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## **SMART AND SIMPLE DESIGN OF SEISMIC RESISTANT REINFORCED CONCRETE FRAME**

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**Abstract.** *In this paper a simple and a more sophisticated design procedure to design moment resisting concrete frames, is presented. This methodology allows to design structures having a smart behavior when subjected to seismic excitation. In fact, the structure develops the maximum number of dissipative zones by means of a particular collapse mechanism: the global one. The proposed procedure is based on the application of the kinematic theorem of the plastic collapse through the evaluation of the sum of the plastic moments of the columns required, at each storey, to prevent undesired failure modes such as soft-storey mechanism. In this work the authors show how the classical design methodology based on the beam-column hierarchy criterion does not allow to obtain a global mechanism. Beam-column hierarchy criterion, commonly suggested by seismic codes, appears only as a very rough approximation when compared to Theory of Plastic Mechanism Control (TPMC) and its theoretical background. By applying this classic methodology, if a global mechanism is obtained, the design of the column section and its reinforcement are not optimized. In fact, only with the theory already recalled we can obtain the minimum of section columns able to provide the development of a global mechanism. Significant improvements proposed by this approach include, among others, the possibility to account for different amount of reinforcement, not only at the top and bottom of the beam section, but also at the beam ends (left and right). A practical application of the TPMC process for the design of a multi-storey frame is presented, with push-over analysis that investigate the actual collapse mechanism of the designed structure. All the obtained results confirm the capability of the design procedure to achieve a collapse mechanism of global type. The importance of this theory and therefore of the design structures is the possibilities to maximize the energy dissipation capacity and global ductility because all the dissipative zones are involved in the corresponding yielding pattern.*

## 1. INTRODUCTION

In order to avoid undesired collapse mechanisms, hierarchy criterion, reported in all the modern seismic codes, suggests that at any joint, the sum of the flexural strength of the columns is greater than the sum of the flexural strength of the beams converging in the same joint [1, 2]. In a collapse mechanism of global type the energy dissipation capacity and global ductility supply are maximized because all the dissipative zones are involved in the corresponding pattern of yielding while all the other structural parts remain in elastic range. In fact, with reference to moment resisting frames, the maximum number of plastic hinges is obtained when two plastic hinges develops in each bay and they are usually located at beam ends. In a seismic resistant concrete frame, beams and columns are identified as dissipative and non-dissipative zones, respectively. These are the basic principles of capacity design approach, independently of the structural scheme and the constructional material [3, 5]. Unfortunately, the beam-column hierarchy criterion does not assure the development of a collapse mechanism of global type but it is, generally, only able to prevent “soft-storey” mechanisms; in fact, it is a non-rigorous application of capacity design principles. In addition, several research are devote in recent years to the understand of seismic collapse mode of reinforced concrete frames, the induced loss and the retrofiting technics to be adopted in order to obtain a better and more dissipative collapse mechanism both in case of existing structures [6, 9] and in case of new structures [10, 17].

In order to overcome this problem, a more sophisticated design procedure, based on the kinematic theorem of plastic collapse and on second order plastic analysis (i.e. the concept of mechanism equilibrium curve) has been presented. “Theory of Plastic Mechanism Control” (TPMC) has been obtained as a powerful tool for the seismic design. In particular, it consists on the extension of the kinematic theorem of plastic collapse to the concept of mechanism equilibrium curve. In fact, for any given structural typology, the design conditions to be applied in order to prevent undesired collapse mechanisms can be derived by imposing that the mechanism equilibrium curve, corresponding to the global mechanism, has to be located below those corresponding to all the other undesired mechanisms up to a displacement level compatible with the local ductility supply of dissipative zones. This design approach has been applied to different steel structural typologies such as MRFs with RBS connections, EB-Frames, dissipative truss-moment frames, MRF-CBF dual systems and MRFs equipped with friction dampers [18, 26]. Starting from the above background, the TPMC is developed also with reference to the reinforced concrete frames [27, 28]. In the present paper, a worked example and a validation of the proposed design procedure are presented.

Furthermore, the simplicity of this smart method will be emphasized by means of a worked example aiming to show its practical application which can be carried out even by means of hand calculations. In addition, static inelastic analyses are carried out to control the fulfilment of the desired collapse mechanism typology, i.e. a collapse mechanism of global type.

## 2. DESIGN WITH THEORY OF PLASTIC MECHANISM CONTROL

In general, three main collapse mechanism typologies that the structure is able to exhibit can be recognized: these mechanisms, depicted in Figure 1, are to be considered undesired because they do not involve all the dissipative zones.

Type-1 mechanism starts from the first storey level. Plastic hinges develop at the beam ends of all the  $i_m$ -th number of storeys involved, at the base section of the first storey columns and at the top section of the  $i_m$ -th storey columns. Type- 2 mechanism starts to the top of the structure and involves an  $i_m$ -th number of storey. Plastic hinges develop at all the beam ends and the base section of the  $i_m$ -th storey columns. Type-3 mechanism involves only one storey, and plastic hinges develop at the top and base section of the same storey columns. The design goal is the global mechanism, a particular case of type-2 mechanism, involving all the storeys.

TPMC allows the theoretical solution of the problem of designing a structure failing in global mode, i.e. assuring that plastic hinges develop only at beam ends while all the columns remain in elastic range with the only exception of base sections at first storey columns. At the base of the TPMC there is the concept of linearized mechanism equilibrium curve for each considered mechanism, expressed by:

$$\alpha = \alpha_0 - \gamma\delta \quad (1)$$

$\alpha$  depends on two parameters: the kinematically admissible multiplier of horizontal forces  $\alpha_0$  and the mechanism equilibrium curve  $\gamma$ ; both derived according to rigid-plastic theory, using the principle of virtual work.

For the better comprehension of the following, the adopted notation is reported to Table 1.

Within the framework of a kinematic approach, for any given collapse mechanism, the mechanism equilibrium curve can be easily derived by equalling the external work to the internal work. In the case of global type mechanism, for a virtual rotation  $d\theta$  of plastic hinges of the columns at first storey, the internal work can be expressed, for earthquake from Left to Right (LR) as:

$$W_{i,LR} = \left[ \sum_{i=1}^{n_c} M_{c,i,1} + \sum_{k=1}^{n_s} \sum_{j=1}^{n_b} M_{b,jk,L}^+ + \sum_{k=1}^{n_s} \sum_{j=1}^{n_b} M_{b,jk,R}^- \right] d\theta = [M_{c,1} + M_{b,Rd,L}^+ + M_{b,Rd,R}^-] d\theta \quad (2)$$

For earthquake from Right to Left (RL):

$$W_{i,RL} = \left[ \sum_{i=1}^{n_c} M_{c,i,1} + \sum_{k=1}^{n_s} \sum_{j=1}^{n_b} M_{b,jk,L}^- + \sum_{k=1}^{n_s} \sum_{j=1}^{n_b} M_{b,jk,R}^+ \right] d\theta = [M_{c,1} + M_{b,Rd,L}^- + M_{b,Rd,R}^+] d\theta \quad (3)$$

The external work due to the horizontal forces, is:

$$W_e = \left[ \alpha \sum_{k=1}^{n_s} F_k h_k \right] d\theta = [\alpha M_F] d\theta \quad (4)$$

This value is the same for both LR and RL seismic input direction.

Therefore, the application of the virtual work principle provides the kinematically admissible multiplier that, for LR earthquake, can be written as:

$$\alpha_{0,LR}^{(g)} = \frac{[M_{c,1} + M_{b,Rd,L}^+ + M_{b,Rd,R}^-]}{M_F} \quad (5)$$

and for RL earthquake:

$$\alpha_{0,RL}^{(g)} = \frac{[M_{c,1} + M_{b,Rd,L}^- + M_{b,Rd,R}^+]}{M_F} \quad (6)$$

In addition, in order to account for second-order effects, the external second-order work due to vertical load is also evaluated.

$$W_v = \sum_{k=1}^{n_s} V_k h_k \frac{\delta}{H_o} d\theta = M_V \frac{\delta}{H_o} d\theta \quad (7)$$

This quantity, as for external work  $W_e$  of Eq. (4), is not dependent on seismic input direction. By accounting for this value, the virtual work principle can be written, for LR earthquake, as:

$$W_{i,LR} = W_e + W_v \quad (8)$$

By substituting Eqs. (2), (4) and (7) in Eq. (8) the following relation can be obtained:

$$[M_{c,1} + M_{b,Rd,L}^+ + M_{b,Rd,R}^-]d\theta = [\alpha M_F]d\theta + M_V \frac{\delta}{H_o} d\theta \quad (9)$$

By means of simple steps it is immediately recognized the form of the linearized mechanism equilibrium curve expressed by Eq. (1). So for LR earthquake:

$$\alpha_{LR}^{(g)} = \alpha_{0,LR}^{(g)} - \gamma^{(g)}\delta = \frac{M_{c,1} + M_{b,Rd,L}^+ + M_{b,Rd,R}^-}{M_F} - \frac{1}{M_F} \frac{M_V}{H_o} \delta \quad (10)$$

In the same way for RL earthquake the virtual work principle provides:

$$W_{i,RL} = W_e + W_v \quad (11)$$

and by substituting Eqs. (3), (4) and (7) in Eq. (11):

$$[M_{c,1} + M_{b,Rd,L}^- + M_{b,Rd,R}^+]d\theta = [\alpha M_F]d\theta + M_V \frac{\delta}{H_o} d\theta \quad (12)$$

So for RL earthquake:

$$\alpha_{RL}^{(g)} = \alpha_{0,RL}^{(g)} - \gamma^{(g)}\delta = \frac{M_{c,1} + M_{b,Rd,L}^- + M_{b,Rd,R}^+}{M_F} - \frac{1}{M_F} \frac{M_V}{H_o} \delta \quad (13)$$

Therefore, the slope of the mechanism equilibrium curve  $\gamma$ , can be easily obtained. In the case of global mechanism it is given by:

$$\gamma^{(g)} = \frac{1}{H_o} \frac{M_V}{M_F} = \frac{1}{h_{n_s}} \frac{M_V}{M_F} \quad (14)$$

also this parameter is not dependent on the seismic input direction.

Therefore, the linearized mechanism equilibrium curves of global mechanism  $\alpha_{LR} = \alpha_{0,LR}^{(g)} - \gamma^{(g)}\delta$  and  $\alpha_{RL} = \alpha_{0,RL}^{(g)} - \gamma^{(g)}\delta$  are completely defined.

For each considered mechanism (Figure 1) a mechanism equilibrium curve can be obtained. In particular, for the  $i_m$ -th mechanism ( $i_m = 1, 2, \dots, n_s$ ) of the  $t$ -th mechanism typology ( $t = 1, 2, 3$ ) the application of kinematic theorem of plastic collapse provides for LR earthquake:

$$\alpha_{i_m,LR}^{(t)} = \alpha_{0,i_m,LR}^{(t)} - \gamma_{i_m}^{(t)} \delta \quad t = 1,2,3 \quad i_m = 1,2, \dots, n_s \quad (15)$$

While for RL earthquake:

$$\alpha_{i_m,RL}^{(t)} = \alpha_{0,i_m,RL}^{(t)} - \gamma_{i_m}^{(t)} \delta \quad t = 1,2,3 \quad i_m = 1,2, \dots, n_s \quad (16)$$

where  $\alpha_{0,im}^{(t)}$  and  $\gamma_{im}^{(t)}$  represent, respectively, the kinematically admissible multiplier and the slope of mechanism equilibrium curve of the  $i_m$ -th mechanism of the  $t$ -th mechanism typology. As a consequence, the unknowns of the design problem are the column sections. They could be determined by means of design conditions expressing that the kinematically admissible multiplier corresponding to the global mechanism is the minimum among all kinematically admissible multipliers corresponding to all other mechanisms (Figure 1). Obviously, this design condition is able to assure the desired collapse mechanism only in case of rigid-plastic behaviour, while actual structures are characterized by elastic displacements before the development of a plastic mechanism. Due to these elastic displacements, second-order effects of vertical loads cannot be neglected. These effects can be taken into account by imposing that the mechanism equilibrium curve corresponding to the global mechanism has to lie below those corresponding to all other mechanisms i.e. the upper bound theorem of plastic design is to be satisfied for each value of the displacements  $\delta$  (Figure 2).

However, the fulfilment of this requirement is necessary only up to a selected ultimate displacement  $\delta_u$ , which has to be compatible with the ductility supply of structural members.

This condition corresponds to impose that, for LR earthquake:

$$\alpha_{LR}^{(g)} = \alpha_{0,LR}^{(g)} - \gamma^{(g)} \delta_u \leq \alpha_{0,im,LR}^{(t)} - \gamma_{im}^{(t)} \delta_u = \alpha_{im,LR}^{(t)} \quad (17)$$

and for RL earthquake:

$$\alpha_{RL}^{(g)} = \alpha_{0,RL}^{(g)} - \gamma^{(g)} \delta_u \leq \alpha_{0,im,RL}^{(t)} - \gamma_{im}^{(t)} \delta_u = \alpha_{im,RL}^{(t)} \quad (18)$$

for  $i_m = 1, 2, 3, \dots, n_s$  and  $t = 1, 2, 3$ .

Therefore, there are  $6n_s$  design conditions to be satisfied for a structural scheme having  $n_s$  storeys. With reference to  $i_m$ -th mechanism of type-1, the kinematically admissible multiplier of seismic horizontal forces is given, for LR earthquake, by:

$$\alpha_{0,im,LR}^{(1)} = \frac{M_{c,1} + \sum_{k=1}^{i_m-1} \sum_{j=1}^{n_b} M_{b,jk,L}^+ + \sum_{k=1}^{i_m-1} \sum_{j=1}^{n_b} M_{b,jk,R}^- + M_{c,im}}{\sum_{k=1}^{i_m} F_k h_k + h_{i_m} \sum_{k=i_m+1}^{n_s} F_k} \quad (19)$$

and for RL earthquake by:

$$\alpha_{0,im,RL}^{(1)} = \frac{M_{c,1} + \sum_{k=1}^{i_m-1} \sum_{j=1}^{n_b} M_{b,jk,L}^- + \sum_{k=1}^{i_m-1} \sum_{j=1}^{n_b} M_{b,jk,R}^+ + M_{c,im}}{\sum_{k=1}^{i_m} F_k h_k + h_{i_m} \sum_{k=i_m+1}^{n_s} F_k} \quad (20)$$

while the slope of the mechanism equilibrium curve, which is the same for both directions, is:

$$\gamma_{im}^{(1)} = \frac{\mathbf{1} \sum_{k=1}^{i_m} V_k h_k + h_{i_m} \sum_{k=i_m+1}^{n_s} V_k}{h_{i_m} \sum_{k=1}^{i_m} F_k h_k + h_{i_m} \sum_{k=i_m+1}^{n_s} F_k} \quad (21)$$

With reference to  $i_m$ -th mechanism of type-2 the kinematically admissible multiplier of seismic horizontal forces is given, for LR earthquake, by:

$$\alpha_{0,im,LR}^{(2)} = \frac{\mathbf{M}_{c,im} + \sum_{k=i_m}^{n_s} \sum_{j=1}^{n_b} \mathbf{M}_{b,jk,L}^+ + \sum_{k=i_m}^{n_s} \sum_{j=1}^{n_b} \mathbf{M}_{b,jk,R}^-}{\sum_{k=i_m}^{n_s} \mathbf{F}_k (h_k - h_{i_m-1})} \quad (22)$$

and for RL earthquake by:

$$\alpha_{0,i_m,RL}^{(2)} = \frac{\mathbf{M}_{c,i_m} + \sum_{k=i_m}^{n_s} \sum_{j=1}^{n_b} \mathbf{M}_{b,jk,L}^- + \sum_{k=i_m}^{n_s} \sum_{j=1}^{n_b} \mathbf{M}_{b,jk,R}^+}{\sum_{k=i_m}^{n_s} \mathbf{F}_k(\mathbf{h}_k - \mathbf{h}_{i_m-1})} \quad (23)$$

while the slope of the mechanism equilibrium curve is:

$$\gamma_{i_m}^{(2)} = \frac{\mathbf{1}}{\mathbf{h}_{n_s} - \mathbf{h}_{i_m-1}} \frac{\sum_{k=i_m}^{n_s} \mathbf{V}_k(\mathbf{h}_k - \mathbf{h}_{i_m-1})}{\sum_{k=i_m}^{n_s} \mathbf{F}_k(\mathbf{h}_k - \mathbf{h}_{i_m-1})} \quad (24)$$

Finally, with reference to  $i_m$ -th mechanism of type-3, the kinematically admissible multiplier of horizontal forces, is given by:

$$\alpha_{0,i_m}^{(3)} = \frac{2\mathbf{M}_{c,i_m}}{(\mathbf{h}_{i_m} - \mathbf{h}_{i_m-1}) \sum_{k=i_m}^{n_s} \mathbf{F}_k} \quad (25)$$

In this case the expression is the same for both directions of earthquake because the beams are not involved in this collapse mechanism.

In addition, the corresponding slope of the mechanism equilibrium curve is given by:

$$\gamma_{i_m}^{(3)} = \frac{\sum_{k=i_m}^{n_s} \mathbf{V}_k}{(\mathbf{h}_{i_m} - \mathbf{h}_{i_m-1}) \sum_{k=i_m}^{n_s} \mathbf{F}_k} \quad (26)$$

It is important to underline that, for any given geometry of the structural system, the slope of mechanism equilibrium curve attains its minimum value when the global type mechanism is developed. In fact, it is easy to check that  $\gamma^{(g)}$ , which is equal to  $\gamma_1^{(2)}$ , is always the minimum value among all the  $\gamma_{i_m}^{(t)}$ . This issue assumes a paramount importance in TPMC allowing the extension of the kinematic theorem of plastic collapse to the concept of mechanism equilibrium curve by simply checking relations (17) and (18) for the value  $\delta = \delta_u$ , as depicted in Figure 2.

### 3. WORKED EXAMPLE

In order to show the practical application of the proposed design procedure, the seismic design of a four-bay four-storey moment resisting frame is presented in this section. The inelastic behaviour of the designed structure is successively examined by means of a push-over static inelastic analysis, confirming the fulfilment of the design goal, i.e. the location of the yielding zones at the beam ends with the only exception of the base section of first-storey columns. The structural scheme of the frame to be designed is shown in Figure 3. The interstorey height is equal to 3m. The characteristic values of the vertical loads acting on the beams are equal to 21.89 kN/m and 8 kN/m for permanent ( $G_k$ ) and live ( $Q_k$ ) actions, respectively. The structural materials adopted are concrete C25/30 and reinforcement of steel grade B450C. According to Eurocode 8, the value of the period of vibration to be used for preliminary design is:

$$T = 0.075 H^{3/4} = 0.075 \cdot 12^{3/4} \approx 0.48 \text{ s} \quad (27)$$

where  $H$  is the total height of the frame.

With reference to the design spectrum for stiff soil conditions (soil class A of Eurocode 8) and by assuming a behaviour factor  $q$  equal to 3.9, the horizontal seismic forces are those depicted in Figure 3.

In the following, the numerical development of the design steps for the structural scheme described above is provided.

*a) Selection of the design top sway displacement*

The selection of the maximum top sway displacement up to which the global mechanism has to be assured is a very important design issue, because the value of this displacement governs the magnitude of second order effects accounted for in the design procedure. A good criterion to choose the design ultimate displacement  $\delta_u$  is to relate it to the plastic rotation supply of beams or beam-to-column connections by assuming  $\delta_u = \theta_u \cdot h_{ns}$  (where  $\theta_u$  can be assumed equal to 0.01 rad).

As a consequence, the design value of the top sway displacement has been assumed equal to:

$$\delta_u = 0.01 \cdot h_{ns} = 0.01 \cdot 12 = 0.12 \text{ m} \quad (28)$$

*b) Design of beam sections to withstand vertical loads.*

The load acting on the frame in the vertical load combination is:

$$Q_{SLU} = 1.3 G_k + 1.5 Q_k = 40.46 \text{ kN/m} \quad (29)$$

For the design of the beams has been considered a bending moment equal to:

$$M_{max} = \frac{Q_{SLU} \cdot L^2}{8} \quad (30)$$

Therefore, by imposing the base of the section equal to  $b=30$  cm, is possible to calculate the height of the beam through the following design relation:

$$d = r \sqrt{\frac{M_{Sd}}{b}} \quad (31)$$

where  $d$  is the effective depth of the cross-section,  $M_{Sd}$  is the design value of the applied internal bending moment and  $r$  is a coefficient, function of different factors (normalized neutral axis depth  $\xi$ , reinforcement ratio between compressive and tension bars  $\rho$ , design value of concrete compressive strength  $f_{cd}$ , design value of yield strength of steel  $f_{sd}$ ).

Assuming  $\xi = 0.25$  and  $\rho = 0.25$  a value of  $r = 0.19$  is obtained. As a consequence the amount of reinforcement is given by:

$$A_s = \frac{M_{Sd}}{0.85 \cdot h \cdot f_{sd}} \quad (32)$$

Obviously the number of steel bars in the beam is such that:

$$M_{Rd} > M_{Sd} \quad (33)$$

where  $M_{Rd}$  is the design value of the resistant moment and  $M_{Sd}$  is the design value of the applied internal bending moment. The reinforcement at the beam ends are reported in Table 2.

*c) Computation of the slopes of mechanism equilibrium curve  $\gamma_{i_m}^{(t)}$ .*

By means of Eqs. (21), (24) and (26) the slopes of mechanism equilibrium curves are computed. These values are reported in Table 3.

In particular it is important to underline that the slope value corresponding to the global mechanism  $\gamma^{(g)} = \gamma_1^{(2)}$ , is the minimum among all the  $\gamma_{i_m}^{(t)}$  values:

$$\gamma^{(g)} = 0.0034 \text{ cm}^{-1} \quad (34)$$

*d) Computation of the required sum of plastic moments of columns at first storey  $M_{c,1}$ .*

The required sums of plastic moments of columns at first storey for LR and RL earthquake are equal to  $\mathbf{M}_{c,1,LR} = 1381.52 \text{ kNm}$  and  $\mathbf{M}_{c,1,RL} = 1373.18 \text{ kNm}$ , respectively.

*e) Distribution among the columns proportionally to their number.*

According to the global mechanism, axial forces in the columns at collapse state depend both from the distributed loads acting on the beams and from the shear action due to the development of plastic hinges at the beam ends, as depicted in Figure 4 (with reference to the earthquake from Left to Right). So that, the total load transmitted by the beams to the columns is the sum of two contributions. The first one,  $N_q$ , is related to the vertical loads acting in the seismic load combination (i.e. the sum of  $ql/2$  type contributions).

In Table 4 the axial forces due to vertical loads, for both directions of earthquake, are reported for each storey and for each column.

The second one,  $N_{M,LR}$  (or  $N_{M,RL}$ ), is related to the shear actions due to the plastic hinges developed at the beam ends i.e. the sum of  $(M_{b,jk,L}^+ + M_{b,jk,R}^-)/l$  for earthquake from left to right (or the sum of  $(M_{b,jk,L}^- + M_{b,jk,R}^+)/l$  for earthquake from right to left). In Table 5 these contributions are reported.

Therefore the required bending moment for each column  $M_{c,i,1}$ , the section, the upper and lower reinforcement and the axial force, for both directions of the earthquake are reported in Table 6.

The sums of obtained column plastic moments at first storey are:  $M_{c,Rd,1,LR} = 1708.70 \text{ kNm}$  for LR earthquake and  $M_{c,Rd,1,RL} = 1696.97 \text{ kNm}$  for RL earthquake which are greater than the required one. As a consequence the value of  $\alpha_0^{(g)}$  for LR and RL earthquake are equal to  $\alpha_{0,LR}^{(g)} = 2.3856$  and  $\alpha_{0,RL}^{(g)} = 2.3815$ , respectively.

*f) Computation of the required sum of plastic moments of columns  $M_{c,im}^{(t)}$  at any storey to avoid undesired mechanism.*

The sum of the plastic moments of columns governing the column design at each storey is given, by the underlined values, in Table 7 for earthquake from Left to Right and in Table 8 for earthquake from Right to Left.

*g) Design of column sections at each storey.*

The required sum of column plastic moments  $M_{c,i,i_m}$ , the section, the upper and lower reinforcement, the axial force for both directions of the earthquake are reported in Table 9

*h) Checking of technological condition*

By observing Table 6 and Table 9 it can be noted that there are some column sections at the first storey which are smaller than the corresponding ones required at the second storey, therefore, a technological condition at the first storey is not satisfied. As a consequence, the

values of  $M_{c,Rd,1,LR}$  and  $M_{c,Rd,1,RL}$  need to be updated and the procedure needs to be repeated from the step e).

#### 4. VALIDATION OF THE DESIGN PROCEDURE

In order to validate the design procedure, a static non-linear analysis has been carried out by means SAP2000 computer program [29]. This analysis has the primary aim to confirm the development of the desired collapse mechanism typology and to evaluate the obtained energy dissipation capacity, testing the accuracy of the proposed design methodology.

Regarding the structural modelling, the mechanical non-linearities, have been concentrated at beam and column ends by means of plastic hinge elements. The constitutive law of such plastic hinge elements is provided by a rigid plastic moment-rotation curve. The type of hinge depends on the element considered i.e. by its internal action. In fact, for the beams and the columns M3 and P-M3 hinge type have been considered, respectively. In case of P-M3 hinge type, the interaction domain P-M has been evaluated for each column and used in SAP2000 computer program. The results of the push-over analysis are mainly constituted by base shear – top sway displacement curve which is depicted in Figure 5.

In this figure is possible to observe, through continuous lines, the push-over curve and the linearized mechanism equilibrium curve of global mechanism, for earthquake from Left to Right, whose expression is:

$$\alpha_{LR}^{(g)} = 2.3856 - 0.003388 \delta \quad (35)$$

Dash-dot lines are used for earthquake to Right to Left and the expression is:

$$\alpha_{RL}^{(g)} = 2.3815 - 0.003388 \delta \quad (36)$$

As already underlined there is a mechanism equilibrium curve for both direction of earthquake. The two mechanism equilibrium curves are different but only for what concern the  $\alpha_0$  value (even if the difference is very small) while the slope is the same.

As it was expected, also the LR push-over curve is different from RL one. This difference can be easily understood if we consider that the axial forces in the columns are different in the two considered push-over and, as a consequence, also the plastic moment is different.

Notwithstanding, in this case, the two curves are very close one each other but there is no proof of the fact that this represent a general result. So that both curves should be always considered when a non-symmetric moment-resisting frame is analyzed.

A further confirmation, even the most important, of the fulfilment of the design objective is represented by the pattern of yielding developed at the occurrence of the design ultimate displacement. In fact, developed plastic hinges are shown in Figure 6 and their pattern is in perfect agreement with the global mechanism.

In order to fulfill the serviceability requirements the interstorey drift have been checked with reference to the limit reported in the Eurocode 8. In particular the considered limit refers to buildings having non structural elements of brittle materials attached to the structure:

$$d_r \leq 0.005 h \quad (37)$$

where  $d_r$  is the design interstorey drift, evaluated as the difference between the lateral displacements  $d_s$  at the top and bottom of the storey under consideration,  $\nu$  is the reduction factor which takes into account the lower return period of the seismic action associated with the damage limitation requirement and  $h$  is the corresponding interstorey height. The value of the reduction factor  $\nu$  depend on the importance class of the building (a value of  $\nu = 0.5$  was adopted). In Table 10 the interstorey limits are reported together the absolute  $d_s$  and relative  $d_r$  displacements. If this serviceability requirement is not verified the structural stiffness can be improved by increasing the beam sections or the ultimate design displacement. In fact, in both cases the final results will be a more rigid structure with respect to the one obtained in the worked example herein presented. In addition, it useful to note that in the design procedure also the effect of modified section could be considered. In fact, in order to increase the ductility of a reinforced concrete section an external confinement can be used [30,31].

## 5. CONCLUSIONS

In this paper a simple and a more sophisticated design procedure to design moment resisting concrete frames, is presented. On the base of the extension of the kinematic theorem of plastic collapse to the concept of mechanism equilibrium curve, the Theory of Plastic Mechanism Control allows to develop a collapse mechanism of global type through the evaluation of the sum of plastic moments of the columns required at each storey. The closed form solution of the design conditions makes the design procedure very easy to be applied. The classical design methodology based on the beam-column hierarchy criterion appears only as a very rough approximation when compared to TPMC and its theoretical background. The simplicity and the reliability of the proposed design procedure has been also demonstrated through its application to a four-bays, four-storey frame, leading to the fulfilment of the design objective, i.e. the development of a collapse mechanism of global type, as it has been confirmed by the results of both push-over static inelastic analysis. The proposed methodology can be considered as belonging to the Performance Based Seismic Design philosophy [32]. In fact, in order to satisfy the limit states of “Life Safe” or “Near Collapse” the designer has to promote a dissipative collapse mechanism avoiding the so called “soft storey mechanism”. The proposed procedure constitutes a rigorous application of the capacity design principles. In fact, beams are designed in order to bear external loads, while columns are designed according to the maximum internal actions transmitted by the dissipative zones. In the next future the research activity will be devoted to account also for the influence of joists on the collapse mechanism. This influence is always ignored, but it could change the designed collapse mechanism [33].

## REFERENCES

- [1] EN 1998-1, “*Eurocode 8: Design of Structures for Earthquake Resistance – Part 1: general Rules, Seismic Actions and Rules for Buildings*”, CEN, 2004.
- [2] New Zealand Standard Code of Practice for the Design of Concrete Structures, NZS 3101: Part 1; Commentary NZS 3101: Part 2; Standard Association of New Zealand, Wellington, New Zealand, 1982.

- [3] Lee H.S., “*Revised rule for concept of strong-column weak-girder design*”, J. struct. eng. ASCE, 122, 359-364, 1996.
- [4] Paulay T., “*Seismic design of ductile moment resisting reinforced concrete frames, columns: evaluation of actions*”, Bull. New Zealand Natl. Soc. Earthquake Eng., 10, 85-94, 1977.
- [5] Paulay T., “*Deterministic Design Procedure for Ductile Frames in Seismic Areas*”, ACI Publication SP-63, American Concrete Institute, Detroit, pp. 357-381, 1980.
- [6] R. Montuori, V. Piluso, “*Reinforced concrete columns strengthened with angles and battens subjected to eccentric load*”. Eng Struct 2009; 31:539–50. Doi: 10.1016/j.engstruct.2008.10.005.
- [7] M. D'Aniello, G. Della Corte, F. M. Mazzolani, “*Seismic upgrading of RC buildings by steel eccentric braces: Experimental results vs numerical modeling*” - Proceedings of the 5th International Conference on Behaviour of Steel Structures in Seismic Areas - Stessa 200, Pages 809-814 Yokohama; Japan; 14- 17, August 2006.
- [8] M. D'Aniello, G. Della Corte, F. M. Mazzolani, “*Seismic upgrading of RC buildings by buckling restrained braces: Experimental results vs numerical modeling*” - Proceedings of the 5th International Conference on Behaviour of Steel Structures in Seismic Areas - Stessa 200, Pages 809-814 Yokohama; Japan; 14- 17, August 2006.
- [9] G. Della Corte, M. D'Aniello, R. Landolfo, “*Field testing of all-steel buckling-restrained braces applied to a damaged reinforced concrete building*” - Journal of Structural Engineering (United States), Volume 141, Issue 1, 1 January 2015.
- [10] P. Castaldo, “*Integrated Seismic Design of Structure and Control Systems*”. Springer International Publishing: New York, 2014. Doi: 10.1007/978-3-319-02615-2.
- [11] P. Castaldo, E. Tubaldi, “*Influence of FPS bearing properties on the seismic performance of base-isolated structures*”. Earthquake Engineering and Structural Dynamics 2015; 44 (15): 2817–2836.
- [12] P. Castaldo, B. Palazzo, P. Della Vecchia, “*Seismic reliability of base-isolated structures with friction pendulum bearings*”. Engineering Structures 2015; 95:80-93.
- [13] B. Palazzo, P. Castaldo, P. Della Vecchia, “*Seismic reliability analysis of base-isolated structures with friction pendulum system*”, 2014 IEEE Workshop on Environmental, Energy and Structural Monitoring Systems Proceedings Napoli September 17-18, 2014.
- [14] P. Castaldo, M. De Iuliis, “*Optimal integrated seismic design of structural and viscoelastic bracing-damper systems*”. Earthquake Engineering and Structural Dynamics, Volume 43, Issue 12, Pages 1809-1827, 10 October 2014.

- [15] M. De Iuliis, P. Castaldo, “*An energy-based approach to the seismic control of one-way asymmetrical structural systems using semi-active devices*”, *Ingegneria Sismica - International Journal of Earthquake Engineering*, 2012; XXIX(4): 31-42.
- [16] Castaldo P., Palazzo B., Della Vecchia P., (2016) “*Life-cycle cost and seismic reliability analysis of 3D systems equipped with FPS for different isolation degrees*”, *Engineering Structures* 125; 349–363, <http://dx.doi.org/10.1016/j.engstruct.2016.06.056>.
- [17] Castaldo P., Amendola G., Palazzo B., (2016) “*Seismic fragility and reliability of structures isolated by friction pendulum devices: Seismic reliability-based design (SRBD)*”, accepted for publication in *Earthquake Engineering and Structural Dynamics*, DOI:10.1002/eqe.2798.
- [18] Mazzolani F. M., Piluso V., (1997) “*Plastic Design of Seismic Resistant Steel Frames*”, *Earthquake Engineering and Structural Dynamics*, Vol. 26, pp. 167-191.
- [19] Montuori R., Nastri E., Piluso V., (2014) “*Advances in theory of plastic mechanism control: closed form solution for MR-Frames*”. *Earthquake Engineering & Structural Dynamics*, DOI: 10.1002/eqe.2498, pp. 1035–1054.
- [20] Montuori, R., Nastri, E., Piluso, V.(2014) *Theory of plastic mechanism control for the seismic design of braced frames equipped with friction dampers*. *Mechanics Research Communications*, 58, pp. 112-123. DOI: 10.1016/j.mechrescom.2013.10.020
- [21] Piluso, V., Montuori, R., Troisi, M.(2014) *Innovative structural details in MR-frames for free from damage structures*. *Mechanics Research Communications*, 58, pp. 146-156. DOI: 10.1016/j.mechrescom.2014.04.002
- [22] Montuori, R., Nastri, E., Piluso, V. (2016) *Theory of Plastic Mechanism Control for MRF-EBF dual systems: Closed form solution*. *Engineering Structures*, 118, pp. 287-306. DOI: 10.1016/j.engstruct.2016.03.050
- [23] Montuori, R., Nastri, E., Piluso, V.(2014) *Rigid-plastic analysis and moment-shear interaction for hierarchy criteria of inverted y EB-Frames*. *Journal of Constructional Steel Research*, 95, pp. 71-80. DOI: 10.1016/j.jcsr.2013.11.013
- [24] Montuori, R., Nastri, E., Piluso, V. (2014. )*Theory of plastic mechanism control for eccentrically braced frames with inverted y-scheme*. *Journal of Constructional Steel Research*, 92, pp. 122-135. DOI: 10.1016/j.jcsr.2013.10.009
- [25] Longo, A., Montuori, R., Piluso, V. (2012.) *Failure mode control and seismic response of dissipative truss moment frames*. *Journal of Structural Engineering (United States)*, 138 (11), pp. 1388-1397. DOI: 10.1061/(ASCE)ST.1943-541X.0000569
- [26] Longo, A., Montuori, R., Piluso, V. (2012) *Theory of plastic mechanism control of dissipative truss moment frames*. *Engineering Structures*, 37, pp. 63-75. DOI: 10.1016/j.engstruct.2011.12.046

- [27] Montuori R., Muscati R., “*A General Procedure for Failure Mechanism Control of Reinforced Concrete Frames*”, *Engineering Structures* 118 (2016), 137-155.
- [28] Montuori, R., Muscati, R.(2015) “*Plastic design of seismic resistant reinforced concrete frame*”. *Earthquake and Structures*, 8 (1), pp. 205-224. DOI: 10.12989/eas.2015.8.1.205
- [29] CSI 2007. *SAP 2000: Integrated Finite Element Analysis and Design of Structures*. Analysis Reference. Computer and Structure Inc. University of California, Berkeley.
- [30] Montuori, R., Piluso, V., Tisi, A.(2012)*Comparative analysis and critical issues of the main constitutive laws for concrete elements confined with FRP* *Composites Part B: Engineering*, 43 (8), pp. 3219-3230. DOI: 10.1016/j.compositesb.2012.04.001
- [31] Montuori, R., Piluso, V., Tisi, A.(2013) *Ultimate behaviour of FRP wrapped sections under axial force and bending: Influence of stress-strain confinement model* *Composites Part B: Engineering*, 54 (1), pp. 85-96. DOI:10.1016/j.compositesb.2013.04.059
- [32] SEAOC. *Vision 2000 - A Framework for Performance Based Design*, Volumes I, II, III. Structural Engineers Association of California, Vision 2000 Committee. Sacramento, California, 1995.
- [33] Montuori, R., Nastri, E., Piluso, V.(2016) *Modelling of floor joists contribution to the lateral stiffness of RC buildings designed for gravity loads*. *Engineering Structures*, 121, pp. 85-96. DOI: 10.1016/j.engstruct.2016.04.046.

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