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Sponsorship for the Sustainability of Historical-Architectural Heritage: Application of a Model's Original Test Finalized to Maximize the Profitability of Private Investors

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Abstract: This paper deals with private sponsorship as a tool for the redevelopment of Italy's vast wealth of historical-architectural public heritage sites. Italian law provides for the stipulation of sponsorship agreements by and between public sector agencies or entities (which need financial resources to restore or re-qualify property) and private sector investors (which guarantee the capital sought by the public sector in exchange for significant returns on their investments) raising various economic issues, particularly in connection with the profitability thresholds for private sector investors in return for the public sector's use of their financial resources. In this paper, the authors focus on how private sector investors determine how much of their businesses turnover constitutes the optimal percentage level of overall income that may be invested in sponsorships to maximize business profitability. For this purpose, a model based on past works on the subject (Bucci et al., 2003) has been chosen by the authors. Such model gives a solution for verifying a sponsorship's profitability. This model is static and is applicable to single-product companies that invest in sponsorships, under the theories of monopolistic competition and of Cobb-Douglas production function. Our objectives are to present this model, explaining in detail the mathematical steps, simplifying the model where possible in order to reduce the levels of complexity in its application, and finally to apply it to real case scenarios of cultural sponsorships.

Keywords: redevelopment of public assets; sponsorship; economic models; business profit

1. Introduction

The cultural heritage sector is of primary importance in Italy, a country that has as many as 53 sites identified as World Heritage Sites by UNESCO (United Nations Educational, Scientific and Cultural Organization), with almost 60,000 archaeological sites and nearly 5000 museums.

Despite the importance of Italy's huge cultural heritage sector, the economic crisis has forced the government to make significant cuts in public funding. In 2014, public spending on cultural services, part of which is represented by expenditure on the protection and enhancement of cultural heritage, was equal to 0.32% of GDP: this value is lower than the EU average (0.46%) and, in proportion, less than half than that of France (0.77%). In a ranking based on this measure, Italy is 24th among the 28 countries of the European Union, only ahead of Portugal, the United Kingdom, Cyprus, and Greece [1], making recourse to private funding indispensable in the protection of the country's cultural heritage.

In recent years, the Italian government has been regulating and promoting innovative public-private partnerships to support the protection of cultural heritage, including sponsorships (regulated by the Decree of the Ministry of Cultural Heritage and Activities and Tourism of

19 December 2012, *Approval of the technical rules on sponsorship of cultural products*). By means of private sponsorships, private sector investors may now provide public entities or agencies with funding (or, alternatively, with products and/or services), with the public sector agency or entity retaining governance over the regeneration/restoration work, ensuring the protection, enhancement, and management of the assets concerned [2–4].

In statistical terms, sponsorships for culture and entertainment increased in 2013 to 159 million, 6% more than in 2012 [5].

There are many examples of private funding operations aimed at regeneration and restoration of world famous monuments. The best-known case is certainly the sponsorship by Diego Della Valle's Tod's for the restoration of the Coliseum in Rome [6,7].

Generally speaking, for-profit private companies invest in sponsorships expecting substantial returns on their investments in terms of brand awareness and advertising.

Regarding this aspect, two purely economic issues related to each other are substantial.

The first is the economic return (in monetary and image terms) that such companies can obtain.

The second is the definition of financial indicators through which a state agency or entity managing or owning a particular cultural heritage site can establish adequate prices for the exploitation of intellectual property (IP) rights. On this issue, there is little academic research; this subject needs to be investigated further, also considering past experiences of public administrations, and it will soon be the subject of an ad hoc study.

From a disciplinary point of view, the problem of evaluating the effectiveness of sponsorships conducted under present laws has not been definitively addressed being absent a single specific base theory.

This lack in current scientific draw is particularly felt in the field of value-based management, as the value of companies is increasingly influenced by intangible elements compared to accounting elements [8,9]. In this regard, the paper proposes that there may be a link between capital invested in sponsorships and achievable operating results, expressed also in financial terms. As such, the disruption in real operating results can be concretely justified and proved.

The adaptation of sponsorship mechanisms and their outcomes in the definition of business models for companies operating in the sector is still needing to be investigated.

With regard to this issue, one of the theoretical models proposed by current scientific studies [10] is presented by the authors of this paper in order to study its functioning, highlight its features, and apply it to real cases of sponsorships. In particular, sponsorships have been considered as intangible production factors which, together with labor and capital, form part of the production functions that contribute to a company's productivity. Starting from this theory, we have tried to evaluate optimal levels of sponsorship such as to maximize a company's profits by considering a Cobb-Douglas production function.

The above model was applied to Caffè Moreno s.r.l., a company active in cultural sponsorships through its participation in the Monumentando Napoli project (Italy). Through this initiative, the City of Naples sought interested parties to enter into a technical sponsorship contract for the restoration of some monuments of the city. In particular, Caffè Moreno s.r.l. funded the restoration of the "Fontana del Carciofo" in Piazza Trieste e Trento.

2. State of the Art

The theoretical models through which the first economic issue (quantification of the economic return to the sponsoring company) is described are, in general, aimed at achieving two objectives: the a posteriori evaluation of the effects of the sponsorship activities in terms of economic return [11–17] and the a priori determination of the optimal number of events to be sponsored to maximize company profits.

Through the a posteriori models, a series of indexes (*contact cost*, *number of attendance*, *average attendance time*, *interest index*, *attention index*) are determined, which are sponsorship performance indicators. These indicators are usually synthesized in a single index, which is the *efficiency of sponsorship action*. However, these models, though widely used by companies, may in some cases lead to contradictory results, especially if too small sample sizes are chosen.

Regarding a priori models concerned with determining how much to effectively invest in a given sponsorship in order to maximize profits, only those models proposed by Bucci, Castellani, and Figini are taken into account in this paper, such as "L'investimento in sponsorizzazione delle imprese: un'analisi economica in termini statici e dinamici (2003)", because they are the only models that deal with the issue of profit optimization applied to cultural heritage sponsorships. Other similar models, applied to companies that invest in sponsorships, were not found in scientific literature.

In the following subsection, we will proceed with a description of these models.

2.1. The Evaluation of the Sponsorship's Optimal Level to Maximize Company Profits

As anticipated in the introduction, the goal of the paper is to examine and apply one of the ex-ante models proposed by Bucci et al. The purpose of these models is to evaluate the optimal levels of sponsorships in order to maximize the profits of private sector investors. The models proposed by the authors are of two types: static and dynamic. The static models proposed are three: the *static model of perfect competition with N single-product companies*, the *static model of monopoly with a single-product* company, and the *static model of monopolistic competition with N multi-product companies*. The models in question presented by the authors range from the ideal market form of perfect competition to a description of the model closer to the features of real market competition, i.e., monopolistic competition. In these models, the authors have adopted, as their base theory, a production function of the Cobb-Douglas type with constant returns to scale [18,19].

In addition to static models, the authors also propose a dynamic model applied only to a perfect competition market.

The proposed models have been conceived to make possible predictions (ex-ante approach) about the potential benefits of sponsorship investments for a company. In this case, the necessary data must be extrapolated from annual budget estimates. However, such data may also be applied as ex-post verification in order to evaluate whether the profit obtained by a private sponsor that invested or is still investing in sponsorships is satisfactory. Of course, in the latter case the data required for the application of the model must be extrapolated from the balance sheets (ex-post financial statement).

At this point, an important observation is necessary.

As the authors have confirmed in their work, perfect competition models are in contrast to assumptions that allow the use of both sponsorship and other forms of corporate advertising. In fact, sponsorship is a strategic tool that differentiates the image and products of a company from those of its direct competitors. In a perfect competition market, businesses produce homogeneous products that are virtually identical. In addition, companies take on atomistic dimensions that make them perfectly similar to each other. So, according to this view, it would not make sense to talk about sponsorships.

In addition to this, it must be said that the majority of companies investing in sponsorships are multi-product, i.e., they do not only offer one type of good.

The conditions for applying the first model would therefore be undermined (the *static model of perfect competition with N single-product companies*). However, the model in question has the merit of analyzing a company's productivity through a simplified approach that does not require complex mathematical modelling or the use of a large number of data.

On the contrary, the *model of monopolistic competition with N multi-product companies* is the one that more realistically identifies the current production conditions.

In fact, in the monopoly competition market, companies offer a non-homogeneous and differentiated product. Product differentiation leads to a change in consumer preferences as, while being similar, business products are not considered by buyers to be identical. Every single business product is distinct from the others by virtue of small tangible differences (technical characteristics, objective quality, etc.) or intangible differences (brand reputation, perceived quality, advertising, etc.).

Under monopoly competition theories, it is reasonable for an enterprise to consider entering into sponsorship agreements.

The application of the monopolistic competition model to a multi-product company that has actually invested in sponsorship requires, however, the use of a large number of data. In addition, the mathematical functions used to solve the optimization problem are somewhat complex and, for this reason, are not immediately applicable to real case scenarios.

The following sub-section describes the model of monopolistic competition in its original formulation. Subsequently, some simplification theories were introduced with the aim of facilitating concrete applications of the model for case study purposes. Finally, as mentioned in the introduction, the model in question was applied to Caffè Moreno s.r.l., which funded the restoration work of the "Fontana del Carciofo" in Piazza Trieste and Trento in Naples. The model's application and the analysis of the results obtained are shown in the final sections.

2.2. The Static Investment Model in Sponsorship

If we consider sponsorships to be an intangible factor affecting a multi-product company's production function (with capital and labor) and contributing to such company's overall productivity, and assuming that company operates in a monopolistic competition market, then the following hold true:

We call Q_i the quantity of products offered by such multi-product company and p_i the unit price of the same products; in monopolistic competition, each multi-product company can modify both the number of products and the price. Therefore, the price p_i of the i-th products manufactured by each company depends on the quantity Q_i offered:

$$p_i = p\ (Q_i, S) \tag{1}$$

In addition to the quantity offered, demand is also a function of the sponsorship factor, as the latter contributes to the consolidation of the company's good reputation, indirectly influencing the preferences of potential consumers and directing them to the purchase of its products.

$Q_i = Q_i\ (K_i, L_i, S)$ represents the production function of the multi-product company (or manufacturer) where K_i is the *capital*, L_i is the *labor*, and S is the *intangible capital stock* invested in sponsorships (*number of sponsored events*). Through sponsorship, the company promotes its overall image: for this reason, there is not the subscript "*I*", since it is not related to a single product.

The function *of total profits* for the company will be:

$$\pi = \Sigma_i\ [R\ (K_i, L_i, S) - C\ (Q_i)], \tag{2}$$

where $R\ (K_i, L_i, S) = p_i\ (Q_i, S)\cdot Q_i\ (K_i, L_i, S)$ are the *total revenues* and $C\ (Q_i)$ are the *production costs* of the i-th goods.

We call $C_K = rK_i$ capital costs, $C_L = wL_i$ labor costs, and $C_S = sS$ sponsorship costs.

We can therefore write the *production costs* related to the i-th good as follows:

$$C\ (Q_i) = rK_i + wL_i + sS, \tag{3}$$

where r is the *unit cost of capital* (which will be better defined for the application case later), w is the *wage rate*, and s is the *unitary price of sponsorships*.

In order to solve our problem, we assume the following production function (non-linear *Cobb-Douglas* type function):

$$Q_i = K_i^{\alpha}\ L_i^{\beta}\ S^{\gamma}, \tag{4}$$

where the coefficients $0 < \alpha < 1$, $0 < \beta < 1$ and $0 < \gamma < 1$ measure the elasticity of production and the returns to scale of the individual production factors.

We also assume $\alpha + \beta + \gamma = 1$; this means that the technology used to cope with production is characterized by *constant returns to scale*. In economics, the term *return to scale* indicates the relationship between the input and the output variation in a production unit. The returns to scale are *constant* in the event that, as inputs increase (or decrease), outputs also increase (or decrease) in a directly proportional way.

The *function of total profits* becomes the following:

$$\begin{gathered}\pi = \Sigma_i\,[R\,(K, L, S) - C\,(Q_i)] = \Sigma_i\,[p_i\,(Q_i, S)\,Q_i\,(K_i, L_i, S) - C\,(Q_i)] \\ \rightarrow \pi = \Sigma_i\,[p_i\,(Q_i, S)\,K_i{}^{\alpha}\,L_i{}^{\beta}\,S^{\gamma} - (rK_i + wL_i + sS)].\end{gathered} \tag{5}$$

The purpose of companies, including those investing in sponsorship, is the maximization of profits.

In monopolistic competition, the company maximizes profit when *marginal cost* (C_m) equals *marginal revenue* (R_m).

We can therefore solve the partial derivative with respect to S and set the marginal profit π equal to zero to obtain the optimal economic conditions for which $R_m = C_m$ (in this way the total profit related to a certain amount Q^* of product obtained by using a specific level S^* of sponsorship stock is maximized):

$$\partial/\partial S\ \Sigma_{\mathrm{i}}\,[(p_i\,(Q_i, S)\,K_i{}^{\alpha}\,L_i{}^{\beta}\,S^{\gamma} - rK_i - wL_i - sS)] = 0. \tag{6}$$

The resolution of the above derivative, although easy on a theoretical level, is difficult when applying the model to real-life situations. This is due to the several factors, given below.

First of all, we should remember that the price of the *i*-th product is, in the case of monopolistic competition, dependent on both the quantity Q_i offered and by S. In the case of the static hypothesis, the analysis of the production function is carried out at a given time t (which for the case study is the end of 2015): at that time, the price is kept constant, i.e., it does not depend on Q_i.

The static model theory is further substantiated by the observation of the market prices of the products offered by the subject company (Caffè Moreno s.r.l.). Indeed, it was possible to note that the prices of these products did not change significantly over the years in which the sponsorship campaign took place (that is, in the two-year period 2014–2015).

It should also be noted that the prices of the products offered by the subject company being analyzed are independent not only of Q_i but also of S. In fact, for the subject company the model is its first sponsorship experience: it did not therefore have the time to strengthen its reputation to directly influence sales (indeed, reputational effects are visible mostly over time, which allows us to disregard in the representative function of pi also the factor S).

Finally, a last simplification should be considered.

The subject company, which is essentially in the business of roasting and coffee distribution, offers different product types (coffee powder, waffles, capsules), which are differentiated by quality and aroma. In addition, a variety of secondary products (cups, saucers, gadgets, etc.) are offered, although in limited quantities. For this reason, it cannot be regarded as a single-product company. So, the average price per kilogram of the various product types offered by the subject company (coffee powder, waffles, capsules) and of the different qualities and aromas was calculated, and the other secondary products offered (cups, saucers, gadgets, etc.) were converted into equivalent kilograms of coffee. In essence, it was assumed that the subject company produced a single type of product (coffee). Obviously, in connection with such products, the conditions of a single-product company were imposed. Furthermore, productive factors were aggregated considering the total amount of capital and labor used for production purposes (previously, it was extremely difficult to distinguish the number of workers needed to produce a certain product from those needed to produce another one; similar considerations are valid for the capital factor). It was thus possible to eliminate the summation operator from the previous equation.

The theory introduced above allows us to rewrite the equation representing total profit in the following form:

$$\pi = pQ\ (K, L, S) - C(Q) = pK^{\alpha}L^{\beta}S^{\gamma} - (rK + wL + sS). \quad (7)$$

We are now able to compute S and set the marginal profit equal to zero (optimal economic condition):

$$\partial/\partial S\ (pK^{\alpha}\ L^{\beta}\ S^{\gamma} - rK - wL - sS) = 0 \quad (8)$$

We can then write:

$$p\gamma K^{\alpha}L^{\beta}S^{\gamma-1} - s = 0, \quad (9)$$

through which:

$$S^{*} = (s/(p\gamma K^{\alpha}L^{\beta}))^{(1/(\gamma-1))}. \quad (10)$$

This formula represents the optimal sponsorship demand for the subject company, that is, the optimal number of events to be sponsored, keeping constant K and L, for profit maximization.

Q, K, L, p, r, w, and s parameters are assumed as data of the problem.

In particular, p, r, w, and s can be derived from market surveys, whereas Q, L, and K can be obtained from the financial statements.

The only unknowns of the problem remain the α, β, and γ constants, which measure, respectively, the total product share attributable to the capital production factor, the total product share attributable to the labor production factor, and the total product share attributable to the sponsorship production factor.

However, these constants can be obtained by exploiting the log-linearity property of the Cobb-Douglas function (it is a linear function in the logarithms) and the hypothesis for which $\alpha + \beta + \gamma = 1$ (that is, the assumption of constant returns to scale). The production function can also be written in the following form:

$$\log Q = \alpha\log K + \beta\log L + \gamma\log S. \quad (11)$$

Let us suppose that the coefficient γ is known. Therefore, β is equal to:

$$\beta = 1 - \gamma - \alpha. \quad (12)$$

We can now replace the expression of β in (9):

$$\log Q = \alpha\log K + (1 - \gamma - \alpha)\log L + \gamma\log S. \quad (13)$$

Once the parameter γ is set, this equation allows us to compute α:

$$\alpha = \frac{\log Q\ -\ (1\ -\ \gamma)\log L\ -\ \gamma\log S}{\log K\ -\ \log L}. \quad (14)$$

We remember that the parameter γ can assume all the values between 0 and 1. By virtue of this, it is easy to obtain all possible α and β values by applying (14) and (12) and varying γ between the two extreme values of the range.

By varying γ (and hence also α and β) we will obtain from (10) different S^{*} values which allow for the maximization of corporate profit.

In particular, knowing the possible values of S^{*}, it is easy to obtain the different levels representative of the amount of products produced by the company which maximizes profit:

$$Q^{*} = K^{\alpha}\ L^{\beta}\ S^{*\gamma}. \quad (15)$$

Finally, knowing the Q^* quantities, you can get the different values of π_{max}:

$$\pi_{max} = p\, Q^*\, (K, L, S) - rK - wL - sS^*. \tag{16}$$

In the next paragraph, the model described will be applied to our subject company.

3. The Application of the Static Investment Model in Sponsorship

As has been said several times, the static investment model in sponsorship outlined above has been applied to the company Caffè Moreno s.r.l., which has been active (between 2014 and 2015) in cultural sponsorship thanks to its participation in the Monumentando project Naples (Italy), sponsoring the restoration of the Fontana del Carciofo. From 2016, for the same project, the company funded the restoration works of two other monuments (the Monument to Armando Diaz and the Obelisk of Portosalvo). The application of the model refers solely to the sponsorship of the Fountain of Artichoke, which for the enterprise is its first experience in this field.

3.1. Monumentando Napoli Project

Monumentando Napoli project was born as part of the institutional work of safeguarding the artistic, historical, and monumental heritage of the city of Naples.

The UNESCO Program Service and Valorization of the Historic City of Naples, as a part of the measures to protect the historic and artistic heritage of the city, has sought, through a public tendering procedure, interested parties to enter into a sponsorship contract with the municipal administration for the design and execution of the restoration works of 27 monuments located on the territory of the city. In the call for tenders the monuments were grouped into 11 lots and for each of them a monetary value was identified for the completion of the planned restoration work [20].

The winner of the public tender was a company called “Uno Outdoor s.r.l.”, a large integrated communications company that was, in fact, the official sponsor of the restoration works of all 27 monuments in Naples; the communications company, however, was free to resell the advertising spaces in question to other companies and, at the same time, to exploit them for self-publicizing purposes.

For this purpose, the sponsor has released some tabs, one for each monument, describing the economic proposal offered to companies which decide to support the restoration of a monument.

Within this framework, Caffè Moreno s.r.l. has financed the restoration works of the “Fontana del Carciofo”, located in Piazza Trieste and Trento. Although the sponsorship costs the official sponsor (Uno Outdoor s.r.l.) €90,000 (of which €65,000 is for the restoration works and the remaining €25,000 is included in the so-called “advertising and propaganda costs”), the company Caffè Moreno s.r.l. has in essence purchased the 150 m^2 of exhibition spaces expected on the scaffoldings of the construction site, paying the communications company an amount of €80,000 for each month of exposure. For this reason, the “real sponsor” of the restoration work of the “Fontana del Carciofo” is Caffè Moreno s.r.l. Also, considering that the work lasted for a total of 6 months (from October 2014 to April 2015), the total cost of sponsorship for Caffè Moreno s.r.l. has been €480,000.

At the top of Figure 1 there is summary information describing the main features of the monument targeted for the interventions. In the table, there are several images of the fountain (before, during, and after the restoration).

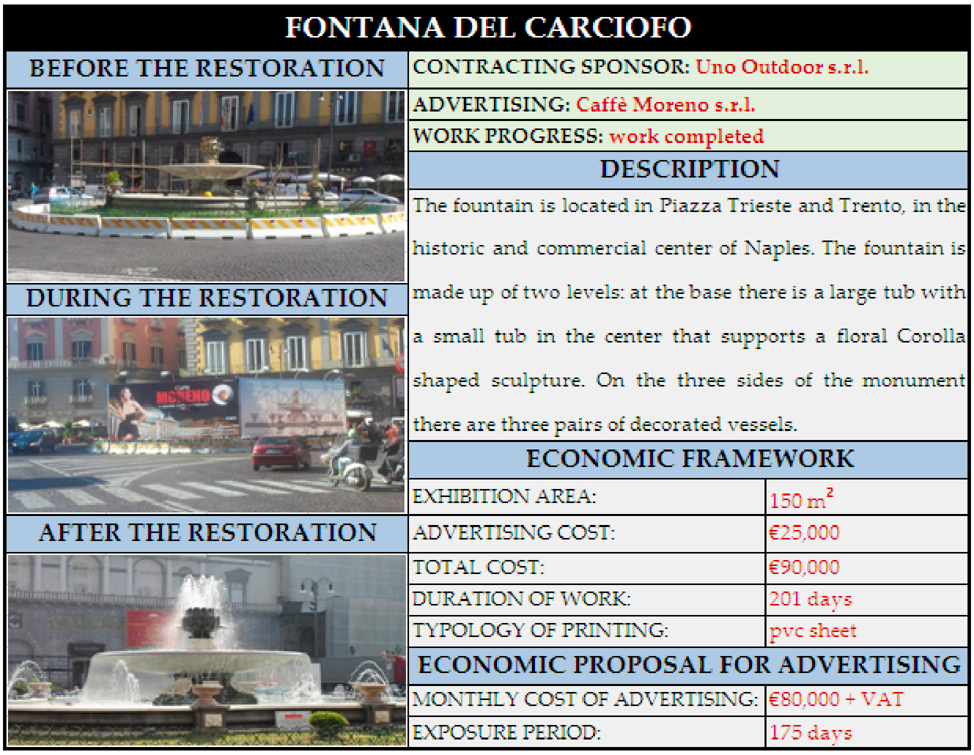

Figure 1. Summary information about the sponsorship of the "Fontana del Carciofo" located in Piazza Trieste and Trento (Naples).

3.2. The Application of the Model

In order to evaluate the actual convenience of the sponsorships promoted by Caffè Moreno s.r.l. with the aim of financing the restoration works of the "Fontana del Carciofo", we applied to this company (and to the sponsorship started following the stipulation of the contract) the static model described in the previous paragraph. The objective was to evaluate whether, with the initiative taken, the company achieved optimal economic conditions. In the event of a negative response, it would be possible to determine exactly the ideal number of restoration projects to be sponsored in order to maximize company profits.

The restoration works of the "Fontana del Carciofo" started in October 2014 and ended in April 2015. However, the advertising on billboards were not left exposed on the yard scaffolding for all of the 7 months, but only for 6. In fact, billboard advertisements with the sponsor's name were only installed on 29 October. Moreover, they were removed on 21 April 2017. So, the advertising phase of the company lasted exactly 5 months and 24 days (approximately 6 months). Consequently, we can say that part of the sponsorship undertaken in 2014 contributed to the increase in revenue for that operating year. Given that the investment in sponsorship affects the operating results of two different years, the data required for a correct application of the model was obtained by averaging those extrapolated from the 2014 and 2015 financial statements [21].

Of course, the average for the S. parameter was also done; the sponsorship costs in 2014 amounted to €160,000, whereas in 2015 they amounted to €360,000. For the model, an average value of S of €240,000 was then chosen.

The following average data in respect of 2014 and 2015 were derived from the income statement:

- production value (PV) = €11,786,374
- production costs (PC) = €10,424,509
- operating income (OI) = PV − PC = €1,361,865
- depreciation and provisions costs (DC) = €1,494,146

- staff costs (C_L) = €1,402,947
- number of employees (n) = 37.5.

From the balance sheet, the following average figure was obtained:

- total assets (A) = €18,750,043.

In order to be able to easily apply the formulas that represent the production function and the static model, some formal simplifications were introduced with respect to the nomenclature of the individual terms of these equations. In particular, with the letter "*R*" we refer to the value of production (PV) and not to the actual revenue of the enterprise. In fact, it must be borne in mind that any production function (including the Cobb-Douglas function) allows for the total amount of products produced by a subject company through the use of production factors to be determined. However, it does not follow automatically that the quantity will be entirely sold, so the value of production does not always coincide with the actual revenue. Nevertheless, to remain consistent with the original nomenclature of the model under analysis and not to create confusion in the reader, we will call R the value of production.

Additionally, we set operating income equal to corporate profit (OI = π). In fact, to get the real profit from the operating results, other items, including income taxes, should be subtracted. However, these are not part of the costs of production and may be omitted.

Within the model, K represents all that is needed for the manufacture of finished products destined for sale, while at the same time, human resources (L) and intangible capital are not taken into account in the form of sponsorship (S). As a result, the K parameter represents the entire section of assets derived from the company's balance sheet, i.e., the total of current assets (cash, inventory, receivables), and fixed assets (tangible, intangible, and financial). K represents the complex of the assets owned by the company and its economic resources (plant and machinery, buildings, equipment, credits, money, inventory products, patents and trademarks, etc.), that is, all that directly or indirectly can be used for productive purposes. It is therefore the total of assets (A). Of course, K parameter was also taken as average value. In particular, it is equal to the average for 2014 and 2015 assets.

The cost of capital (C_K) was also quantified with a deductive approach in the financial statements (in particular, in the income statement), placing it equal to the sum of depreciation and provisions costs (DC).

Other expenditure items (costs for purchasing raw materials and consumables, service costs, expenses for the enjoyment of third party assets, etc.) are included in the total production costs; these are the cost for the acquisition of those products or services that are wholly consumed during the production process and are therefore no longer present at the end of the operating year. They are obtained from the budget data, and are named C_V.

Finally, as anticipated in the previous paragraph, the number of sponsored events in the two-year period 2014–2015 and the average unit cost of the sponsorship activity are known a priori (S = 1 and s = €240,000).

At this point, we can make a brief summary of available data and consequently obtainable data:

- R = €11,786,374
- C_T = €10,424,509
- π = €1,361,865
- C_L = €1,402,947
- C_S = €240,000
- C_K = €1,494,146
- $C_V = = C_T - C_K - C_L - C_S$ = €7,287,416
- K = €18,750,043
- L = 37.5 employees
- S = 1 sponsorship

- p = €/kg 15
- $r = C_K/K \cong 8\%$
- $w = C_L/L$ = €/employees 37,412
- s = €/sponsorships 240,000
- $Q = R/p$ = kg 785,758.

Now only the values of the parameters α, β, and γ remain to be determined.

As mentioned, the problem is not uniquely determined, so it is possible to determine α and β only after fixing a possible value of γ between 0 and 1.

For example, we will proceed to the determination of the parameters S^*, Q^*, and π_{max}, assuming that γ = 0.10.

This allows us to determine α from (13) and β from (11):

$$\alpha = \frac{\log Q - (1-\gamma)\log L - \gamma \log S}{\log K - \log S} = \frac{\log 785{,}758 - (1-0.10)\log 37.5 - 0.10 \log 1}{\log 18{,}750{,}043 - \log 37.5} \cong 0.79 \quad (17)$$

and

$$\beta = 1 - \gamma - \alpha = 1 - 0.10 - 0.79 \cong 0.11. \quad (18)$$

Now we can calculate the optimal level of sponsorship that maximizes the company's profit from (8):

$$S^* = (s/(p\gamma K^{\alpha} L^{\beta}))^{(1/(\gamma-1))} = (240{,}000/(15 \times 0.10 \times 18{,}750{,}043^{0.79} \times 37.5^{0.11}))^{(1/(0.10-1))} \cong 6 \quad (19)$$

This means that, for a given sponsorship productivity level equal to γ = 0.10, the company would have to sponsor five more restoration works with similar features to the one already financed (at parity of the unit cost of the sponsorship). In this case, the optimal amount of product and the maximum profit would be as follows:

$$Q^* = K^{\alpha} L^{\beta} S^{*\gamma} = 18{,}750{,}043^{0.79} \times 37.5^{0.11} \times 6^{0.10} \cong \text{kg } 937{,}747 \quad (20)$$

and

$$\begin{aligned} \pi_{max} &= p\,Q^*(K, L, S) - Cv - rK - wL - sS^* \\ &= 15 \times 937{,}747 - 7{,}287{,}416 - 18{,}750{,}043 \times 0.08 - 37{,}412 \times 37.5 - 240{,}000 \times 6 \\ &= \cong €2{,}475{,}070. \end{aligned} \quad (21)$$

In fact, we can observe how the company, by increasing the number of sponsored restoration works from one to six, increases its profits by about €1,113,205 (i.e., the total profit increases by about 1.8 times). Additionally, the *sponsorship's costs* and *total costs* will increase proportionally:

$$C_{S^*} = sS^* = 240{,}000 \times 6 = €1{,}440{,}000 \quad (22)$$

and

$$C^* = Cv + rK + wL + sS^* = 7{,}287{,}416 + 1{,}494{,}146 + 240{,}000 \times 6 = €11{,}591{,}128. \quad (23)$$

By varying γ between 0 and 1 we have obtained all the possible values of α, β, S^*, Q^*, and π_{max}.

It was thus possible to construct the graphs representative of the functions $S^* = f(\gamma)$, $Q^* = f(\gamma)$ and $\pi_{max} = f(\gamma)$, from which important considerations emerged.

4. Discussion

Before describing the results obtained from the application of the model, shown in Figures 2–4, it is necessary to make some considerations about the exponential terms of the Cobb-Douglas production function.

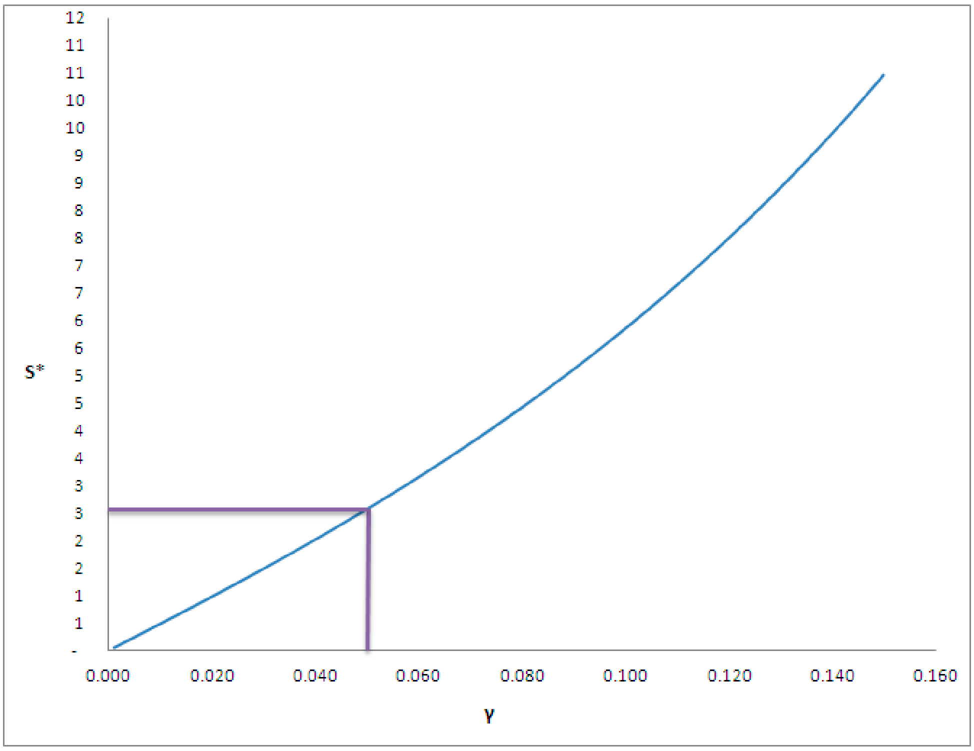

Figure 2. Variation law of S* at γ variation ($0.001 < \gamma < 0.150$).

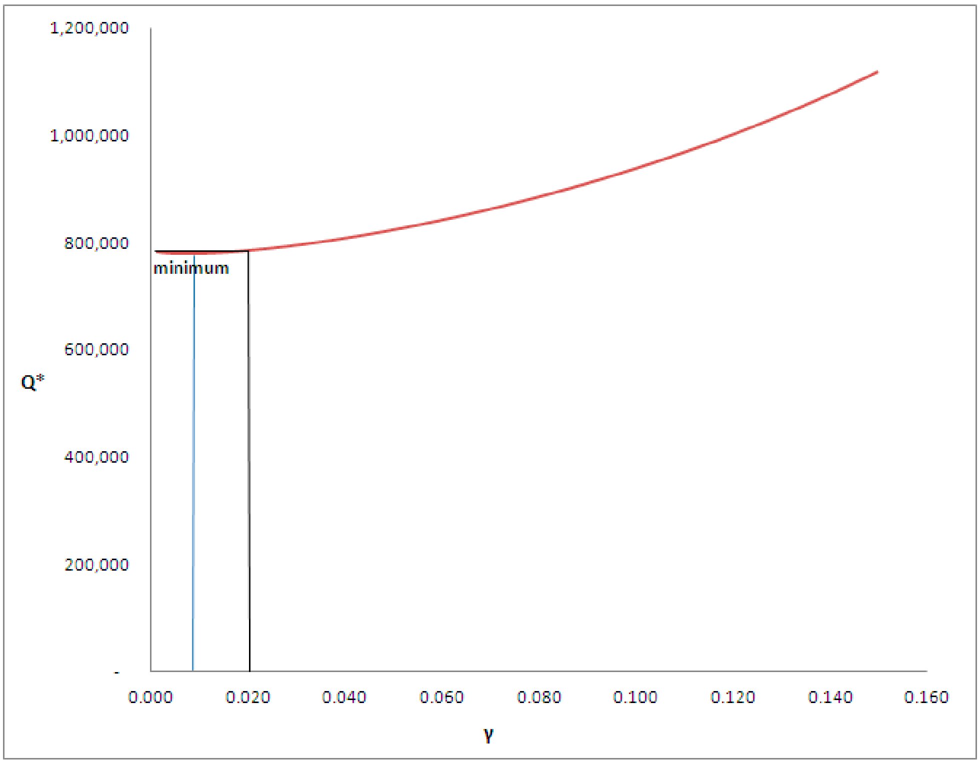

Figure 3. Variation law of Q^* at γ variation ($0.001 < \gamma < 0.150$).

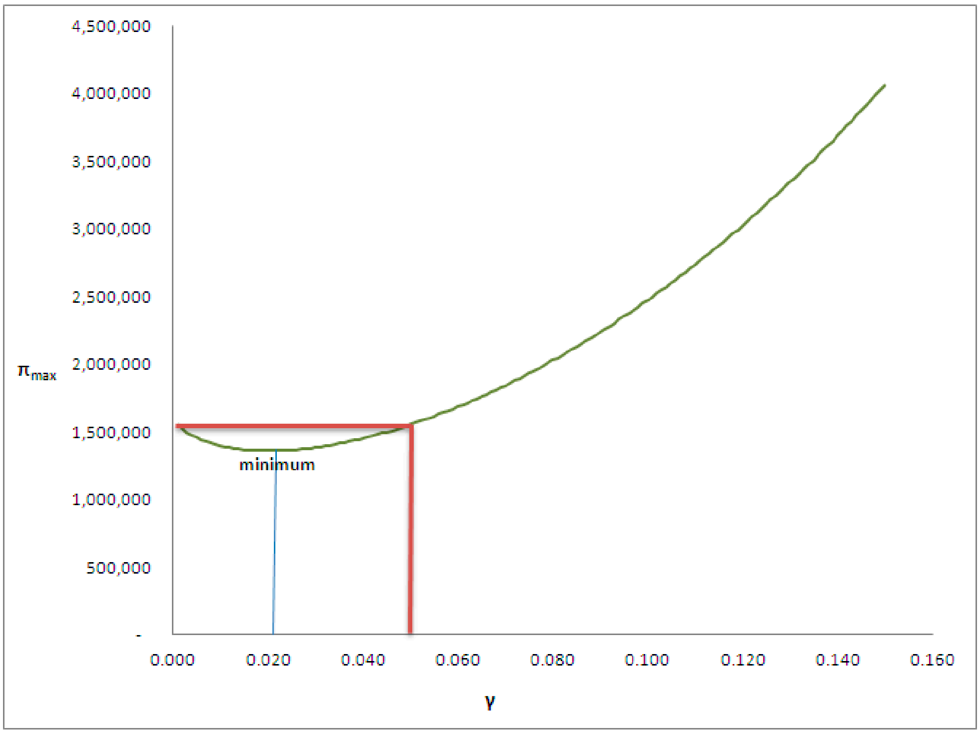

Figure 4. Variation law of π_{max} at γ variation (0.001 < γ < 0.150).

As mentioned above, parameter γ provides information about the amount of that part of total output attributable to the sponsorship production factor. This means that when γ increases, the quantity of products manufactured by the subject company and attributable solely to the sponsorship activity will also increase exponentially. Conversely, the reverse is true when γ decreases.

Therefore, the γ parameter can also be interpreted as a skill parameter: if a subject company is very skilled in sponsorship, it can reach certain thresholds of Q production by employing fewer resources in terms of work and capital.

The α parameter, instead, provides information about the total amount of the products that is attributable to the capital production factor. This means that an increase in α will also increase exponentially the quantity of products manufactured by the subject company and attributable solely to the use of physical and financial capital.

Finally, β represents the parameter that gives information about the quantity of products produced by the subject company which are dependent solely on the contribution of the labor input factor.

Now it is necessary to define in detail the variation laws of S^*, Q^*, and π_{max} at the variation of γ parameter (reported respectively in Figures 2–4).

It is useful to remember again that S^* represents the optimal level of sponsorship, that is, the optimal number of regeneration/restoration works that the subject company has to sponsor, considering that the unit cost for each intervention is equal and constant, to obtain the highest possible level of profit (which we indicate with π_{max}). By substituting the S^* value in the production function, we obtain the Q^* quantity, which represents the number of products manufactured for sale that, if really sold, will allow the company to achieve the maximum total profit level (that is, π_{max}).

Analyzing Figure 2, it is possible to notice how, when γ constantly increases, S^* rises more than in proportion. As previously mentioned, when γ approaches 0, S^* also tends to 0; instead, when γ approaches 1, S^* tends asymptotically to +:.

When close to γ = 0 happens the company sponsors so badly that it is as if it is not sponsoring at all.

The variation laws of Q^* and π_{max} at the variation of γ have a behavior similar to that of S^*, albeit with some differences. In fact, these two functions have a minimum point; for this reason, for values of γ very close to 0, Q^* and π_{max} increase rather than decrease. In particular, these two functions, for $\gamma = 0$, will be equal to: $Q^* = 785{,}758$ kg = Q and $\pi_{max} = 1{,}601{,}865$ €. Instead, for $\gamma \to 1$ both Q^* and π_{max} tend to +:.

Also, at the minimum point of the law of variation of π_{max}, we can notice that the coefficients α, β, and γ assume values for which the number of sponsorships actually made by the company (S) is already sufficient to maximize profit. By proceeding through various attempts and considering several decimal digits, it was then possible to determine the three exponential coefficients of the Cobb-Douglas production function α', β', and γ' that allow $S^* = S$, $Q^* = Q$, and $\pi_{max} = \pi$ to be obtained:

$$\begin{aligned} \gamma' &= 0.0203628, \\ \alpha' &= 0.7638764, \\ \beta' &= 0.2157608. \end{aligned} \tag{24}$$

So, keeping the other data constant, these three values of productivity parameters allow us to get the actual Q quantities and π profits of the subject company, assuming that it is already adopting an optimal behavior. Consequently, if γ' was the value actually used, the company would already be obtaining the maximum possible profit. However, for that value, despite the optimal behavior, it would get a lower profit than it would have in the absence of sponsorship.

With that said, let us take a look at the graph shown in Figure 4.

Through the function $\pi_{max} = f(\gamma)$, it is possible to express a judgment of economic convenience regarding sponsorship activities. Indeed, it is possible to notice that for $\gamma = 0$ the maximum achievable profit, equal to π_{max} = €1,601,865, coincides with what is obtainable (for the same values as Q, L, and K) without any sponsorships (i.e., not considering factor S in the Cobb-Douglas equation). At this value of π_{max}, as seen, corresponds to $Q^* \equiv Q$.

When γ increases, the maximum profit value obtainable begins to decrease until it reaches its minimum for $\gamma \cong 0.020$ (which again corresponds to $Q^* \equiv Q$).

For $\gamma \cong 0.053$ we can see that π_{max} has increased again, being equal to €1,601,865.

Furthermore, we have seen that the profit actually perceived by the company, which is equal to π = €1,361,865, coincides with what we have at the minimum point of the function $\pi_{max} = f(\gamma)$.

It is now clear that, for $\gamma < 0.053$, the company should not invest in sponsorships because for this value it would get lower profits than without sponsorships (Figure 4). Moreover, for $\gamma = 0.053$ we have $S^* \cong 3$ (as can be seen from Figure 2).

We can therefore state that, according to the proposed model, investment in sponsorships is only convenient if the subject company decides to finance at least three restoration works very similar to the one actually sponsored (or, alternatively, a single larger and more visible intervention that, qualitatively, is equivalent to at least three smaller interventions).

The results obtained were confirmed comparing them with the data for the financial statements of 2013, previous to the years of analysis.

There is a profit of €1,556,431 (slightly lower than the one expected by the model, of €1,601,865), which indicates superior performance for the company in the absence of resources dedicated to sponsorship.

Of course, in reading this result we have to consider the limits of the static model assumed and also that the verification of the effects of sponsorship is limited to a few months after its end.

However, the threshold result indicating at least three similar events to the one financed is interesting if it is based on the following considerations.

First of all, the cost for three events (€1,440,000), which weighs on expected revenue (€11,786,374) for about 12%, is consistent with the percentages given by the models for estimating the marketing budget. In fact, the most commonly used percentages are 5–10% for medium-sized companies, 20% for small companies, and 2–5% for large companies [22–24]. The company under consideration, as established by the Recommendation of the Commission of the European Union 96/280/EC, is of medium to small size (it is a small company for the number of employees, less than 49, and a medium-sized company for the revenues, between €7 million and €40 million). So, the value of 12% is acceptable.

Secondly, it should be emphasized that the data obtained (three events) represents a starting point that could be increased with caution through a dynamic assessment of the effects of the initiative.

5. Conclusions

The scarcity of government resources destined for the preservation of public historical-architectural heritage sites has stimulated the search for sources other than the state. Sponsorships by private enterprise constitute viable alternative sources of funding. This paper proposes to show potential sponsors how to invest within the sector. In particular, it intends to show potential sponsors how and why it may be convenient to invest in this sector. The main objective of the proposed study is to provide them with a measure of sponsorship spending that is sustainable and economically convenient.

The study defines the value of the ideal amount (expressed in terms of percentage of turnover) that a company should invest in a sponsorship to maximize profit, all other conditions remaining constant.

Ceteris paribus means that the company cannot increase its know-how in the sponsorship field (e.g., by recruiting specialized consultants). Therefore, it cannot change γ consciously.

The model collects and processes ex-post financial statements for 2014 and 2015. From October 2014 to April 2015 the company supported sponsorship investments (the restoration works ended in April 2015). As the model is static, it assumes that there are no variations in other variables influencing the budget outturn (or rather there are no significant variations) with the exception of the S variable (with a value equal to 1).

The model thus analyzes average financial statements data for the 2014–2015 two-year period, confirming a contraction in operating result compared to 2013 (from approximately €1.6 million to €1.3 million). This contraction is also due to a limited performance of the invested capital in sponsorships (from the analysis of the financial statements it can be seen that production factors remained almost unchanged). In fact, γ is about 0.020; this value is fairly limited because it is lower than the threshold (calculated by the model and equal to about 0.053), beyond which the firm has a higher profit than compared to that obtained in the absence of sponsorship. Of course, the reliability of the analysis must take into account the return times on the investment which are rather short (from October 2014, starting date of the sponsorship, until the end of 2015) and the fact that a more complete result could be obtained analyzing the data of Budget 2016. However, as the analysis period increases, the assumption of static model is lost. Dynamic models, significantly more complex and therefore less immediate in the application, will be analyzed in forthcoming publications.

In the variation law between π_{max} and γ, π_{max} becomes minimum when y is equal to about 0.020.

Starting from this point, we increased γ until π_{max} equaled the profit that one would have obtained in the absence of sponsorship. In this case, we have $S^* = 3$.

This result shall be read as follows.

The model shows us that if a company enters into three sponsorship agreements exactly the same as the one implemented, it would have increased the average profit for the 2014–2015 biennium at 2013 levels. Therefore, increasing the γ, i.e., the productivity parameter, should increase the company's commitment to the sponsorship investment.

It should also be said that, as an alternative to the sponsorship of three or more separate works, one could think of funding a single main intervention through which to conduct a more incisive and decisive sponsorship campaign from the point of view of dissemination of messages (for example, through mass media support).

Of course, the data generated by the model should be considered with some precaution, given the many simplifications introduced (perfect competition, single-product companies, constant returns to scale, static model). In particular, with regard to theories of constant returns, the scale assumed by the authors who theorized the model, may be sufficiently consistent for the company studied. In fact, the subject company is in the business of roasting and distribution of coffee, i.e., industrial activities without high technological specialization and with a corporate size (38 employees) which does not easily facilitate efficiency in distribution of labor. So, the model is consistent with companies that use traditional factors such as labor and capital [25], but may be less reliable in the case of information-intensive production factors, such as sponsorships.

By virtue of this, our next objective will be to study the dynamic model and adapt it to market systems of monopoly competition using a production function with the most representative returns to scale.

In addition, the analyzes carried out should be associated with those of other ex-post models in order to verify the possible correspondence between the expected results from the company, in terms of monetary returns, and the real feelings, perceptions, and opinions of the sponsorship's audience.

These are all aspects that are currently being studied and analyzed on the basis of analyses conducted in neighboring study fields [26–32].

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