Free vibrations of Bernoulli-Euler nano-beams by the stress-driven nonlocal integral model

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A B S T R A C T

Nonlocal theories of Continuum Mechanics are widely used in order to assess size effects in nano-structures. In this paper, free vibrations of nano-beams are investigated by making recourse to the novel stress-driven nonlocal integral model (SDM). Equations of motion governing the dynamics of a Bernoulli-Euler nano-beam are consistently formulated and numerically integrated by Matlab. Selected case studies involving structures of nanotechnological interest are examined. Natural frequencies, evaluated according to the SDM, are compared with those obtained by the Eringen differential law (EDM) and by the gradient elasticity theory (GradEla). SDM provides an effective methodology to describe nonlocal phenomena in NEMS.

1. Introduction

Carbon nanotubes are non-traditional materials of current interest in several applicative sectors, such as engineering, medicine and electronics [1–6]. Methodologies of Continuum Mechanics, advantageously adopted in order to study composite structures [7–13], are nowadays exploited also to assess size effects in devices at nano-scale [14–18]. The key aspect of this approach is in selecting suitable nonlocal elastic relations [19–31]. The Eringen differential law (EDM), introduced in Eq. (3.19) of [32] and applied by Peddieson to Nanotechnology in Ref. [33], is certainly the most popular constitutive strategy for the analysis of bending, vibration and buckling of nano-beams [34–42].

For nonlocal problems formulated in unbounded domains, EDM can be considered equivalent to a strain-driven integral convolution (Eq. (2.2) in Ref. [32]), due to the tacit fulfillment of suitable constitutive boundary conditions of vanishing at infinity. In this context, EDM was effectively exploited by Eringen in order to investigate screw dislocations and Rayleigh surface waves.

In nonlocal structural problems of technical interest, involving bounded domains and standard kinematic boundary constraints, the constitutive boundary conditions (CBCs) [43,44] associated with the Eringen strain-driven integral convolution (EIM) are in contrast with equilibrium equations.

Accordingly, the elastostatic problem of a nano-structure formulated according to EIM admits no solution at all. The unmotivated usual choice of disregarding the CBCs and to model nonlocal phenomena in nano-beams by EDM cannot be considered a significant assumption.

A discussion about inapplicability of Eringen theory to Structural Mechanics and resolutions of improperly claimed paradoxes [45–48] is provided in Refs. [49,50]. These difficulties were by-passed in Refs. [51,52] by adopting the local-nonlocal mixture originally conceived in Refs. [53,54]. An innovative stress-driven nonlocal integral model (SDM) was proposed in Ref. [55] to examine size-dependent static behavior of inflected nano-beams. Unlike the Eringen strain-driven nonlocal integral convolution, SDM provides a new mathematically and mechanically consistent approach for the design and optimization of nano-structures [56].

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Such a strategy is resorted to in the present paper to study size-dependent dynamical behavior of nano-beams.

The plan is the following. The stress-driven integral model (SDM) is formulated in Sect.2 for Bernoulli-Euler nano-beams. Equations governing free vibrations are provided in Sect.3. Fundamental natural frequencies of cantilever, simply-supported, clamped-pinned and doubly-clamped nano-beams are numerically computed and compared with the ones obtained by the gradient elasticity theory (GradEla) [57–60] and by the Eringen differential law (EDM) in Sect.4. Concluding remarks are summarized in Sect.5.

2. Stress-driven nonlocal integral model (SDM)

Let us consider a straight beam under flexure. Length and cross-section are respectively denoted by L and Ω. The bending plane is described by the pair of Cartesian axes (x, y), with x nano-beam axial abscissa. According to the Bernoulli-Euler kinematics, the geometric bending curvature χ_t at time t is related to the transverse displacement v_t by

$$\chi(x,t) := \chi_t(x) = v_t^{(2)}(x), \tag{1}$$

where the apex $(\bullet)^{(n)}$ is the n-th derivative along x, with $n \in \mathbb{N}$. The differential condition of d'Alembert dynamical equilibrium for a freely vibrating nano-beam writes as

$$M^{(2)} = -m\ddot{v}, \tag{2}$$

where M and m are the bending moment and mass per unit length of the nano-beam and a dot $(\dot{\bullet})$ stands for time derivative.

The stress-driven nonlocal elastic model (SDM) proposed in Ref. [55] is described by the following integral convolution law defining the elastic curvature field

$$\chi_{\mathrm{EL}}(x) = \int_0^L \varXi_\lambda(x-\xi)C(\xi)M(\xi)d\xi, \tag{3}$$

with C local elastic compliance, which is the inverse of the elastic stiffness (E is the Young modulus)

$$K := \int_\Omega Ey^2 dA. \tag{4}$$

The dimensionless nonlocal parameter $\lambda > 0$ in Eq. (3) is the ratio between a characteristic length L_c and the nano-beam length L. As proven in Ref. [55], introducing the special kernel

$$\varXi_\lambda(x) := \frac{1}{2L_c}\exp\left(-\frac{|x|}{L_c}\right), \tag{5}$$

the integral law Eq. (3) is equivalent to the differential condition

$$\chi_{\mathrm{EL}} - L_c^2 \chi_{\mathrm{EL}}^{(2)} = CM, \tag{6}$$

with the constitutive boundary conditions

$$\begin{cases} \chi_{\mathrm{EL}}^{(1)}(0) = \dfrac{1}{L_c}\chi_{\mathrm{EL}}(0), \\ \chi_{\mathrm{EL}}^{(1)}(\mathrm{L}) = -\dfrac{1}{L_c}\chi_{\mathrm{EL}}(\mathrm{L}). \end{cases} \tag{7}$$

Hereafter, we assume a purely elastic constitutive behavior. Accordingly, geometric and elastic bending curvature fields are coincident $\chi = \chi_{\mathrm{EL}}$. It is worth noting that the kernel Eq. (5) fulfils symmetry, positivity and limit impulsivity

$$\begin{cases} \varXi_\lambda(x-\xi) = \varXi_\lambda(\xi - x) \geq 0, \\ \lim\limits_{\lambda \to 0} \varXi_\lambda(x) = \boldsymbol{\delta}(x), \end{cases} \tag{8}$$

with $\boldsymbol{\delta}$, the Dirac unit impulse at $0 \in \mathscr{R}$ and the limit being intended in terms of distributions

$$\lim_{\lambda \to 0} \int_{-\infty}^{+\infty} \varXi_\lambda(x-\xi) \cdot f(\xi)d\xi = f(x), \tag{9}$$

for any continuous map $f : \mathscr{R} \mapsto \mathscr{R}$.

3. Free vibrations of nano-beams

Natural frequencies of a elastically homogeneous Bernoulli-Euler nano-beam, formulated according to the SDM recalled in Sect.2, are evaluated as follows. Let us preliminary take the second derivative along the abscissa x of Eq. (6) and prescribe the differential condition of equilibrium Eq. (2)

$$\chi_{\mathrm{EL}}^{(2)} - L_c^2 \chi_{\mathrm{EL}}^{(4)} = -Cm\ddot{v}. \tag{10}$$

Enforcement of the differential condition of kinematic compatibility Eq. (1) provides the differential equation governing free vibrations of a nano-beam

$$v^{(4)} - L_c^2 v^{(6)} = -Cm\ddot{v}. \tag{11}$$

Natural frequencies and mode shapes are got by separating spatial and time variables as

$$v(x,t) = \psi(x)\phi(t). \tag{12}$$

Eq. (11) rewrites thus as

$$\psi^{(4)}(x)\phi(t) - L_c^2\psi^{(6)}(x)\phi(t) = -Cm\psi(x)\ddot{\phi}(t), \tag{13}$$

whence we get

$$-\omega^2 := \frac{\ddot{\phi}(t)}{\phi(t)} = \frac{L_c^2\psi^{(6)}(x) - \psi^{(4)}(x)}{Cm\psi(x)}. \tag{14}$$

The previous condition provides two basic differential equations

$$\begin{cases} \ddot{\phi}(t) + \omega^2\phi(t) = 0, \\ L_c^2\psi^{(6)}(x) - \psi^{(4)}(x) + s\psi(x) = 0, \end{cases} \tag{15}$$

with the scalar s defined by

$$s := \omega^2 Cm. \tag{16}$$

The general integral of Eq. $(15)_1$ (harmonic motion equation) is given by

$$\phi(t) = a\sin(\omega t) + b\cos(\omega t), \tag{17}$$

with the pair (a, b) to be evaluated by prescribing suitable initial conditions.

Evaluation of natural frequencies consists of three steps.

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(1) Integration of Eq. (15)$_2$. The general integral is expressed in terms of a six-dimensional array $\mathbf{p} = \{p_1, p_2, p_3, p_4, p_5, p_6\}$.
(2) Enforcement of six boundary conditions providing a homogeneous algebraic system $\mathbf{A}(\lambda, s)\mathbf{p} = \mathbf{o}$.
(3) Solution of the nonlinear equation in s (see Eq. (16))

$$\det \mathbf{A}_\lambda(s) = 0, \tag{18}$$

for any given positive dimensionless nonlocal parameter λ.

4. Case studies

The stress-driven nonlocal elastic model (SDM) is here adopted in order to get a numerical assessment of fundamental natural frequencies ω_1 for cantilever, simply-supported, clamped-pinned and doubly-clamped nano-beams in terms of the dimensionless nonlocal parameter $\lambda = L_c/L$. Let us introduce the dimensionless fundamental natural frequency

FIG. 1

FIG. 2

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FIG. 3

FIG. 4

$$\omega^* := \omega_1 \mathrm{L}^2 \sqrt{\frac{m}{K}}. \tag{19}$$

The solution methodology exposed in Sect.3 requires the prescription of four kinematic and/or static boundary conditions involving transverse displacement, bending moment and their derivatives, as follows.

Cantilever nano-beam

$$\begin{cases} v(0) = 0, & v^{(1)}(0) = 0, \\ M(\mathrm{L}) = 0, & M^{(1)}(\mathrm{L}) = 0. \end{cases} \tag{20}$$

Simply-supported nano-beam

$$\begin{cases} v(0) = 0, & M(0) = 0, \\ v(\mathrm{L}) = 0, & M(\mathrm{L}) = 0. \end{cases} \tag{21}$$

Clamped-pinned nano-beam

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Table 1
Fundamental natural frequency ω^* of a cantilever nano-beam, ratio Δ_1 between the gap of ω^* of SDM with respect to EDM and ratio Δ_2 between the gap of ω^* of SDM with respect to GradEla vs. nonlocal parameter λ.

λ	ω^*			Δ_1(%)	Δ_2(%)
	SDM	EDM	GradEla		
0.00	3.516013	3.516013	3.516013	0.000000	0.000000
0.01	3.551528	3.516170	3.516819	1.005581	0.986939
0.02	3.587734	3.516620	3.519177	2.022217	1.948076
0.03	3.624609	3.517370	3.523011	3.048836	2.883834
0.04	3.662122	3.518430	3.528240	4.083985	3.794585
0.05	3.700236	3.519798	3.534784	5.126350	4.680666
0.06	3.738905	3.521470	3.542562	6.174554	5.542408
0.07	3.778081	3.523450	3.551491	7.226764	6.380163
0.08	3.817712	3.525750	3.561487	8.280844	7.194333
0.09	3.857741	3.528350	3.572466	9.335562	7.985400
0.10	3.898114	3.531280	3.584342	10.388129	8.753950

Table 2
Fundamental natural frequency ω^* of a simply supported nano-beam, ratio Δ_1 between the gap of ω^* of SDM with respect to EDM and ratio Δ_2 between the gap of ω^* of SDM with respect to GradEla vs. nonlocal parameter λ.

λ	ω^*			Δ_1 (%)	Δ_2 (%)
	SDM	EDM	GradEla		
0.00	9.869630	9.869630	9.869630	0.000000	0.000000
0.01	9.874376	9.864750	9.874279	0.097583	0.000986
0.02	9.888289	9.850170	9.887512	0.386992	0.007864
0.03	9.910724	9.826050	9.908108	0.861730	0.026402
0.04	9.941046	9.792560	9.934875	1.516312	0.062115
0.05	9.978614	9.750050	9.966641	2.344233	0.120126
0.06	10.022791	9.698810	10.002283	3.340416	0.205029
0.07	10.072950	9.639310	10.040743	4.498660	0.320764
0.08	10.128486	9.571950	10.081054	5.814243	0.470514
0.09	10.188824	9.497260	10.122357	7.281723	0.656635
0.10	10.253421	9.415900	10.163914	8.894756	0.880640

Table 3
Fundamental natural frequency ω^* of a clamped-pinned nano-beam, ratio Δ_1 between the gap of ω^* of SDM with respect to EDM and ratio Δ_2 between the gap of ω^* of SDM with respect to GradEla vs. nonlocal parameter λ.

λ	ω^*			Δ_1(%)	Δ_2(%)
	SDM	EDM	GradEla		
0.00	15.418195	15.418195	15.418195	0.000000	0.000000
0.01	15.585374	15.409340	15.438519	1.142387	0.951227
0.02	15.778142	15.382820	15.496887	2.569892	1.814909
0.03	15.995777	15.338930	15.589720	4.282220	2.604647
0.04	16.237326	15.278090	15.713688	6.278507	3.332369
0.05	16.501636	15.200897	15.865658	8.556985	4.008521
0.06	16.787386	15.108080	16.042654	11.115287	4.642204
0.07	17.093136	15.000510	16.241858	13.950364	5.241257
0.08	17.417372	14.879140	16.460628	17.058995	5.812319
0.09	17.758564	14.745000	16.696522	20.437873	6.360862
0.10	18.115212	14.599210	16.947332	24.083506	6.891233

$$\begin{cases} v(0) = 0, & v^{(1)}(0) = 0, \\ v(L) = 0, & M(L) = 0. \end{cases} \tag{22}$$

Doubly-clamped nano-beam

$$\begin{cases} v(0) = 0, & v^{(1)}(0) = 0, \\ v(L) = 0, & v^{(1)}(L) = 0. \end{cases} \tag{23}$$

Moreover, for all the considered structural schemes, the following further two boundary constitutive conditions in terms of transverse displacements (see Eqs. (7) and (1)) are to be enforced

Table 4
Fundamental natural frequency ω^* of a clamped-clamped nano-beam, ratio Δ_1 between the gap of ω^* of SDM with respect to EDM and ratio Δ_2 between the gap of ω^* of SDM with respect to GradEla vs. nonlocal parameter λ.

λ	ω^*			Δ_1 (%)	Δ_2 (%)
	SDM	EDM	GradEla		
0.00	22.373350	22.373350	22.373350	0.000000	0.000000
0.01	22.851842	22.359540	22.426836	2.201755	1.895077
0.02	23.393189	22.318430	22.581172	4.815567	3.595993
0.03	23.997578	22.250410	22.828357	7.852297	5.121792
0.04	24.664325	22.156180	23.161895	11.320294	6.486642
0.05	25.391792	22.036720	23.576160	15.224913	7.701135
0.06	26.177462	21.893210	24.066021	19.568863	8.773535
0.07	27.018092	21.727070	24.626619	24.352211	9.710927
0.08	27.909917	21.539850	25.253232	29.573405	10.520180
0.09	28.848886	21.333250	25.941234	35.229680	11.208611
0.10	29.830868	21.109030	26.686099	41.318044	11.784297

$$\begin{cases} v^{(3)}(0) = \dfrac{1}{L_c} v^{(2)}(0), \\ v^{(3)}(L) = -\dfrac{1}{L_c} v^{(2)}(L). \end{cases} \tag{24}$$

The results are compared with the ones corresponding to the Eringen Differential law (EDM) and to the Gradient Elasticity theory (GradEla). EDM is based on the following constitutive relation

$$M - L_c^2 M^{(2)} = K \chi_{EL}. \tag{25}$$

The ensuing differential problem governing free vibrations of nano-beams is expressed by a fourth-order differential equation in terms of transverse displacements. Fundamental natural frequencies are obtained in literature (see e.g. Ref. [34]) by discarding the higher-order constitutive boundary conditions inferred from the Eringen strain-driven integral nonlocal law [50]. We emphasize that the higher-order constitutive boundary conditions of Eringen strain-driven integral theory are in contrast with equilibrium conditions [49]. The strain-driven model cannot be adopted in order to study size-effects in nano-beams and accordingly its differential counterpart (EDM) cannot be considered as a significant and reliable nonlocal model in structural applications. GradEla is formulated by introducing the following specific elastic energy

$$\psi(\chi_{EL}, \chi_{EL}^{(1)}) := \frac{1}{2} K \chi_{EL}^2 + \frac{1}{2} L_c^2 K \left(\chi_{EL}^{(2)}\right)^2, \tag{26}$$

which is a special case of the model (C_1) presented in Ref. [61] by setting $\alpha = 0$. The corresponding differential equation governing free vibrations of a nano-beam coincides with the one of the SDM Eq. (11). Difference with SDM consists in the higher-order constitutive boundary conditions which take the form (see Table 2 in Ref. [61])

$$\begin{cases} L_c v^{(3)}(0) = 0, \\ L_c v^{(3)}(L) = 0. \end{cases} \tag{27}$$

In Figs. 1–4, plots of dimensionless fundamental natural frequencies ω^*, associated with SDM, GradEla and EDM, vs. the dimensionless nonlocal parameter λ are provided for all kinematic boundary conditions examined above. Numerical evaluations of $\omega^*(\lambda)$ and the ratios Δ_1 and Δ_2 between the gaps of ω^* of SDM with respect to EDM and GradEla are listed in Tables 1–4. As displayed in Fig. 5, SDM exhibits a hardening behavior both in terms of nonlocal parameter λ and of the number of kinematic boundary constraints. The acronyms CF, PP, CP, CC stand for clamp-free, pin-pin, clamp-pin, clamp-clamp respectively. The softest structural response,

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FIG. 5

with respect to all the examined kinematic boundary conditions, is exhibited by cantilever nano-beams.

5. Closing remarks

The outcomes of the present paper may be summarized as follows.

(1) Size-dependent vibrational behavior of Bernoulli-Euler nano-beams has been investigated by adopting the stress-driven integral nonlocal elastic model (SDM).

(2) Fundamental natural frequencies of cantilever, simply-supported, clamped-pinned and doubly-clamped nano-beams have been numerically computed by Matlab. The results are compared with the ones obtained by the Eringen differential law (EDM) and by the gradient elasticity theory (GradEla). SDM provides, for all considered kinematic boundary conditions, the highest frequencies.

(3) In gradient elasticity theories, the problematic choice of physically motivated higher-order boundary conditions is actively debated in literature, as witnessed by the recent valuable papers in Refs. [62–64]. A significant improvement brought by the new stress-driven integral approach (SDM) is that all boundary conditions are naturally and univocally provided by the theory, so that closure of the nonlocal dynamical problem is assured.

(4) In the research field of nano-structures, the stress-driven nonlocal integral model (SDM) provides an effective constitutive remedy to the inapplicable Eringen strain-driven nonlocal integral theory [50].

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FIGURES

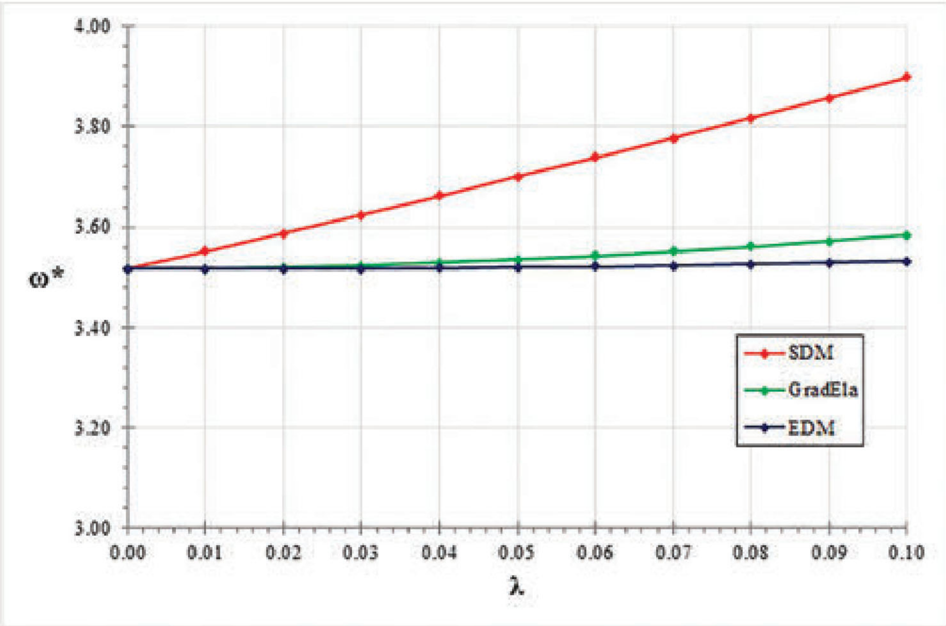

Fig. 1. Cantilever nano-beam: dimensionless fundamental natural frequency ω^* vs. nonlocal parameter λ, evaluated by SDM, EDM and GradEla.

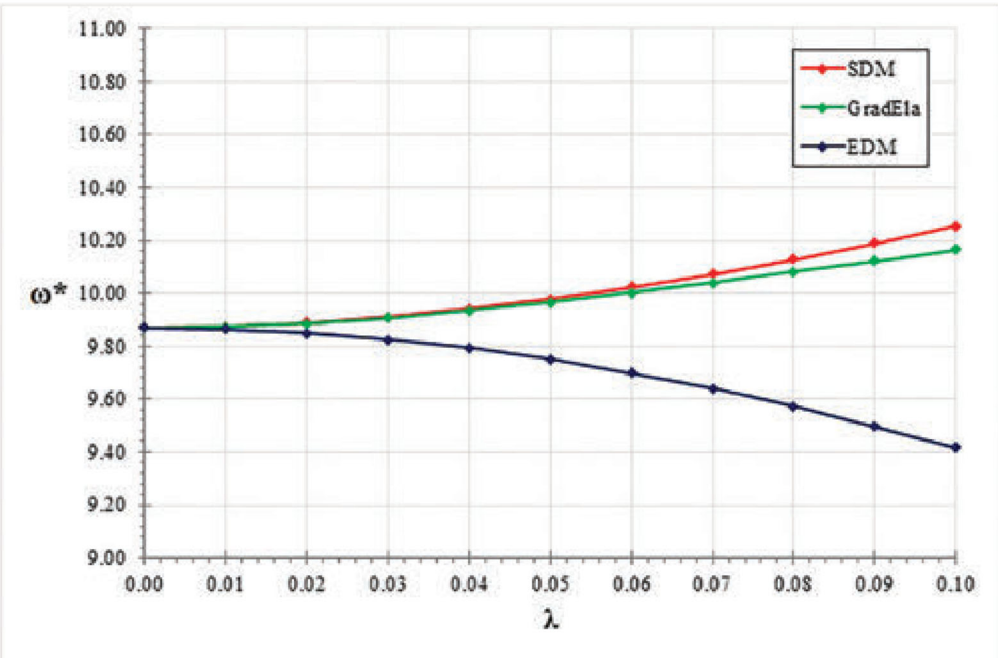

Fig. 2. Simply-supported nano-beam: dimensionless fundamental natural frequency ω^* vs. nonlocal parameter λ, evaluated by SDM, EDM and GradEla.

FIGURES

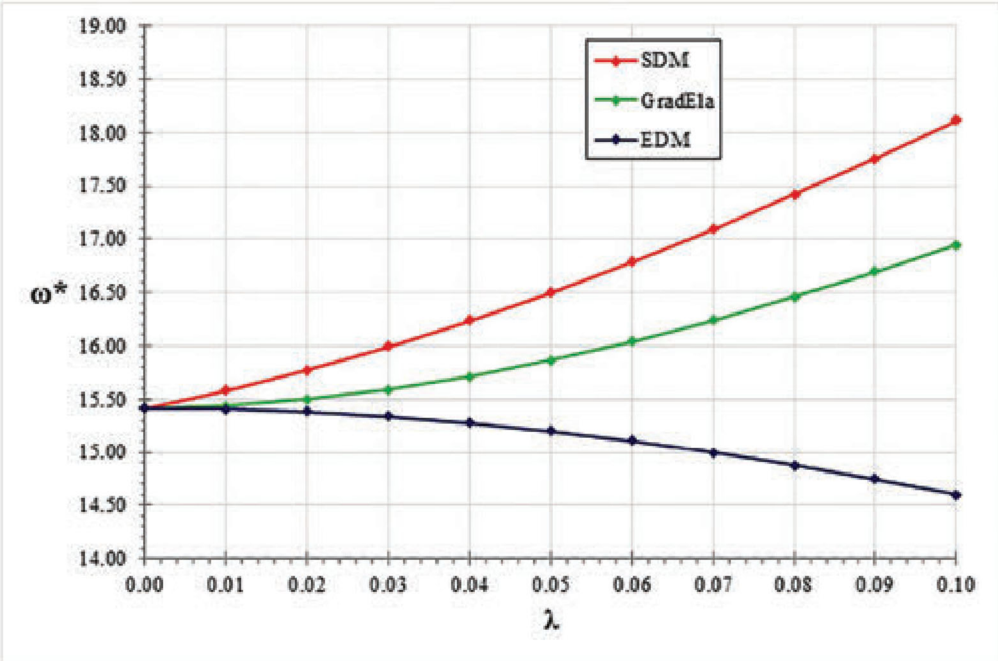

Fig. 3. Clamped-pinned nano-beam: fundamental natural frequency ω^* vs. nonlocal parameter λ, evaluated by SDM, EDM and GradEla.

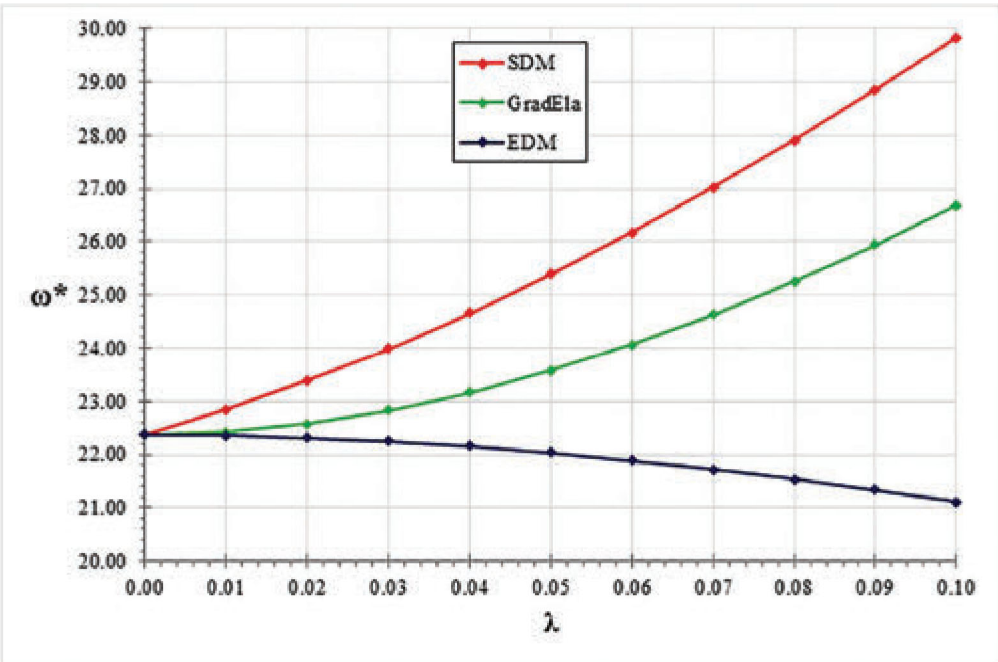

Fig. 4. Clamped-clamped nano-beam: dimensionless fundamental natural frequency ω^* vs. nonlocal parameter λ, evaluated by SDM, EDM and GradEla.

FIGURES

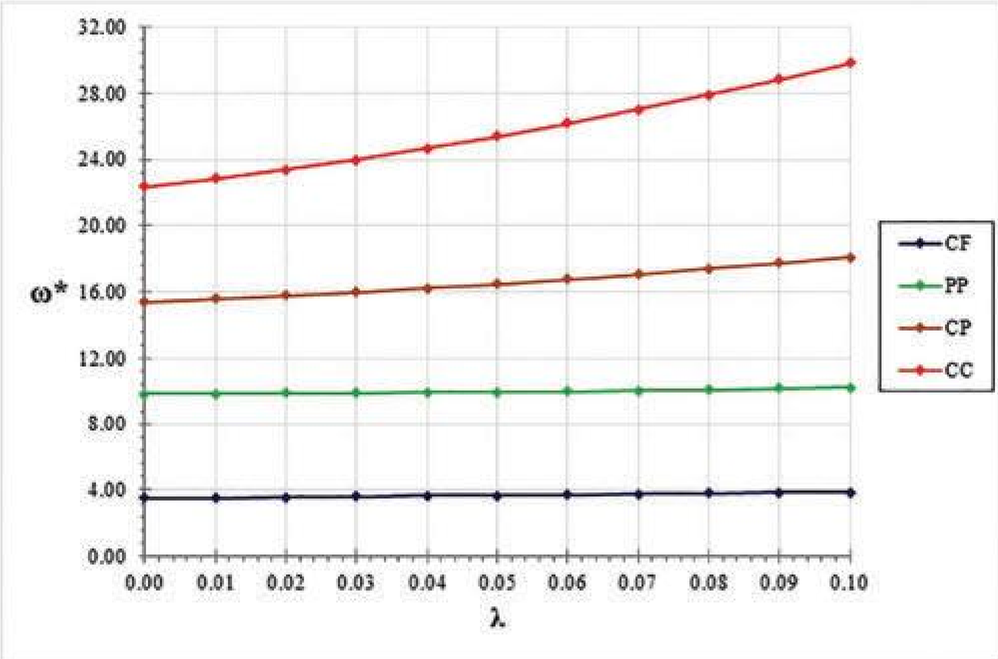

Fig. 5. Dimensionless fundamental natural frequency ω^* vs. nonlocal parameter λ, evaluated by SDM under standard kinematic boundary conditions.