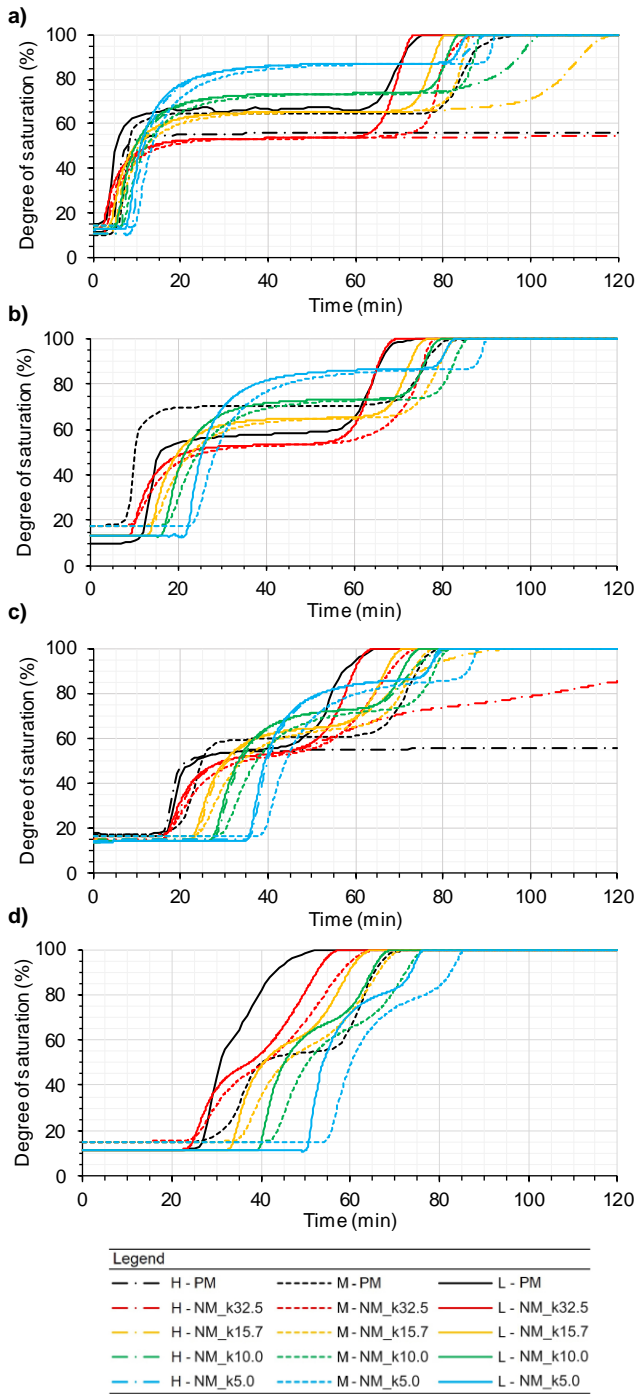
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375 **Fig. 11** Effect of the variation of the  $l$  parameter for the monitored points in the lower part (L, in continuous line), in the middle part  
 376 (M, in dashed line), and in the upper part (H, in dot-dashed line) in terms of saturation degree at depths 6 cm (a), 12 cm (b), 18 cm (c),  
 377 24 cm (d) from ground level. [Legend: PM=Physical model, NM\_xx=Numerical model case xx (see Table 3)]

### 378 Effect of parameter “ $k_{sat}$ ”

379 Fig. 12 shows the effect of the saturated hydraulic conductivity on the slope response in terms of saturation degree.  
 380 Reasonably, this parameter is able to control both the downward wetting front advancement and Bottom-Up saturation  
 381 mechanism. In particular, lower values of  $k_{sat}$  slow down the Bottom-Up saturation and the advancement of wetting front  
 382 occurs at higher degrees of saturation. Particularly interesting is the case of the highest considered value of  $k_{sat}$ , which  
 383 is able to simulate almost perfectly  $t_2^*$  for points L6, M12, L12, L18, L24, and to maintain point H6 into the apparent  
 384 steady state conditions. However, in this case, the numerical values of  $t_1$  are slightly smaller than the physical ones. In  
 385 terms of rate of changes, a lower  $k_{sat}$  provides higher values of both  $i_1$  and  $i_2$ . Indeed, as  $q/k_{sat}$  increases, the  
 386 propagating wetting front tends to become closer to the Top-Down saturation described by the Green-Ampt model, and  
 387 therefore the apparent steady state degree of saturation ( $S_p$ ) increases and the variation of the saturation degree is also  
 388 faster (increasing of  $i_1$ ).

389 Both the  $l$  and  $k_{sat}$  parameters influence only the timing for the porewater pressure to reach the “plateau” and the final  
 390 conditions, but they have no significant effect on the initial porewater pressure conditions.



391  
 392 **Fig. 12** Effect of the variation of the  $k_{sat}$  parameter for the monitored points in the lower part (L, in continuous line), in the middle  
 393 part (M, in dashed line), and in the upper part (H, in dot-dashed line) in terms of saturation degree at depths 6 cm (a), 12 cm (b), 18 cm  
 394 (c), 24 cm (d) from ground level. [Legend: PM=Physical model, NM\_xx=Numerical model case xx (see Table 3)]

### 395 Best-fit simulation

396 According to what was described in the previous sections, once the contribution of each parameter to the hydraulic  
 397 response of the slope is assessed, it is possible to use the experimentally obtained data to calibrate the numerical model  
 398 and obtain the unsaturated hydraulic parameters of the investigated soil. To do so, a series of trial-and-error analyses were  
 399 carried out. The best-fit simulation (NM\_BestFit) is characterized by the set of parameters reported in Table 4.

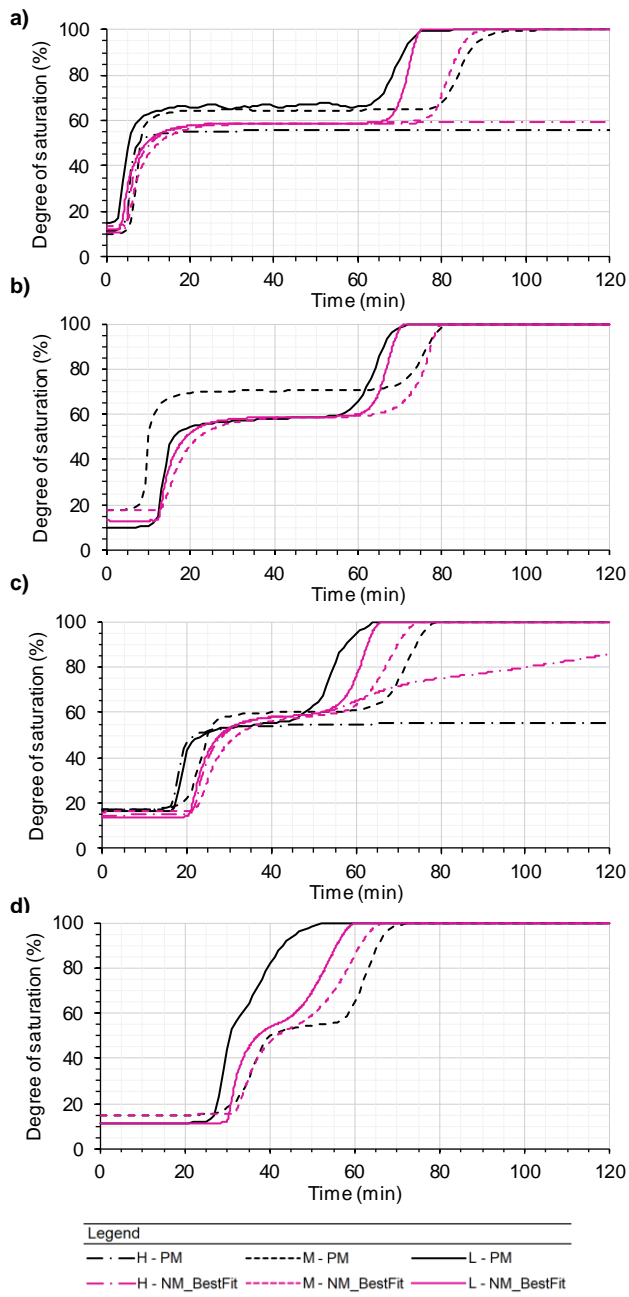
400 **Table 4** Set of best-fit hydraulic parameters for the optimized analysis of the case study

Parameter	NM_BestFit
$S_r$ (%)	5
$S_{sat}$ (%)	100
$n$ (-)	2.7
$a$ (kPa)	0.82
$l$ (-)	0.25
$k_{sat}$ (m/s)	$3.3 \cdot 10^{-4}$

401

402 The results of the analysis are shown in Fig. 13 and Fig. 14. Aside from a few discrepancies, the trends reported for the  
403 numerical model track the physical measurements adequately well. It is evident that the wetting front propagation and the  
404 Bottom-Up saturation timing ( $t_1$  and  $t_2^*$ ) are properly simulated. In addition, the average rate of change of the saturation  
405 degree for the Bottom-Up saturation ( $i_2$ ) also agrees well with the physical evidence. The rate of change of the degree of  
406 saturation of the wetting front propagation ( $i_1$ ) is slightly underestimated and, most notably, the  $S_p$  in the numerical model  
407 value is almost constant for all the points examined, differently from what is reported by the physical measurements.  
408 Given the homogeneous assumption adopted for the numerical model, the latter results are to be expected and, therefore,  
409 the fit for  $S_p$  at different depths and positions cannot be further improved by calibration in this model.

410 It should be noted that the accuracy of the calibrated model parameters is highly dependent on the suitability of the  
411 monitoring equipment to provide instantaneous data on the monitored variables in a given soil type. While there are  
412 sensors and measurement techniques that can cover a wide measurement range in fine-grained soils with low hydraulic  
413 conductivity, long equilibration times and slow response, as well as relatively low accuracy of the sensors when measuring  
414 in the wet range of the SWCC, may indicate seemingly slower changes in the monitored data and generally provide  
415 erroneous data sets for the calibration procedure. While the optimisation process is supported by the parametric analysis  
416 performed, consistency between the measured and calculated results should be ensured for multiple points throughout the  
417 domain and the parameters obtained should be checked for their soundness and representativeness for the soil material  
418 studied.



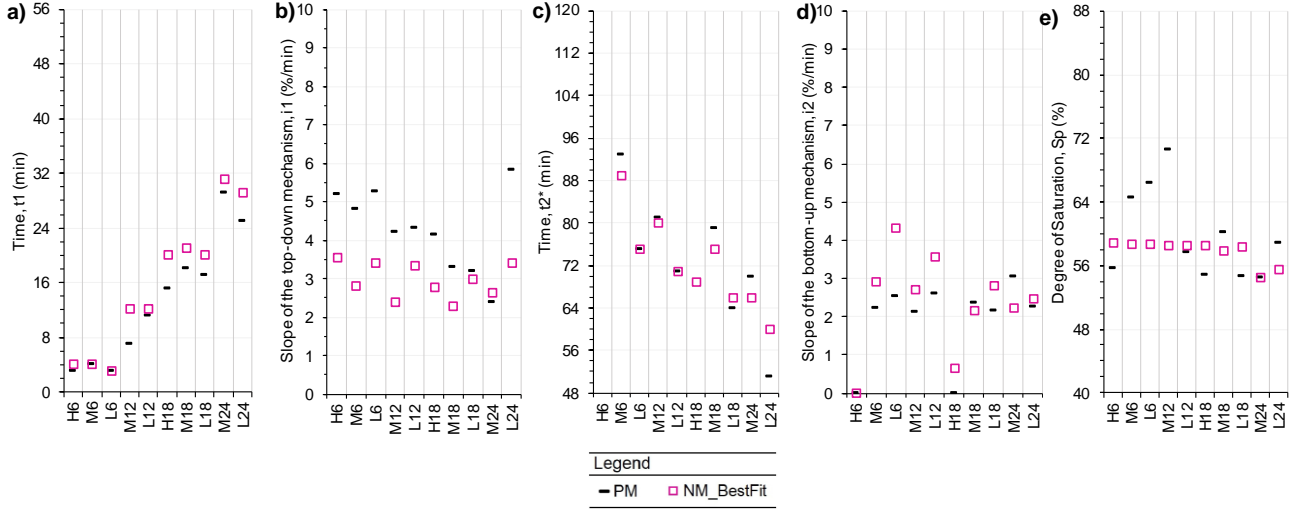
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**Fig. 13** Comparison of Best-Fit simulation (in pink) and measurements (in black) in correspondence of the monitored points in the lower part (L, in continuous line), in the middle part (M, in dashed line), and in the upper part (H, in dot-dashed line) in terms of the saturation degree at depths 6 cm (a), 12 cm (b), 18 cm (c), 24 cm (d) from the ground level



423

424 **Fig. 14** Comparison of the optimized analysis (NM\_BestFit, pink) with the physical measurements (PM, black) in the monitored  
 425 points (see Fig. 3 for ID and location) for the following control parameters: a)  $t_1$ ; b)  $i_1$ ; c)  $t_2^*$ ; d)  $i_2$ ; e)  $S_p$

## 426 Evaluating the performance of the numerical analysis

427 To evaluate the agreement between the numerical model results and the physical measurements, as well as to prove the  
 428 validity of the procedure herein proposed and discussed, two different approaches have been considered. The first one  
 429 assesses the errors between numerical and physical data in a standard way, as is most commonly done for time series  
 430 comparison, adopting the following two global error metrics:

- 431 • Root Mean Square Error:

$$432 \quad RMSE = \sqrt{\frac{1}{N} \cdot \sum_{i=1}^N (S_i^{NM} - S_i^O)^2}$$

- 433 • Mean Prediction Error:

$$434 \quad MPE = \frac{100\%}{N} \cdot \sum_{i=1}^n \frac{S_i^{NM} - S_i^O}{S_i^O}$$

435 where  $S_i^{NM}$  is the computed value of the degree of saturation at time  $t_i$ ,  $S_i^O$  is the measured value of the degree of saturation  
 436 at time  $t_i$ , and  $N$  is the number of points considered in the time series.

437 In a second approach, the errors between the numerical models and the physical observations are computed adopting five  
 438 residual values, considering five of six key control parameters previously defined, as follows:

$$439 \quad e(t_1) = t_1^{NM} - t_1^O$$

$$440 \quad e(i_1) = i_1^{NM} - i_1^O$$

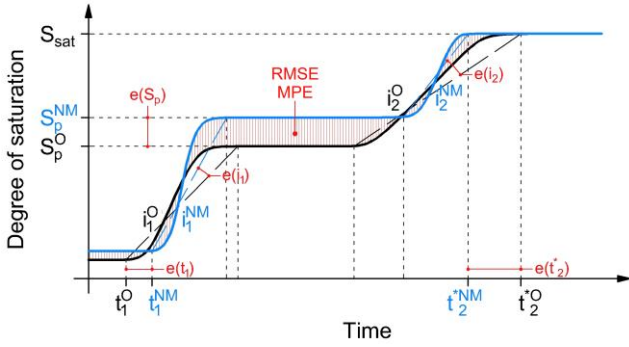
$$441 \quad e(t_2^*) = t_2^{*NM} - t_2^{*O}$$

$$442 \quad e(i_2) = i_2^{NM} - i_2^O$$

$$443 \quad e(S_p) = S_p^{NM} - S_p^O$$

444 where the apex  $NM$  refers to the values of the control parameters in the numerical model and the apex  $O$  refers to the  
 445 values of the control parameters in the observations.

446 Fig. 15 graphically depicts the meaning of these errors. In particular, the RMSE and MPE global metrics are related to  
 447 the area comprised between the numerical results and the physical measurements time series, whereas the five adopted  
 448 residual values represent the linear distance between the observed and the simulated values for  $t_1$ ,  $t_2^*$ , and  $S_p$ , and the  
 449 angular difference between the two values for  $i_1$  and  $i_2$ .



450

451 **Fig. 15** Schematic comparison between the numerical results (in blue) and the physical monitoring (in black) in terms of degree of  
 452 saturation time series, with a graphical representation of how the errors are considered in this study

453 Table 5 reports the values of the RMSE and MPE global error metrics for each monitored point and their average value  
 454 (AVRG). Both indicators suggest a similar trend between the examined points, revealing that the model performs best at  
 455 points H6 and L12 (i.e., RMSE equal to 3.26 and 3.72, and MPE equal to 3.64 and 0.66, respectively), while the worst  
 456 performance is observed at points H18, M12 and L24. This is consistent with the evaluation carried out by means of the  
 457 five residual values herein proposed (Table 6), which also indicates that points H6, L12, and M24 are the best simulated  
 458 ones. Considering the single residuals, and their physical meaning: the shallowest points (i.e., H6, M6, L6, L12) show an  
 459 almost perfect match with the wetting front arrival time,  $t_1$ ; points L18 and M24 well relate to the duration of the wetting  
 460 front advancement, represented by  $i_1$ ; and points M18 and L24 are the ones showing the best modelling fit with the  
 461 duration of the final saturation phase, related to the control parameter  $i_2$ . Good agreement between the numerical and  
 462 physical models with respect to  $S_p$  was observed for all monitored points except for M12, for which an error of about  
 463 12% is computed.

464

465 **Table 5** Global error metrics adopted for evaluating the performance of the models in terms of the degree of saturation data at different  
 466 monitored points, also considering their average values (AVRG)

Metrix	H6	M6	L6	M12	L12	H18	M18	L18	M24	L24	AVRG
RMSE (%)	3.26	6.08	7.75	14.10	3.72	17.27	6.58	7.75	5.66	13.00	8.52
MPE (%)	3.64	-3.32	-9.24	-13.75	0.66	15.13	-3.57	-7.11	2.55	-9.15	-2.42

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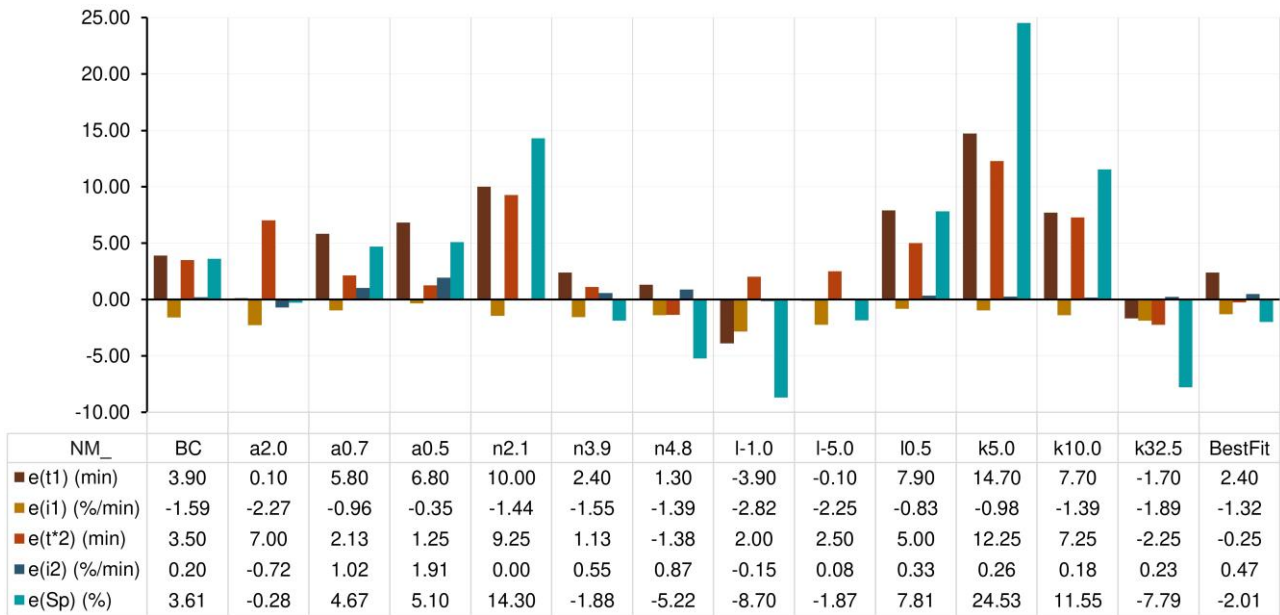
468 **Table 6** Model performance evaluation by means of five residual values representative of five key control indicators related to the  
 469 degree of saturation data at different monitoring points, also considering their average values (AVRG)

Metrix	H6	M6	L6	M12	L12	H18	M18	L18	M24	L24	AVRG
$e(t_1)$ (min)	1	0	0	5	1	5	3	3	2	4	2.40
$e(i_1)$ (%/min)	-1.65	-1.99	-1.86	-1.86	-0.99	-1.38	-1.00	-0.20	0.22	-2.44	-1.32
$e(t_2^*)$ (min)	-	-4	0	-1	0	-	-4	2	-4	9	-0.25
$e(i_2)$ (%/min)	-	0.71	1.75	0.60	0.95	-	-0.21	0.63	-0.85	0.20	0.47
$e(S_p)$ (%)	3.18	-6.00	-7.69	-12.09	0.85	3.71	-2.39	3.67	-0.05	-3.30	-2.01

470

471 Finally, the average values of the five residuals are compared in Fig. 16 for all the analyses considered. As it is clearly  
 472 visible, the range of errors is mostly contained within a relatively small range of values, even though significant residuals  
 473 are computed for the analyses NM\_n2.1, NM\_k4.3, and NM\_k8.6. Although the best fit model seems to be similar to  
 474 NM\_n3.9 and NM\_n4.8, these three models are considerably different in the topmost part of the slope (H), where only  
 475 the NM\_BestFit model does not reach the fully saturated conditions (in agreement with the physical measurements).

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**Fig. 16** Average residual error values for the different cases of parametric analysis and for the best-fit model

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## Conclusions

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A new methodological approach has been proposed to calibrate the unsaturated hydraulic properties of a coarse-grained soil undergoing a wetting process, using water content measurements on a well-monitored physical model of a slope subjected to constant rainfall infiltration. The soil considered in the study has negligible volumetric deformations when subjected a wetting process, and thus the interpretation of the experimental results justifies the use of an uncoupled numerical modelling approach. The procedure is built around the wetting front concept and uses six control parameters to guide the trial-and-error calibration of the numerical model of the slope. In particular, the first derivative of the evolution over time of the degree of saturation is used to define, objectively by means a properly set threshold value, the key times of the infiltration process in the locations where monitoring data are available. Unlike more traditional error evaluation methodologies, the proposed approach focuses on the peculiar characteristics of the rainfall infiltration and soil saturation mechanisms, to effectively guide the calibration procedure from a geotechnical point of view. The validity of the outcome of the calibration has been also verified adopting standard time-series error metrics, i.e., comparing the time-dependent degree of saturation measures in the different monitored points with the corresponding model results. In conclusion, this study clearly highlighted and quantified the significant influence of SWCC and HFC in controlling the soil response in a slope subjected to constant and prolonged rainfall infiltration.

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## Acknowledgements

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502

## Data availability statement

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The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

505

## Declarations

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The authors have no relevant financial or non-financial interests to disclose. The authors declare no conflict of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.

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Symbol	Description
$a$	Van Genuchten parameter
$C_c$	Capillarity coefficient
$CU$	Uniformity coefficient
$D_{10}$	Particle diameter corresponding to 10% cumulative undersize particle size distribution
$D_{60}$	Particle diameter corresponding to 60% cumulative undersize particle size distribution
$d$	Rainfall duration
$D_{r,i}$	Initial relative density
$e(x)$	Residual error of the control parameter $x$
$e_{max}$	Maximum void ratio
$e_{min}$	Minimum void ratio
$G_s$	Specific gravity
$h$	Ponding head
$h_p$	Ponding head
$i_1$	Rate of change of saturation degree for downward wetting front propagation
$i_2$	Rate of change of saturation degree for bottom-up saturation
$k_{sat}$	Saturated hydraulic conductivity
$l$	Hydraulic conductivity function parameter
$m$	Van Genuchten parameter
$n$	Van Genuchten parameter
$n_s$	Soil porosity
$n_{s,i}$	Initial soil porosity
$q$	Rainfall intensity
$q_0$	Surface flux at steady state conditions
$q_1$	Surface flux under transient conditions
$S$	Degree of saturation
$S'$	First derivative of the degree of saturation
$S_0$	Initial degree of saturation
$S_e$	Effective degree of saturation
$S_{in}$	Degree of saturation below the wetting front
$S_p$	Degree of saturation for the apparent steady state conditions
$S_{q_0}$	Degree of saturation at $q_0$
$S_{q_1}$	Degree of saturation at $q_1$
$S_r$	Residual degree of saturation
$S_{sat}$	Degree of saturation at fully saturated state
$t^*$	Time of abruptly changing of degree of saturation
$t_1$	Time of initial change in saturation degree
$t_2^*$	Time of fully saturated conditions

Symbol	Description
$t_{sat}$	Saturation time
$w_i$	Initial gravimetric water content
$z_{sat}$	Depth of the saturation front
$z_w$	Depth of the wetting front

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711 **Appendix B. List of abbreviations**

Abbreviation	Description
<i>AVRG</i>	Average value
<i>HFC</i>	Hydraulic Conductivity Function
<i>MPE</i>	Mean Predicted Error
<i>N</i>	Number of observations
<i>NM</i>	Numerical Model
<i>PM</i>	Physical Model
<i>RMSE</i>	Root Mean Square Error
<i>ST</i>	Schematic Trend
<i>SWCC</i>	Soil Water Characteristic Curve
<i>TS</i>	Threshold value

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