Abstract

Quantum resource theory focuses on understanding and quantifying thevarious types of physical resources to perform specific tasks, protocols, and operations in quantum engineering and quantum technologies that cannot be achieved by classical means.

The main goal of this PhD thesis is to explore how quantum resources can be exploited and characterized in different contexts of quantum technologies and quantum engineering. We begin by reviewing the resource theories of two fundamental features of quantum systems – coherence and entanglement – and their operational relevance. Building on these concepts, we introduce a geometric framework for defining bona fide measures of quantum nonlocality, a necessary step toward establishing a consistent resource theory of Bell nonlocality. The framework provides a unified geometric measure applicable to arbitrary Bell inequalities and measurement scenarios, allowing for explicit evaluation when- ever the local set is characterized. Structural results are obtained for several important classes of states: the closest local states to Werner and isotropic states are shown to share the same symmetry as the original state, and, in the two-qubit case, the closest local state to any Bell-diagonal state is itself Bell-diagonal. Leveraging these results, for such states and for the Clauser-Horne-Shimony-Holt (CHSH) and Collins-Gisin-Linden-Massar-Popescu (CGLMP) inequalities, we derive explicit expressions for the geometric measures of nonlocality using various distance metrics. Building on these concepts, we derive an exact uncertainty–purity relation valid for arbitrary finite-dimensional quantum states. While standard uncertainty relations are formulated for pairs of non-commuting observables, generalized uncertainty relations involving multiple observables have also been developed. In this thesis, the total uncertainty is defined over all local SU (N) observables and it is shown to equal the sum of the purities of the global and reduced states, expressing an uncertainty conservation law. This equality reveals a direct quantitative link between uncertainty, purity, and entanglement, from which we derive a general criterion for k-separability that serves as a necessary–and for pure states, sufficient–condition, as well as experimentally friendly for detecting entanglement.

In the second part, we address the problem of quantum state discrimination, a fundamental task in quantum information and metrology. After reviewing the principles of optimal discrimination and the Helstrom bound – illustrated through the well-known example of quantum illumination – we extend the analysis to novel scenarios. In particular, we study the discrimination between two-level quantum systems using different optical resources, showing that also in this case entangled two-mode squeezed light achieves the best performance compared to coherent and single-mode squeezed states. Finally, we apply the same theoretical framework in particle physics to address one of the fundamental open problems in the field: the nature of the neutrino, namely whether it is a Dirac or a Majorana particle. We investigate this question within our resource-theoretic approach, analyzing the ultimate discrimination between the two hypotheses based on the neutrinos' internal degrees of freedom, such as spin and chirality, and determining the optimal measurement strategy that maximizes their distinguishability.

The third part of the thesis focuses on the simulation of complex classical systems using quantum architectures. First, we establish a connection between multiphoton quantum interference in linear-optical networks and Hopfield-like Hamiltonians of classical neural networks. In particular, we demonstrate that a system of N_{ph} indistinguishable photons distributed over M modes realizes a p-body Hopfield Hamiltonian with $p = 2N_{ph}$, enabling the photonic simulation of memory retrieval and spin-glass phases. Building on this idea, we propose a general and platform-independent framework for simulating Hopfield neural networks using generic n-qubit systems. In this approach, the effective coupling terms naturally emerge from the coherence properties of the quantum density matrix, revealing quantum coherence as the essential resource enabling an exponential mapping from n qubits to 2^n classical neurons. Moreover, within the same formalism, we develop a specific protocol that achieves an exponential quantum speedup in simulating the network dynamics, thereby establishing a direct operational manifestation of the underlying resource.

Overall, the thesis establishes deep conceptual and operational connections between quantum resources and their applications in quantum discrimination, quantum metrology, and quantum simulation. These results contribute to the development of a unified framework for quantum resource engineering, providing both theoretical insight and practical tools for the design of future quantum technologies.