A GENERAL PROCEDURE FOR FAILURE MECHANISM CONTROL OF REINFORCED CONCRETE FRAMES

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Abstract. *In this paper new advances for designing moment resisting concrete frames, failing in a global mode (TPMC: Theory of Plastic Mechanism Control), are presented.*
The theory has been developed in the nineties with reference to moment-resisting frames (MRFs) and progressively extended to several steel structural typologies commonly adopted as seismic-resistant structural system.
The procedure is based on the application of the kinematic theorem of the plastic collapse.
In particular, the outcome of the theory is the evaluation of the sum of the plastic moments of the columns required, at each storey, to prevent undesired failure modes such as soft-storey mechanism. In the proposed method the second-order effects, due to vertical loads, can play an important role in the seismic design of reinforced concrete frames; they can be taken into account by mean the mechanism equilibrium curve of the considered collapse mechanism.
The advances presented in this paper are constituted by the possibility to consider different values of reinforcement, not only at the top and bottom part of the beam section, but also at the beam ends (left and right). The practical application of the TPMC with reference to the design of a multi-storey frame is now presented. In addition both push-over analyses and non-linear dynamic analyses have been made to investigate the actual collapse mechanism of the designed structure. All the obtained results confirm the ability of the design procedure to obtain a collapse mechanism of global type.

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1 INTRODUCTION

Nowadays, the primary purpose of the structural design consists in avoiding collapse mechanism characterized by the development of the minimum energy dissipation capacity of the structure, such as “soft-storey” mechanism, assuring the development of a collapse mechanism of global type. In particular, such kind of mechanism is characterized by the location of plastic hinges at all the beam ends and at the base sections of first storey columns.

Relatively to the moment resisting frames, the maximum number of plastic hinges is obtained when two plastic hinges develops in each bay and they are usually located at beam ends. However, for particular load conditions, plastic hinges can develop also in the mid span of the bay. In a collapse mechanism of global type the energy dissipation capacity and global ductility supply are maximized because all the dissipative zones are involved in the corresponding pattern of yielding. Conversely, all the other structural parts remain in elastic range. In fact, in the design of structures, it is important to identify both dissipative and non dissipative zones. The first ones are designed according to the internal actions arising from the seismic forces provided by the codes; the second ones are proportioned on the basis of the maximum internal actions transmitted by the dissipative zones. In a seismic resistant concrete frame, beams are identified as dissipative zones while columns are identified as non-dissipative zones. These are the basic principles of capacity design approach, independently of the structural scheme and the constructional material [1 - 3]. In order to avoid undesired collapse mechanisms hierarchy criterion, reported in all the modern seismic codes, suggests that at any joint, the sum of the flexural strength of the columns is greater than the sum of the flexural strength of the beams converging in the same joint [4, 5]. Unfortunately, the beam-column hierarchy criterion, being based on simple joint equilibrium, is only able to prevent “soft-storey” mechanisms, but it does not assure the development of a collapse mechanism of global type; in fact it is a non-rigorous application of capacity design principles [6 - 15]. In addition, several research are devote in recent years to the understand of seismic collapse mode of reinforced concrete frames, the induced loss and the retrofitting technics to be adopted in order to obtain a better and more dissipative collapse mechanism both in case of new structures [16-23]and in case of existing structures. [24-31]

For this reason, a more sophisticated design procedure, based on the kinematic theorem of plastic collapse and on second order plastic analysis (i.e. the concept of mechanism equilibrium curve) has been presented in 1997 [32] with reference to steel moment resisting frames. Starting from this first work, the “Theory of Plastic Mechanism Control” (TPMC) has been obtained as a powerful tool for the seismic design. In particular, it consists on the extension of the kinematic theorem of plastic collapse to the concept of mechanism equilibrium curve. In fact, for any given structural typology, the design conditions to be applied in order to prevent undesired collapse mechanisms can be derived by imposing that the mechanism equilibrium curve, corresponding to the global mechanism, has to be located below those corresponding to all the other undesired mechanisms up to a displacement level compatible with the local ductility supply of dissipative zones. This design approach was successively extended to different steel structural typologies such as MRFs with RBS connections, EB-Frames, dissipative truss-moment frames, MRF-CBF dual systems and

MRFs equipped with friction dampers [33-38]. So, it can be concluded that structures in high seismicity zones are normally designed in order to avoid the yielding of columns because the desired goal is always the development of a global mechanism.

Starting from the above background, the “Theory of Plastic Mechanism Control” is developed also with reference to the reinforced concrete frames. In the design procedure presented in [39] there is a very important limitation: the reinforcement at the beam ends is exactly the same. This hypothesis is very far from practical applications, because in an actual reinforced concrete moment resisting frames is almost unlikely that, for each beam, the reinforcements at its ends are equal. In the present paper his limitation is resolved and relations are modified accordingly. Furthermore, the simplicity of the proposed method will be emphasized by means of a worked example aiming to show its practical application which can be carried out even by means of hand calculations. In addition, static and dynamic inelastic analyses are carried out to control the fulfilment of the desired collapse mechanism typology, i.e. a collapse mechanism of global type.

1. THEORY OF PLASTIC MECHANISM CONTROL

In general, three main collapse mechanism typologies that the structure is able to exhibit can be recognized: these mechanisms, depicted in Figure 1, are to be considered undesired because they do not involve all the dissipative zones.

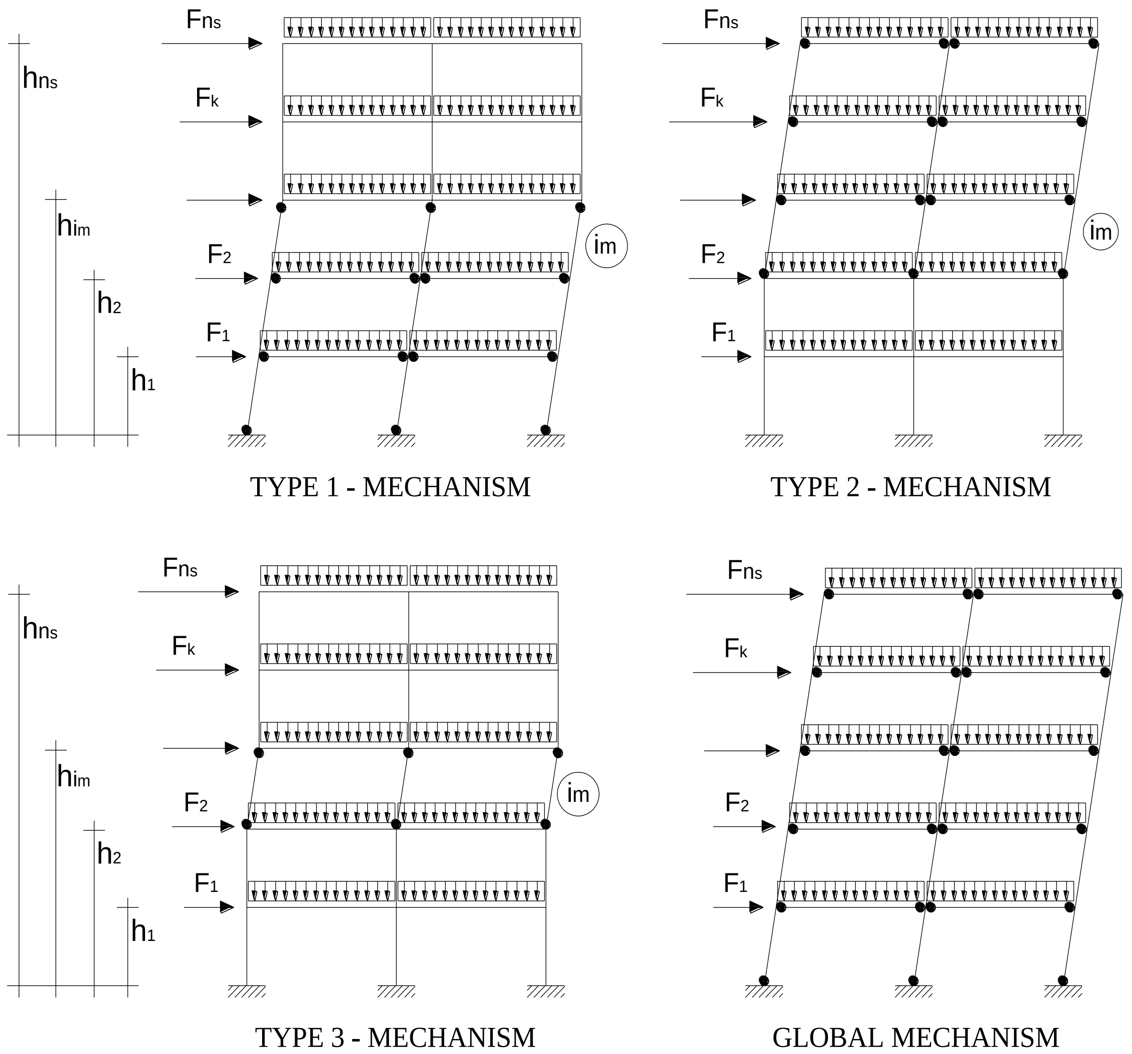

Figure 1: Collapse mechanism typologies

Type-1 mechanism starts from the first storey level and involves an i_m-th number of storeys, for this reason plastic hinges develop at the beam ends of all the storeys involved, at the base section of the first storey columns and at the top section of the i_m-th storey columns. Type- 2 mechanism is a particular kind of mechanism which starts to the top of the structure and involves an i_m-th number of storey. In particular, all the beam ends and the base section of the i_m-th storey columns develop plastic hinges. The global mechanism, representing the design goal, is a particular case of type-2 mechanism involving all the storeys. Finally, type-3 mechanism involves only one storey, so that plastic hinges develop at the base and top section of the same storey columns. It has to be considered the worst mechanism because involves only the columns which are to be considered less dissipative than the beam sections. TPMC allows the theoretical solution of the problem of designing a structure failing in global mode, i.e. assuring that plastic hinges develop only at beam ends while all the columns remain in elastic range with the only exception of base sections at first storey columns. In order to apply the TPMC it is of paramount importance the introduction of the concept of linearized mechanism equilibrium curve for each considered mechanism. The mathematical expression of this curve can be written as:

$$\alpha = \alpha_0 - \gamma\delta \tag{1}$$

where $\boldsymbol{\alpha_0}$ is the kinematically admissible multiplier of horizontal forces and $\boldsymbol{\gamma}$ is the slope of the mechanism equilibrium curve. Both parameters can be derived, according to rigid-plastic theory, using the principle of virtual work. Within the framework of a kinematic approach, for any given collapse mechanism, the mechanism equilibrium curve can be easily derived by equalling the external work to the internal work. In addition, in order to account for second-order effects, the external second-order work due to vertical load is also evaluated.

It is important to underline that in this paper the procedure is complete because it allows to overcome all the limitations of the previous version. Now it is possible to consider a generic frame characterized by a non-symmetric configuration both in section reinforcement (different top and bottom reinforcement) and in beam reinforcement (different left and right ends reinforcement). Obviously it means that we have to consider both directions of the earthquake because the sum of plastic design resistance of beams is different for two directions.

For the better comprehension of the following, the adopted notation is reported to Table 1.

Table 1: Notation

n_c	number of columns	$M_{c,i,im}$	plastic moment of the i-th column at i_m-th storey
n_b	number of bays	$M_{c,im} = \sum_{i=1}^{n_c} M_{c,i,im}$	sum of plastic moments of columns at i_m-th storey
n_s	number of storeys	$M_V = \sum_{k=1}^{n_s} V_k h_k$	second-order work due to vertical loads in global mechanism
i_m	index of mechanism	$M_F = \sum_{k=1}^{n_s} F_k h_k$	external work due to horizontal forces in the global mechanism
H_o	sum of the interstorey heights of the storeys involved by the generic mechanism	$F = \sum_{k=1}^{n_s} F_k$	sum of the horizontal forces
h_k	height of the k-th storey (with k=1, 2, .., n_s)	$M_{b.jk}$	plastic design resistance of beam at j-th bay of the k-th storey
e	index of beam ends (e=L=left end, e=R=right end)	$M_{b,Rd,e} = \sum_{k=1}^{n_s} \sum_{j=1}^{n_b} M_b$	sum of the plastic design resistances of beam ends (for e end) in the global mechanism

It is important to note that the quantities involved in the evolution of internal work, for each considered mechanism, depend on the direction of the seismic input. In fact, due to the non-symmetric configuration of reinforcement (top and bottom at each end of the beam) we have four different plastic moments for each beam. When a collapse mechanism involves the generic beam, two of them are to be used for earthquake direction from left to right (LR) and, the other two, are to be used for earthquake from right to left (RL) (Figure 2).

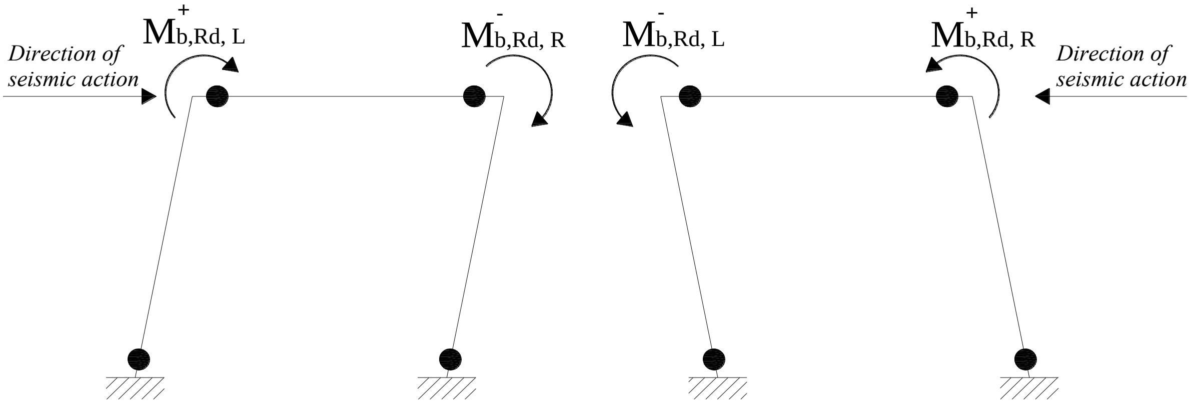

Figure 2: Plastic moment to be considered for LR and RL earthquake direction for each beam involved in the considered collapse mechanism

Concerning the evaluation of the kinematically admissible multiplier of horizontal forces, corresponding to the generic mechanism, it is easy to recognise that, in the case of global type mechanism, for a virtual rotation $d\theta$ of plastic hinges of the columns at first storey, the internal work can be expressed, for earthquake from Left to Right (LR) as:

$$W_{i,LR} = \left[\sum_{i=1}^{n_c} M_{c,i,1} + \sum_{k=1}^{n_s}\sum_{j=1}^{n_b} M^{+}_{b.jk,L} + \sum_{k=1}^{n_s}\sum_{j=1}^{n_b} M^{-}_{b.jk,R}\right] d\theta = \left[M_{c,1} + M^{+}_{b,Rd,L} + M^{-}_{b,Rd,R}\right] d\theta \quad (2)$$

For earthquake from Right to Left (RL):

$$W_{i,RL} = \left[\sum_{i=1}^{n_c} M_{c,i,1} + \sum_{k=1}^{n_s}\sum_{j=1}^{n_b} M^{-}_{b.jk,L} + \sum_{k=1}^{n_s}\sum_{j=1}^{n_b} M^{+}_{b.jk,R}\right] d\theta = \left[M_{c,1} + M^{-}_{b,Rd,L} + M^{+}_{b,Rd,R}\right] d\theta \quad (3)$$

The external work due to the horizontal forces, is:

$$W_e = \left[\alpha \sum_{k=1}^{n_s} F_k h_k\right] d\theta = [\alpha M_F] d\theta \quad (4)$$

This value is the same for both LR and RL seismic input direction.

Therefore the application of the virtual work principle provides the kinematically admissible multiplier that, for LR earthquake, can be written as:

$$\alpha^{(g)}_{0,LR} = \frac{\left[M_{c,1} + M^{+}_{b,Rd,L} + M^{-}_{b,Rd,R}\right]}{M_F} \quad (5)$$

and for RL earthquake:

$$\alpha^{(g)}_{0,RL} = \frac{\left[M_{c,1} + M^{-}_{b,Rd,L} + M^{+}_{b,Rd,R}\right]}{M_F} \quad (6)$$

In order to compute the slope of the mechanism equilibrium curve, it is necessary to evaluate the second-order work due to vertical loads. With reference to Figure 3, it can be observed that the horizontal displacement of the k-th storey involved in the generic mechanism is given by $u_k = r_k sin\theta$, where r_k is the distance of the k-th storey from the centre of rotation C and θ the angle of rotation. The top sway displacement is given by $\delta = H_o sin\theta$, where H_o is the sum of the interstorey heights of the storeys involved by the generic mechanism. In the case of global type mechanism, as shown in Figure 1, all the storeys participate to the collapse mechanism, so that $H_o = h_{ns}$. The relationship between vertical and horizontal virtual displacements is given by (Figure 3):

$$dv_k = du_k tan\theta \approx du_k sin\theta = du_k \frac{\delta}{H_o} \tag{7}$$

It shows that, as the ratio dv_k/du_k is independent from the considered storey, vertical and horizontal virtual displacement vectors have the same shape. In fact, the virtual horizontal displacements are given by:

$$du_k = r_k cos\theta d\theta \approx r_k d\theta \tag{8}$$

By substituting Eq. (8) in Eq. (7), the virtual vertical displacements are given by:

$$dv_k = \frac{\delta}{H_o} r_k d\theta \tag{9}$$

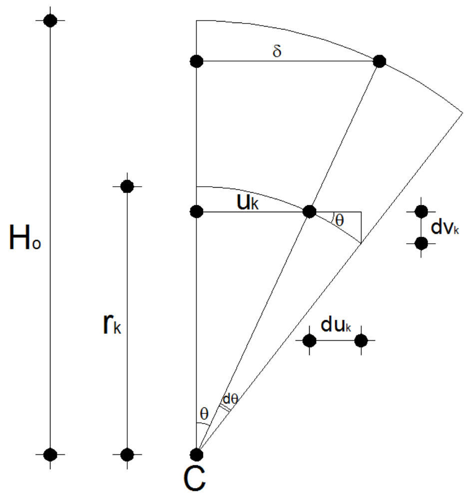

Figure 3: Second order vertical displacements

and, therefore, they have the same shape r_k of the horizontal ones. As a consequence, the second-order work due to vertical loads for the global mechanism is given by:

$$W_v = \sum_{k=1}^{n_s} V_k h_k \frac{\delta}{H_o} d\theta = M_V \frac{\delta}{H_o} d\theta \tag{10}$$

This quantity, as for external work W_e of Eq. (4), is not dependent on seismic input direction. By accounting for this value, the virtual work principle can be written, for LR earthquake, as:

$$W_{i,LR} = W_e + W_v \tag{11}$$

By substituting Eqs. (2), (4) and (10) in Eq. (11) the following relation can be obtained:

$$\left[M_{c,1} + M_{b,Rd,L}^{+} + M_{b,Rd,R}^{-}\right] d\theta = [\alpha M_F] d\theta + M_V \frac{\delta}{H_o} d\theta \tag{12}$$

In the same way for RL earthquake the virtual work principle provides:

$$W_{i,RL} = W_e + W_v \tag{13}$$

and by substituting Eqs. (3), (4) and (10) in Eq. (13):

$$\left[M_{c,1} + M_{b,Rd,L}^{-} + M_{b,Rd,R}^{+}\right] d\theta = [\alpha M_F] d\theta + M_V \frac{\delta}{H_o} d\theta \tag{14}$$

By means of simple steps it is immediately recognized the form of the linearized mechanism equilibrium curve expressed by Eq. (1). So for LR earthquake:

$$\alpha_{LR}^{(g)} = \alpha_{0,LR}^{(g)} - \gamma^{(g)}\delta = \frac{M_{c,1} + M_{b,Rd,L}^{+} + M_{b,Rd,R}^{-}}{M_F} - \frac{\frac{1}{H_o} M_V}{M_F}\delta \tag{15}$$

while for RL earthquake:

$$\alpha_{RL}^{(g)} = \alpha_{0,RL}^{(g)} - \gamma^{(g)}\delta = \frac{M_{c,1} + M_{b,Rd,L}^{-} + M_{b,Rd,R}^{+}}{M_F} - \frac{\frac{1}{H_o} M_V}{M_F}\delta \tag{16}$$

Therefore, the slope of the mechanism equilibrium curve γ, can be easily obtained. In the case of global mechanism it is given by:

$$\gamma^{(g)} = \frac{\frac{1}{H_o} M_V}{M_F} = \frac{\frac{1}{h_{n_s}} M_V}{M_F} \tag{17}$$

also this parameter is not dependent on the seismic input direction.

Therefore, the linearized mechanism equilibrium curves of global mechanism $\alpha_{LR} = \alpha_{0,LR}^{(g)} - \gamma^{(g)}\delta$ and $\alpha_{RL} = \alpha_{0,RL}^{(g)} - \gamma^{(g)}\delta$ are completely defined. It can be useful to underline that the linearization of equilibrium curve is due to the small displacement theory adopted in Eq. (8). In fact, due to this assumption, second-order work due to vertical loads is linear and as a consequence, also the mechanism equilibrium curve is linear.

For each considered mechanism (Figure 1) a mechanism equilibrium curve can be obtained. In particular, for the $\mathrm{i_m}$-th mechanism ($\mathrm{i_m} = 1, 2, \dots, n_s$) of the t-th mechanism typology ($t = 1, 2, 3$) the application of kinematic theorem of plastic collapse provides for LR earthquake:

$$\alpha_{im,LR}^{(t)} = \alpha_{0,im,LR}^{(t)} - \gamma_{im}^{(t)}\,\delta \qquad t = 1,2,3 \qquad i_m = 1,2,\dots n_s \tag{18}$$

While for RL earthquake:

$$\alpha_{im,RL}^{(t)} = \alpha_{0,im,RL}^{(t)} - \gamma_{im}^{(t)}\,\delta \qquad t = 1,2,3 \qquad i_m = 1,2,\dots n_s \tag{19}$$

where $\alpha_{0,im}^{(t)}$ and $\gamma_{im}^{(t)}$ represent, respectively, the kinematically admissible multiplier and the slope of mechanism equilibrium curve of the i_m-th mechanism of the t-th mechanism typology. In the proposed method the beam section properties are assumed to be known quantities because they are designed to resist vertical loads. As a consequence, the unknowns of the design problem are the column sections. They could be determined by means of design conditions expressing that the kinematically admissible multiplier corresponding to the global mechanism is the minimum among all kinematically admissible multipliers corresponding to all other mechanisms (Figure 1). Obviously, this design condition is able to assure the desired collapse mechanism only in case of rigid-plastic behaviour, while actual structures are characterized by elastic displacements before the development of a plastic mechanism. Due to these elastic displacements, second-order effects of vertical loads cannot be neglected. These effects can be taken into account by imposing that the mechanism equilibrium curve corresponding to the global mechanism has to lie below those corresponding to all other mechanisms i.e. the upper bound theorem of plastic design is to be satisfied for each value of the displacements δ (Figure 4).

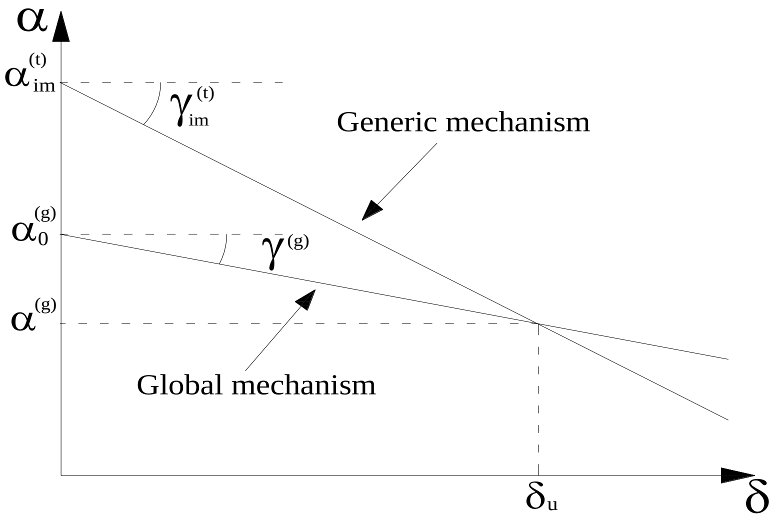

Figure 4: Design condition

However, the fulfilment of this requirement is necessary only up to a selected ultimate displacement δ_u, which has to be compatible with the ductility supply of structural members.

This corresponds to impose the following conditions, for LR earthquake:

$$\alpha^{(g)}_{LR} = \alpha^{(g)}_{0,LR} - \gamma^{(g)}\,\delta_u \,\leq\, \alpha^{(t)}_{0,im,LR} - \gamma^{(t)}_{im}\,\delta_u = \alpha^{(t)}_{im,LR} \tag{20}$$

and for RL earthquake:

$$\alpha^{(g)}_{RL} = \alpha^{(g)}_{0,RL} - \gamma^{(g)}\,\delta_u \,\leq\, \alpha^{(t)}_{0,im,RL} - \gamma^{(t)}_{im}\,\delta_u = \alpha^{(t)}_{im,RL} \tag{21}$$

for $i_m = 1, 2\ 3, \dots, n_s$ and $t = 1, 2, 3$.

Therefore, there are $\boldsymbol{6n_s}$ design conditions to be satisfied for a structural scheme having $\boldsymbol{n_s}$ storeys. With reference to $\boldsymbol{i_m}$-th mechanism of type-1, the kinematically admissible multiplier of seismic horizontal forces is given, for LR earthquake, by:

$$\alpha^{(1)}_{0,i_{m,LR}} = \frac{M_{c,1} + \sum_{k=1}^{i_m-1}\sum_{j=1}^{n_b} M^{+}_{b.jk,L} + \sum_{k=1}^{i_m-1}\sum_{j=1}^{n_b} M^{-}_{b.jk,R} + M_{c,im}}{\sum_{k=1}^{i_m} F_k h_k + h_{i_m}\sum_{k=i_m+1}^{n_s} F_k} \tag{22}$$

and for RL earthquake by:

$$\alpha^{(1)}_{0,i_{m,RL}} = \frac{M_{c,1} + \sum_{k=1}^{i_m-1}\sum_{j=1}^{n_b} M^{-}_{b.jk,L} + \sum_{k=1}^{i_m-1}\sum_{j=1}^{n_b} M^{+}_{b.jk,R} + M_{c,im}}{\sum_{k=1}^{i_m} F_k h_k + h_{i_m}\sum_{k=i_m+1}^{n_s} F_k} \tag{23}$$

while the slope of the mechanism equilibrium curve, which is the same for both directions, is:

$$\gamma^{(1)}_{i_m} = \frac{1}{h_{i_m}}\frac{\sum_{k=1}^{i_m} V_k h_k + h_{i_m}\sum_{k=i_m+1}^{n_s} V_k}{\sum_{k=1}^{i_m} F_k h_k + h_{i_m}\sum_{k=i_m+1}^{n_s} F_k} \tag{24}$$

With reference to i_m-th mechanism of type-2 the kinematically admissible multiplier of seismic horizontal forces is given, for LR earthquake, by:

$$\alpha_{0,i_{m,LR}}^{(2)} = \frac{M_{c,im} + \sum_{k=i_m}^{n_s} \sum_{j=1}^{n_b} M_{b.jk,L}^{+} + \sum_{k=i_m}^{n_s} \sum_{j=1}^{n_b} M_{b.jk,R}^{-}}{\sum_{k=i_m}^{n_s} F_k (h_k - h_{i_m - 1})} \tag{25}$$

and for RL earthquake by:

$$\alpha_{0,i_{m,RL}}^{(2)} = \frac{M_{c,im} + \sum_{k=i_m}^{n_s} \sum_{j=1}^{n_b} M_{b.jk,L}^{-} + \sum_{k=i_m}^{n_s} \sum_{j=1}^{n_b} M_{b.jk,R}^{+}}{\sum_{k=i_m}^{n_s} F_k (h_k - h_{i_m - 1})} \tag{26}$$

while the slope of the mechanism equilibrium curve is:

$$\gamma_{i_m}^{(2)} = \frac{1}{h_{n_s} - h_{i_m - 1}} \frac{\sum_{k=i_m}^{n_s} V_k \left(h_k - h_{i_m - 1}\right)}{\sum_{k=i_m}^{n_s} F_k (h_k - h_{i_m - 1})} \tag{27}$$

It is useful to note that, for $i_m = 1$ Eqs. (25), (26) and (27) are coincident with Eq. (5), (6) and (17) respectively, because in such case the mechanism is coincident with the global one. In addition, these relations for $i_m = 1$ include the term $h_{i_m - 1} = h_0$ which is to be assumed equal to zero. Finally, with reference to i_m-th mechanism of type-3, the kinematically admissible multiplier of horizontal forces, is given by:

$$\alpha_{0,i_m}^{(3)} = \frac{2M_{c,im}}{\left(h_{i_m} - h_{i_m - 1}\right) \sum_{k=i_m}^{n_s} F_k} \tag{28}$$

In this case the expression is the same for both directions of earthquake because the beams are not involved in this collapse mechanism.

In addition, the corresponding slope of the mechanism equilibrium curve is given by:

$$\gamma_{\mathrm{i_m}}^{(3)} = \frac{\sum_{\mathrm{k=i_m}}^{\mathrm{n_s}} \mathrm{V_k}}{\left(\mathrm{h_{i_m}} - \mathrm{h_{i_m - 1}}\right) \sum_{\mathrm{k=i_m}}^{\mathrm{n_s}} \mathrm{F_k}} \tag{29}$$

It is important to underline that, for any given geometry of the structural system, the slope of mechanism equilibrium curve attains its minimum value when the global type mechanism is developed. In fact, it is easy to check that $\gamma^{(g)}$, which is equal to $\gamma_1^{(2)}$, is always the minimum value among all the $\gamma_{i_m}^{(t)}$. This issue assumes a paramount importance in TPMC allowing the extension of the kinematic theorem of plastic collapse to the concept of mechanism equilibrium curve by simply checking relations (20) and (21) for the value $\delta = \delta_u$, as depicted in Figure 4.

2. DESIGN ALGORITHM

The above mentioned relations can be used to design concrete frames failing in global mode and, therefore, having a mechanism equilibrium curve given by Eq. (1), with the kinematically admissible multiplier of horizontal forces given by Eq. (5) for LR earthquake and by Eq. (6) for RL earthquake and the slope given by relation (17). In particular the design algorithm is constituted by the following steps:

a) Selection of a design top sway displacement δ_u compatible with the ductility supply of structural members. To this scope the plastic rotation capacity of beams can be assumed equal to 0.01 rad so that $\delta_u = 0.01 \cdot h_{ns}$ where h_{n_s} is the height of the structure.

b) Design of beam sections to withstand vertical loads acting in the non-seismic load combination. The preliminary design of beam can be made by considering a bending moment belonging to the range $qL^2/8 \div qL^2/10$ being q the load acting on the beam in the vertical load combination.

c) Computation, by means of Eqs. (24), (27) and (29), of the slopes of mechanism equilibrium curves $\gamma_{i_m}^{(t)}$ which are known quantities because they depend on loads (vertical and horizontal) and frame geometry.

d) For each considered i_m value, Eqs. (20) and (21) provides the following relations where the unknown quantities are represented by the sum of required plastic moments of columns at i_m-th storey and at first storey ($\mathrm{M_{c,im}}$ and $\mathrm{M_{c,1}}$) for the two seismic directions. It is important to note that for $i_m = 1$ and $t = 2$, Eqs. (20) and (21) are an identity because global mechanism is obtained. Furthermore, for $i_m = 1$, type 1 and type 3 mechanisms are coincident. This observation can be immediately derived from Figure 1 and, in addition, it is easy to check that $\alpha_{0,1}^{(1)} = \alpha_{0,1}^{(3)}$ and $\gamma_1^{(1)} = \gamma_1^{(3)}$.

As a consequence, for $i_m = 1$ there is only a design condition where the only unknown is represented by $M_{c,1}$. In fact, for LR earthquake, by substituting in Eq. (20) the values of $\alpha_{0,LR}^{(g)}$, $\alpha_{0,1,LR}^{(3)}\left(or\ \alpha_{0,1,LR}^{(1)}\right)$ and $\gamma_{1,LR}^{(3)}\left(or\ \gamma_{1,LR}^{(1)}\right)$ we obtain:

$$M_{c,1,LR} \geq \frac{M_{b,Rd,L}^{+} + M_{b,Rd,R}^{-} + \left(\gamma_1^{(3)} - \gamma^{(g)}\right) \cdot M_F \cdot \delta_u}{2\dfrac{M_F}{h_1 F} - 1} \tag{30}$$

For RL earthquake, by substituting in Eq. (21) the values of $\alpha_{0,RL}^{(g)}$, $\alpha_{0,1,RL}^{(3)}\left(or\ \alpha_{0,1,RL}^{(1)}\right)$ and $\gamma_{1,RL}^{(3)}\left(or\ \gamma_{1,RL}^{(1)}\right)$ it can be obtained:

$$M_{c,1,RL} \geq \frac{M_{b,Rd,L}^{-} + M_{b,Rd,R}^{+} + \left(\gamma_1^{(3)} - \gamma^{(g)}\right) \cdot M_F \cdot \delta_u}{2\dfrac{M_F}{h_1 F} - 1} \tag{31}$$

In this way the sum of the required plastic moments at the columns of the first storey are known, for both directions of the seismic input. The above relations are of paramount importance from the practical point of view, because it allows to design first storey columns by means of a closed form solution easy to be applied by simple calculations. In fact, in equations (30) and (31) all the quantities on the right side are immediately known when external horizontal forces are known and the reinforcement for all beams have been evaluated.

e) The sum of the required plastic moments of columns can be distributed among the columns in different ways which are at the discretion of the designer. In this case, the following simple rule can be adopted:

$$M_{c,i,1,LR} = \frac{M_{c,1,LR}}{n_c} \qquad i = 1, 2, \dots, n_c \tag{32}$$

and for the opposite earthquake in the opposite direction:

$$M_{c,i,1,RL} = \frac{M_{c,1,RL}}{n_c} \qquad i = 1, 2, \dots, n_c \tag{33}$$

It is important to underline that the way of distributing the sum of required plastic moments, expressed by upper equations, is not mandatory, in fact, any other distribution among the columns of storey 1, having as sum the value $M_{c,i,1}$, is perfectly equivalent.

f) Design of the columns at first storey. It starts by considering a section able to resist to vertical loads at ultimate limit state ($N_{V,SLU}$). The base of the section is calculated with the following equation:

$$h = \frac{N_{V,SLU}}{\nu \cdot b \cdot f_{cd}} = \frac{N_{V,SLU}}{0.5 \cdot b \cdot f_{cd}} \tag{34}$$

For a given value of b and h, the reinforcements of the section can be designed.

The shape of the M-N interaction domain, for a concrete frame, does not give, immediately, the value of the design axial force because, differently as happens in steel members, the maximum axial force does not necessarily implicate the worst condition. The problem can be solved by considering two values of axial forces:

- a first value N_{T_LR}, given by the sum of the axial forces due to vertical loads, in the seismic load combination ($N_{q,LR}$) and the axial forces related to the shear actions due to the plastic hinges developed at the beam ends for earthquake from left to right (N_{M_LR}), (Figure 6):

$$N_{T\_LR} = N_{q,LR} + N_{M\_LR} \tag{35}$$

- a second value N_{T_RL}, given by the same equation in which appear $N_{q,RL}$ and the axial forces related to the shear actions due to the plastic hinges developed at the beam ends for earthquake from right to left (N_{M_RL}), (Figure 6):

$$N_{T\_RL} = N_{q,RL} + N_{M\_RL} \tag{36}$$

For a generic column and for a fixed direction of the earthquake, if the axial load is given by Eq. (35), then, for the opposite direction of horizontal forces, the axial load contribution is given by Eq. (36). To each of these axial forces is associated the corresponding value of the design moment expressed by the Eqs. (32) and (33) respectively.

In conclusion the design points are:

$$A\left(N_{T\_LR}\,,\ M_{c,i,1,LR}\right) \qquad\qquad B\left(N_{T\_RL}\,, M_{c,i,1,RL}\right) \tag{37}$$

g) Design of the reinforcement of columns at first storey. If the maximum percentage of reinforcement prescribed by the regulations is reached, then the procedure is repeated by the step f) by increasing the section dimensions. Once the columns are designed, the obtained value of $M_{c,1}$, namely $M_{c,Rd,1}$, is generally greater than the required minimum value provided by Eqs. (32) and (33). Therefore, the kinematically admissible multiplier is to be evaluated accordingly, i.e. by means of Eq (5) for LR earthquake and by the Eq. (6) for the RL earthquake, by replacing the term $M_{c,1}$ with the value $M_{c,Rd,1}$ resulting from designed sections.

By applying this design algorithm the columns at first storey are designed to resist to both directions of earthquake. Obviously, the sum of the resistant plastic moments are different because the axial forces in the columns change with the direction of the seismic input. So that in Eq. (5) we will have a value $M_{c,Rd,1,LR}$ which, generally, is different from the value $M_{c,Rd,1,RL}$ that will be replaced in Eq. (6).

h) Computation of the required sum of plastic moments of columns $M_{c,im}$ for $i_m > 1$ imposing that the i_m-th mechanism equilibrium curves of type 1, 2 and 3 have to be located above the curve of global one, i.e. by applying relation (20) and (21). In fact, for a fixed value of i_m, relation (20) and (21) provide three values of $M_{c,im,LR}^{(t)}$ and $M_{c,im,RL}^{(t)}$, respectively, for $t = 1,2,3$. In particular, in order to avoid the i_m-th mechanism of type 1, the minimum required value of $M_{c,im}$ is for LR earthquake:

$$M_{c,im,LR}^{(1)} \geq \left(\alpha_{0,LR}^{(g)} - \gamma^{(g)}\delta + \gamma_{i_m}^{(1)}\delta_u\right)\left(\sum_{k=1}^{i_m} F_k h_k + h_{i_m}\sum_{k=i_m+1}^{n_s} F_k\right) - M_{c,Rd,1} + \\ - \sum_{k=1}^{i_m-1}\sum_{j=1}^{n_b} M_{b.jk,L}^{+} - \sum_{k=1}^{i_m-1}\sum_{j=1}^{n_b} M_{b.jk,R}^{-} \tag{38}$$

while for RL earthquake:

$$M_{c,im,RL}^{(1)} \geq \left(\alpha_{0,RL}^{(g)} - \gamma^{(g)}\delta + \gamma_{i_m}^{(1)}\delta_u\right)\left(\sum_{k=1}^{i_m} F_k h_k + h_{i_m}\sum_{k=i_m+1}^{n_s} F_k\right) - M_{c,Rd,1} + \\ - \sum_{k=1}^{i_m-1}\sum_{j=1}^{n_b} M_{b.jk,L}^{-} - \sum_{k=1}^{i_m-1}\sum_{j=1}^{n_b} M_{b.jk,R}^{+} \tag{39}$$

In addition, in order to avoid the i_m-th mechanism of type-2, the minimum required value of $M_{c,im}$, for LR earthquake is:

$$M_{c,im,LR}^{(2)} \geq \left(\alpha_{0,LR}^{(g)} - \gamma^{(g)}\delta + \gamma_{i_m}^{(2)}\delta_u\right)\sum_{k=i_m}^{n_s} F_k\left(h_k - h_{i_m-1}\right) + \\ - \sum_{k=i_m}^{n_s}\sum_{j=1}^{n_b} M_{b.jk,L}^{+} - \sum_{k=i_m}^{n_s}\sum_{j=1}^{n_b} M_{b.jk,R}^{-} \tag{40}$$

while for RL earthquake:

$$M_{c,im,RL}^{(2)} \geq \left(\alpha_{0,RL}^{(g)} - \gamma^{(g)}\delta + \gamma_{i_m}^{(2)}\delta_u\right)\sum_{k=i_m}^{n_s} F_k\left(h_k - h_{i_m-1}\right) + \\ - \sum_{k=i_m}^{n_s}\sum_{j=1}^{n_b} M_{b.jk,L}^{-} - \sum_{k=i_m}^{n_s}\sum_{j=1}^{n_b} M_{b.jk,R}^{+} \tag{41}$$

Finally to avoid the i_m-th mechanism of type-3, the minimum required value of $M_{c,im}$ is:

$$M_{c,im,LR}^{(3)} \geq \left(\alpha_{0,LR}^{(g)} - \gamma^{(g)}\delta + \gamma_{i_m}^{(3)}\delta_u\right)\frac{\left(h_{i_m} - h_{i_m-1}\right)}{2}\sum_{k=i_m}^{n_s} F_k \tag{42}$$

while for RL earthquake:

$$M_{c,im,RL}^{(3)} \geq \left(\alpha_{0,RL}^{(g)} - \gamma^{(g)}\delta + \gamma_{i_m}^{(3)}\delta_u\right)\frac{\left(h_{i_m} - h_{i_m-1}\right)}{2}\sum_{k=i_m}^{n_s} F_k \tag{43}$$

The relations now written have been derived from Eqs. (20) and (21) for $\boldsymbol{i_m > 1}$ and $\boldsymbol{t = 1}$, $\boldsymbol{t = 2}$ and $\boldsymbol{t = 3}$, respectively.

In order to develop a collapse mechanism of global type for a LR earthquake we have to satisfy relations (38), (40) and (42), so that the required sum of plastic moments for the $\mathbf{i_m}$-th storey is given by:

$$M_{c,im,LR} = max\left\{M_{c,im,LR}^{(1)}, M_{c,im,LR}^{(2)}, M_{c,im,LR}^{(3)}\right\} \qquad \text{for} \quad i_m > 1 \tag{44}$$

Similarly, a collapse mechanism of global type for a RL earthquake can develop if relations (39), (41) and (43) are satisfied, so that the required sum of plastic moments for the $\mathrm{i_m}$-th storey is given by:

$$M_{c,im,RL} = max\left\{M_{c,im,RL}^{(1)}, M_{c,im,RL}^{(2)}, M_{c,im,RL}^{(3)}\right\} \qquad \text{for} \quad i_m > 1 \tag{45}$$

i) The sum of the required plastic moments of columns at each storey, is distributed among all the storey columns with the same procedure as for the columns on the first storey i.e. according to the following relation for LR earthquake:

$$M_{c,i,im,LR} = \frac{M_{c,im,LR}}{n_c} \qquad i = 1, 2, \dots, n_c \tag{46}$$

while for RL earthquake:

$$M_{c,i,im,RL} = \frac{M_{c,im,RL}}{n_c} \qquad i = 1, 2, \dots, n_c \tag{47}$$

j) Design of columns at each storey. The procedure is the same as that explained in the points f) and g). Even for the design of columns of the upper storeys there are two design points related to the directions of the earthquake.

$$A\left(N_{T\_LR,im}, M_{c,i,im,LR}\right) \qquad\qquad B\left(N_{T\_RL,im}, M_{c,i,im,RL}\right) \tag{48}$$

k) To take into account of technological condition is imposed, starting from the base, that the column sections cannot increase along the building height. If this condition requires the change of sections at first storey then the procedure needs to be repeated from point f). In fact, in this case, a new values of $M_{c,Rd,1,LR}$ and $M_{c,Rd,1,RL}$ are obtained and, as a consequence, the value of the sum of plastic moment of columns at each storey. On the contrary, if the condition only requires the change of sections at the upper storeys, i.e. without the involvement of first storey columns, then the design step j) is to be repeated in order to consider the new section dimensions.

The check of technological condition could look redundant because it is common for both axial force and shear demand to increase gradually going from the top to the base of the structure. But,when the proposed procedure is applied, as reported in the worked example, the required sum of plastic moments of k-th storey can be bigger than the required sum of plastic moments at $(k-1)$-th storey, so that also step k) is to be applied.

3. WORKED EXAMPLE

In order to show the practical application of the proposed design procedure, the seismic design of a four-bay five-storey moment resisting frame is presented in this section.

The inelastic behaviour of the designed structure is successively examined by means of a push-over static and also a dynamic inelastic analysis, confirming the fulfilment of the design goal, i.e. the location of the yielding zones at the beam ends with the only exception of the base section of first-storey columns. The structural scheme of the frame to be designed is shown in Figure 5. The interstorey height is equal to 3m. The characteristic values of the vertical loads acting on the beams are equal to 19.5 kN/m and 12 kN/m for permanent (G_k) and live (Q_k) actions, respectively. The structural materials adopted are concrete C25/30 and reinforcement of steel grade B450C. According to Eurocode 8, the value of the period of vibration to be used for preliminary design is:

$$T = 0.075\, H^{3/4} = 0.075 \cdot 15^{3/4} \approx 0.57\, s \tag{49}$$

where H is the total height of the frame.

With reference to the design spectrum for stiff soil conditions (soil class A of Eurocode 8) and by assuming a behaviour factor q equal to 3.9, the horizontal seismic forces are those depicted in Figure 5.

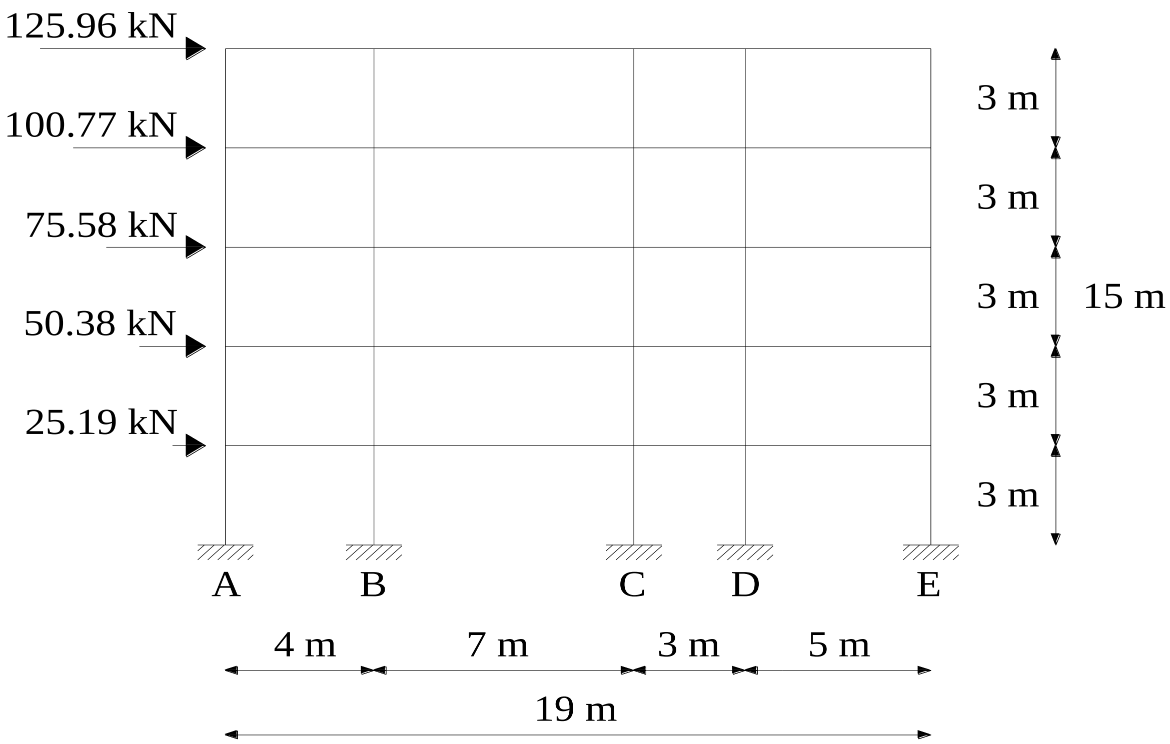

Figure 5: Structural scheme of the designed frame

In the following, the numerical development of the design steps for the structural scheme described above is provided.

a) Selection of the design top sway displacement

The selection of the maximum top sway displacement up to which the global mechanism has to be assured is a very important design issue, because the value of this displacement governs the magnitude of second order effects accounted for in the design procedure. A good criterion to choose the design ultimate displacement δ_u is to relate it to the plastic rotation

supply of beams or beam-to-column connections by assuming $\delta_u = \theta_u \cdot h_{ns}$ (where θ_u can be assumed equal to 0.01 rad).
As a consequence, the design value of the top sway displacement has been assumed equal to:

$$\delta_u = 0.01 \cdot h_{ns} = 0.01 \cdot 15 = 0.15\ m \tag{50}$$

b) Design of beam sections to withstand vertical loads.

The load acting on the frame in the vertical load combination is:

$$Q_{SLU} = 1.3\ G_k + 1.5\ Q_k = 43.35\ kN/m \tag{51}$$

For the design of the beams has been considered a bending moment equal to:

$$M_{max} = \frac{Q_{SLU} \cdot L^2}{8} \tag{52}$$

Therefore, by imposing the base of the section equal to b=30 cm, is possible to calculate the height of the beam through the following design relation:

$$d = r\sqrt{\frac{M_{Sd}}{b}} \tag{53}$$

Assuming $\xi = 0.25$ and $\rho = 0.25$ a value of $r = 0.19$ is obtained. As a consequence the amount of reinforcement is given by:

$$A_s = \frac{M_{Sd}}{0.85 \cdot h \cdot f_{sd}} \tag{54}$$

Obviously the number of steel bars in the beam is such that:

$$M_{Rd} > M_{Sd} \tag{55}$$

The reinforcement at the beam ends are reported in Table 2.

Table 2: Reinforcement at the beam ends (L = left and R = right) for the first storey.

PART OF SECTION	*L_{AB} = 4m*		*L_{BC} = 7m*		*L_{CD} = 3m*		*L_{DE} = 5m*	
	e = L	*e = R*	*e = L*	*e = R*	*e = L*	*e = R*	*e = L*	*e = R*
Top	*3 Φ20*	*3 Φ20*	*5 Φ20*	*5 Φ20*	*5 Φ20*	*4 Φ20*	*4 Φ20*	*4 Φ20*
Bottom	*3 Φ20*	*4 Φ20*	*6 Φ20*	*5 Φ20*	*5 Φ20*	*5 Φ20*	*5 Φ20*	*4 Φ20*

These reinforcement are the same at all the storeys.

c) Computation of the slopes of mechanism equilibrium curve $\gamma_{i_m}^{(t)}$.

By means of Eqs. (24), (27) and (29) the slopes of mechanism equilibrium curves are computed. These values are reported in Table 3 and they are the same for both directions of seismic input.

Table 3: Slopes of mechanism equilibrium curves (cm^{-1})

STOREY i_m	$\gamma_{i_m}^{(1)}$	$\gamma_{i_m}^{(2)}$	$\gamma_{i_m}^{(3)}$
1	0.0194	0.0032	0.0194
2	0.0090	0.0036	0.0166
3	0.0057	0.0045	0.0145
4	0.0041	0.0062	0.0129
5	0.0032	0.0116	0.0116

In particular it is important to underline that the slope value corresponding to the global mechanism $\gamma^{(g)} = \gamma_1^{(2)}$, is the minimum among all the $\gamma_{i_m}^{(t)}$ values:

$$\gamma^{(g)} = 0.003167\ cm^{-1} \tag{56}$$

d) Computation of the required sum of plastic moments of columns at first storey $M_{c,1}$.
As previously pointed out, the required sum of plastic moments of columns at first storey is provided by Eq. (30) for LR earthquake and by Eq. (31) for RL earthquake. In the examined case, these sums are equal to $\mathbf{M_{c,1,LR} = 1823.58\ kNm}$ and $\mathbf{M_{c,1,RL} = 1839.71\ kNm}$, respectively.

e)-f)-g) They have to be distributed among the columns proportionally to their number.

According to the global mechanism, axial forces in the columns at collapse state depend both from the distributed loads acting on the beams and from the shear action due to the development of plastic hinges at the beam ends, as depicted in Figure 6 (with reference to the earthquake from Left to Right).

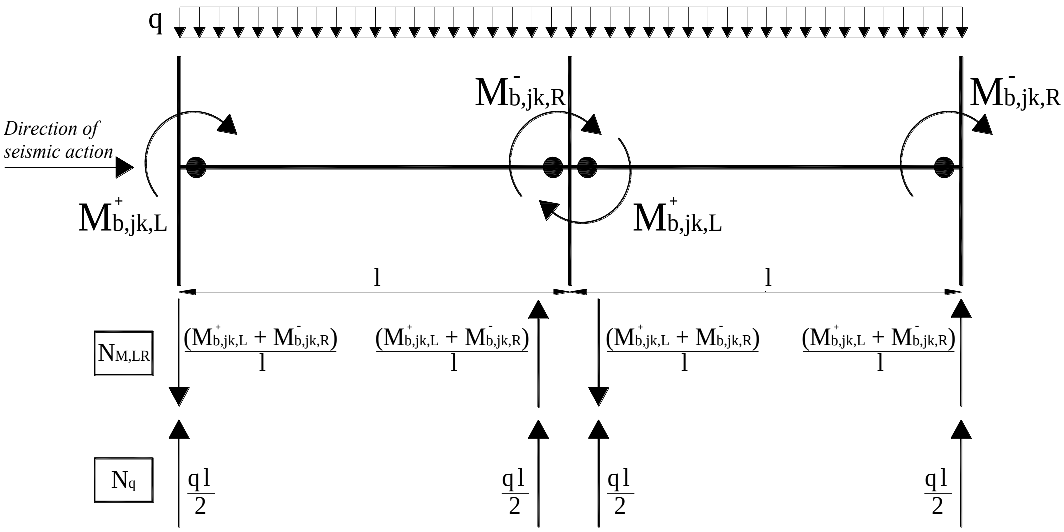

Figure 6: Loads transmitted by the beams to the columns at collapse state for LR earthquake

So that, the total load transmitted by the beams to the columns is the sum of two contributions. The first one, N_q, is related to the vertical loads acting in the seismic load combination (i.e. the sum of $ql/2$ type contributions). In Table 4 the axial forces due to vertical loads, for both directions of earthquake, are reported for each storey and for each column.

Table 4: Axial forces acting in the columns related to the vertical loads for both directions of earthquake

Storey	Column A		Column B		Column C		Column D		Column E	
i_m	$N_{q,LR}$ (kN)	$N_{q,RL}$ (kN)	$N_{q,LR}$ (kN)	$N_{q,RL}$ (kN)	$N_{q,LR}$ (kN)	$N_{q,RL}$ (kN)	$N_{q,LR}$ (kN)	$N_{q,RL}$ (kN)	$N_{q,LR}$ (kN)	$N_{q,RL}$ (kN)
1	231.00	288.75	635.25	462.00	577.50	577.50	462.00	635.25	288.75	231.00
2	184.80	231.00	508.20	369.60	462.00	462.00	369.60	508.20	231.00	184.80
3	138.60	173.25	381.15	277.20	346.50	346.50	277.20	381.15	173.25	138.60
4	92.40	115.50	254.10	184.80	231.00	231.00	184.80	254.10	115.50	92.40
5	46.20	57.75	127.05	92.40	115.50	115.50	92.40	127.05	57.75	46.20

The second one, $N_{M,LR}$ (or $N_{M,RL}$), is related to the shear actions due to the plastic hinges developed at the beam ends i.e. the sum of $(M^{+}_{b.jk,L} + M^{-}_{b.jk,R})/l$ for earthquake from left to right (or the sum of $(M^{-}_{b.jk,L} + M^{+}_{b.jk,R})/l$ for earthquake from right to left). In Table 5 these contributions are reported.

Table 5: Axial forces acting in the columns related to the shear actions for both directions of earthquake

Storey	Column A		Column B		Column C		Column D		Column E	
i_m	$N_{M,LR}$ (kN)	$N_{M,RL}$ (kN)	$N_{M,LR}$ (kN)	$N_{M,RL}$ (kN)	$N_{M,LR}$ (kN)	$N_{M,RL}$ (kN)	$N_{M,LR}$ (kN)	$N_{M,RL}$ (kN)	$N_{M,LR}$ (kN)	$N_{M,RL}$ (kN)
1	-318.16	-724.18	-205.66	77.45	-109.12	132.28	-107.41	144.50	740.35	369.95
2	-254.52	-576.11	-164.53	69.77	-87.30	87.24	-85.93	123.15	592.28	295.96
3	-190.89	-432.09	-123.39	52.32	-65.47	65.43	-64.45	92.36	444.21	221.97
4	-127.26	-288.06	-82.26	34.88	-43.65	43.62	-42.97	61.58	296.14	147.98
5	-63.63	-144.03	-41.13	17.44	-21.82	21.81	-21.48	30.79	148.07	73.99

Therefore the required bending moment for each column $\mathbf{M_{c,i,1}}$, the section, the upper and lower reinforcement and the axial force, for both directions of the earthquake are reported in Table 6.

Table 6: Final design of the column sections at first storey

STOREY	Column	$\mathrm{M_{c,i,1,LR}}$ [kNm]	$\mathrm{M_{c,i,1,RL}}$ [kNm]	b x h	$\mathrm{A_s = A'_s}$	$\mathrm{N_{LR}}$ [kN]	$\mathrm{N_{RL}}$ [kN]
1°	A	364.71	367.94	30x50	6 Φ 24	-87.16	-435.44
	B			30x50	6 Φ 20	429.59	539.45
	C			30x60	6 Φ 16	468.38	709.78
	D			30x50	6 Φ 20	354.59	779.75
	E			30x50	5 Φ 20	1029.10	600.95

The sums of obtained column plastic moments at first storey are: $M_{c,Rd,1,LR} = 2030.28\, kNm$ for LR earthquake and $M_{c,Rd,1,RL} = 2032.28\, kNm$for RL earthquake which are greater than the required one.

As a consequence the value of $\alpha_0^{(g)}$ obtained from Eq. (5) for LR earthquake and from Eq. (6) for RL earthquake are equal to $\alpha^{(g)}_{0,LR} = 3.0468$ and $\alpha^{(g)}_{0,RL} = 3.0648$, respectively.

h) Computation of the required sum of plastic moments of columns $M^{(t)}_{c,im}$ at any storey, to avoid undesired mechanism by means of equations (38) (or (39) for RL earthquake), (40) (or (41) for RL earthquake) and (42) (or (43) for RL earthquake).

So for earthquake from Left to Right the sum of column plastic moments required at each storey to avoid undesired mechanisms is reported in Table 7.

Table 7: Sum of plastic moments of column required at each storey to avoid undesired mechanism

STOREY i_m	$M^{(1)}_{c,im;LR}$ [kNm]	$M^{(2)}_{c,im,LR}$ [kNm]	$M^{(3)}_{c,im,LR}$ [kNm]
1	<u>2030.28</u>	-	2030.28
2	<u>2681.75</u>	730.98	1706.37
3	<u>3240.35</u>	-343.36	1448.49
4	<u>3124.07</u>	-967.79	1078.14
5	<u>2107.97</u>	-917.35	595.31

And for earthquake from Right to Left the same sum is reported in Table 8.

Table 8: Sum of plastic moments of column required at each storey to avoid undesired mechanism

STOREY i_m	$M^{(1)}_{c,im,RL}$ [kNm]	$M^{(2)}_{c,im,RL}$ [kNm]	$M^{(3)}_{c,im,RL}$ [kNm]
1	$\underline{2032.28}$	-	2032.28
2	$\underline{2717.82}$	721.43	1719.63
3	$\underline{3277.82}$	-358.10	1459.86
4	$\underline{3157.26}$	-983.93	1086.66
5	$\underline{2129.30}$	-929.21	600.04

The sum of the plastic moments of columns governing the column design at each storey (Eqs. (44) and (45)) is given in Table 7 and Table 8 by the underlined values. It can be recognized that, in the examined case, the need to avoid type-1 mechanism always governs the design of columns.

i)-j) Design of column sections at each storey.

The required sum of column plastic moments M_{c,i,i_m}, the section, the upper and lower reinforcement, the axial force for both directions of the earthquake are reported in Table 9.

Table 9: Design of column sections at each storey

STOREY	Column	$\mathrm{M_{c,im,LR}}$ [kNm]	$\mathrm{M_{c,im,RL}}$ [kNm]	b x h	$\mathrm{A_s = A'_s}$	$\mathrm{N_{LR}}$ [kN]	$\mathrm{N_{RL}}$ [kN]
2°	A	536.35	543.56	30x60	5 Φ 28	-69.72	-345.11
	B			30x50	6 Φ 24	343.67	439.37
	C			30x50	6 Φ 24	374.70	549.24
	D			30x60	7 Φ 20	283.67	631.35
	E			30x50	6 Φ 24	823.28	480.76
3°	A	648.07	655.56	30x70	5 Φ 28	-52.29	-258.84
	B			30x60	5 Φ 28	257.76	329.52
	C			30x60	6 Φ 24	281.03	411.93
	D			30x60	5 Φ 28	212.75	473.51
	E			30x60	6 Φ 24	617.46	360.57
4°	A	624.81	631.45	30x60	4 Φ 32	-34.86	-172.56
	B			30x60	5 Φ 28	171.84	219.68
	C			30x60	6 Φ 24	187.35	274.62
	D			30x60	5 Φ 28	141.83	315.68
	E			30x60	6 Φ 24	411.64	240.38
5°	A	421.59	425.86	30x50	6 Φ 24	-17.43	-86.28
	B			30x50	6 Φ 24	85.92	109.84
	C			30x50	6 Φ 24	93.68	137.31
	D			30x50	6 Φ 24	70.92	157.84
	E			30x50	6 Φ 24	205.82	120.19

k) Checking of technological condition

By observing Table 6 and Table 9 it can be noted that there are some column sections at the first storey which are smaller than the corresponding ones required at the second storey, therefore, a technological condition at the first storey is not satisfied. As a consequence, the values of $M_{c,Rd,1,LR}$ and $M_{c,Rd,1,RL}$ need to be updated and the procedure needs to be repeated from the step e). In Table 10 and Table 11 the new value of required sum of plastic moments of columns $M^{(t)}_{c,im}$ at any storey are reported for both directions of earthquake.

Table 10: Sum of plastic moments of column required at each storey for LR earthquake

STOREY i_m	$M_{c,im}^{(1)}$ [kNm]	$M_{c,im}^{(2)}$ [kNm]	$M_{c,im}^{(3)}$ [kNm]
1	2125.80	-	2125.80
2	2636.60	800.45	1718.52
3	3216.04	-298.20	1458.91
4	3115.39	-943.47	1085.95
5	2107.97	-908.66	599.65

Table 11: Sum of plastic moments of column required at each storey for RL earthquake

STOREY i_m	$M_{c,im}^{(1)}$ [kNm]	$M_{c,im}^{(2)}$ [kNm]	$M_{c,im}^{(3)}$ [kNm]
1	2098.26	-	2098.26
2	2686.63	769.42	1728.02
3	3261.03	-326.91	1467.06
4	3151.27	-967.13	1092.06
5	2129.30	-923.21	603.04

In Table 12 are reported the final value of the columns.

Table 12: Design of column sections at each storey for earthquake

STOREY	Column	$M_{c,im,LR}$ [kNm]	$M_{c,im,RL}$ [kNm]	b x h	$A_s = A'_s$	N_{LR} [kN]	N_{RL} [kN]
1°	A	364.71	367.94	30x70	7 Φ 20	-87.16	-435.44
	B			30x60	7 Φ 16	429.59	539.45
	C			30x60	6 Φ 16	468.38	709.78
	D			30x60	7 Φ 16	354.59	779.75
	E			30x60	6 Φ 16	1029.10	600.95
2°	A	527.32	537.32	30x70	6 Φ 24	-69.72	-345.11
	B			30x60	7 Φ 20	343.67	439.37
	C			30x60	7 Φ 20	374.70	549.24
	D			30x60	7 Φ 20	283.67	631.35
	E			30x60	7 Φ 20	823.28	480.76
3°	A	643.20	652.20	30x70	5 Φ 28	-52.29	-258.84
	B			30x60	5 Φ 28	257.76	329.52
	C			30x60	6 Φ 24	281.03	411.93
	D			30x60	5 Φ 28	212.75	473.51
	E			30x60	6 Φ 24	617.46	360.57
4°	A	623.07	630.25	30x60	4 Φ 32	-34.86	-172.56
	B			30x60	5 Φ 28	171.84	219.68
	C			30x60	6 Φ 24	187.35	274.62
	D			30x60	5 Φ 28	141.83	315.68
	E			30x60	6 Φ 24	411.64	240.38
5°	A	421.59	425.86	30x50	6 Φ 24	-17.43	-86.28
	B			30x50	6 Φ 24	85.92	109.84
	C			30x50	6 Φ 24	93.68	137.31
	D			30x50	6 Φ 24	70.92	157.84
	E			30x50	6 Φ 24	205.82	120.19

4. VALIDATION OF THE DESIGN PROCEDURE

In order to validate the design procedure, a static non-linear analysis (push-over) has been carried out to investigate the actual seismic response of the designed frame by means SAP2000 computer program [40]. This analysis has the primary aim to confirm the development of the desired collapse mechanism typology and to evaluate the obtained energy dissipation capacity, testing the accuracy of the proposed design methodology. Regarding the structural modelling, the mechanical non-linearities, have been concentrated at beam and column ends by means of plastic hinge elements. The constitutive law of such plastic hinge elements is provided by a rigid plastic moment-rotation curve. The type of hinge depends on the element considered i.e. by its internal action. In fact, for the beams and the columns M3 and P-M3 hinge type have been considered, respectively. In case of P-M3 hinge type, the interaction domain P-M has been evaluated for each column and used in SAP2000 computer program. The results of the push-over analysis are mainly constituted by base shear–top sway displacement curve which is depicted in Figure 7. In the same figure also a straight line is given, i.e. the one corresponding to the linearized mechanism equilibrium curve of global mechanism whose expression, for the designed frame and for earthquake from Left to Right, is:

$$\alpha_{LR}^{(g)} = 3.0468 - 0.003167\ \delta \tag{57}$$

For earthquake to Right to Left, the expression is:

$$\alpha_{RL}^{(g)} = 3.0648 - 0.003167\ \delta \tag{58}$$

As already underlined there is a mechanism equilibrium curve for both direction of earthquake. The two mechanism equilibrium curves are different but only for what concern the α_0 value while the slope is the same as reported in Figure 7.

As it was expected, also the LR push-over curve is different from RL one. This difference can be easily understood if we consider that the axial forces in the columns are different in the two considered push-over and, as a consequence, also the plastic moment is different.

Notwhitsgtanding, in this case, the two curves are very close one each other but there is no proof of the fact that this represent a general result. So that both curves should be always considered when a non-symmetric moment-resisting frame is analized.

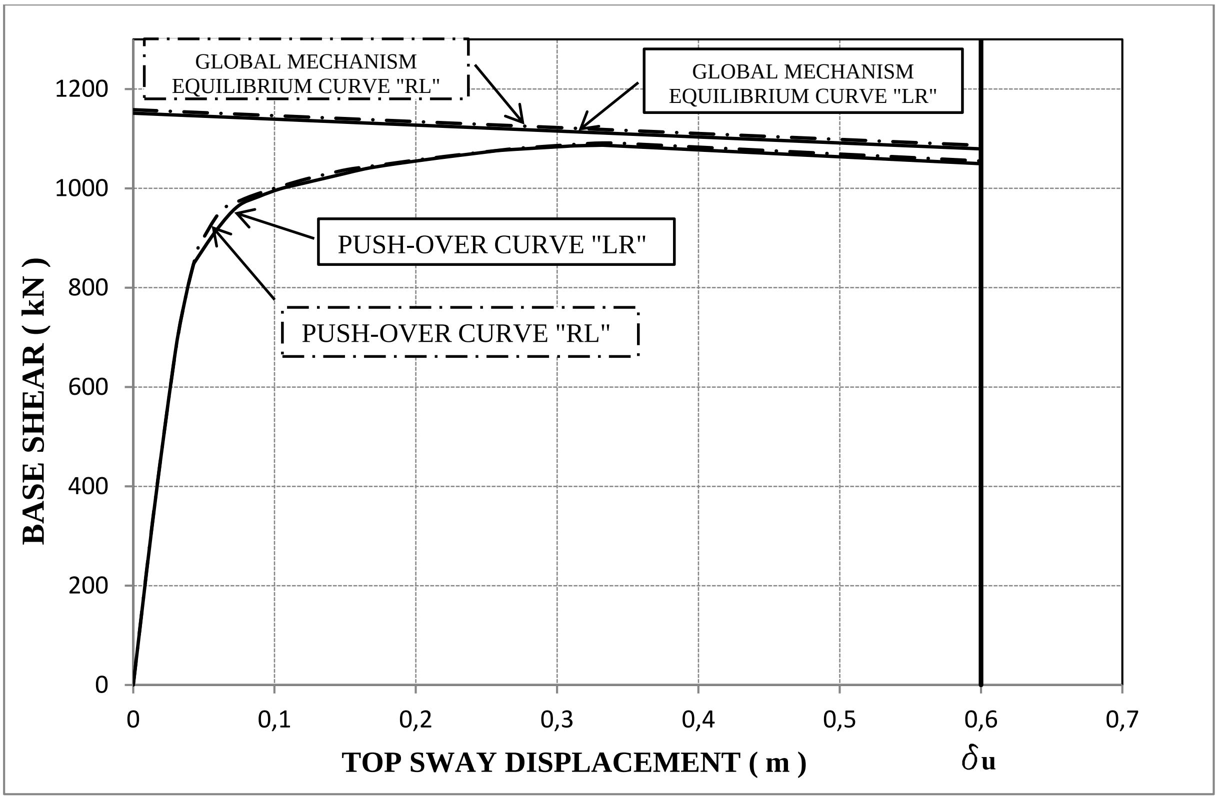

Figure 7: Overlap of the push-over curve with the global mechanism equilibrium curve

Obviously, the base shear is, in this case, obtained by multiplying the value of α, given by Eq. (57) and Eq. (58), for the design base shear corresponding to $\alpha = 1$. The comparison between the push-over curve and the global mechanism equilibrium curve provides a first confirmation of the accuracy of the proposed design procedure. In fact, the last branches of push-over curves parallel to global mechanism equilibrium curves as showed in Figure 7.

A further confirmation, even the most important, of the fulfilment of the design objective is represented by the pattern of yielding developed at the occurrence of the design ultimate displacement. In fact, developed plastic hinges are shown in Figure 8 and their pattern is in perfect agreement with the global mechanism.

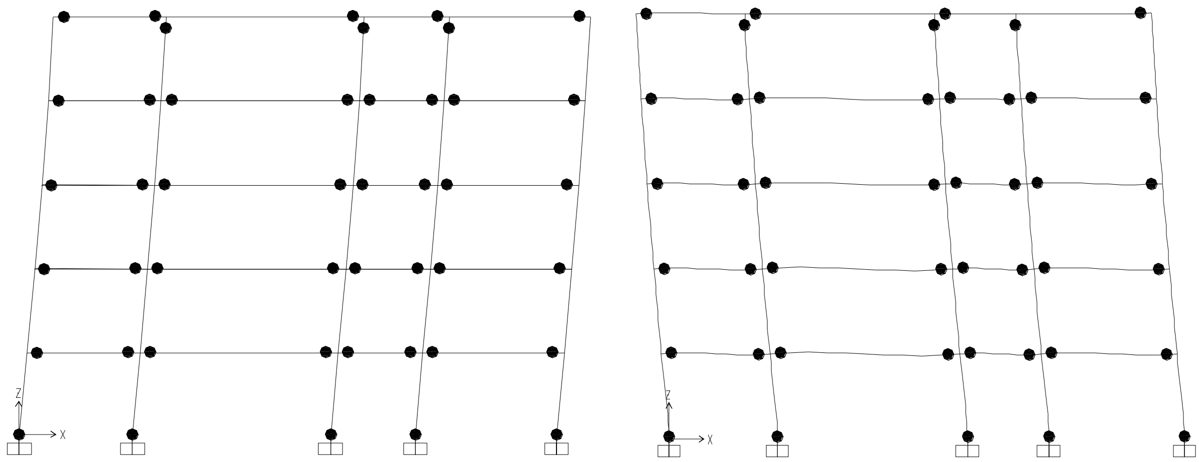

Figure 8: Pattern of yielding of the designed frame at $\delta = \delta_u$ for LR and RL earthquake direction.

In order to fulfill the serviceability requirements the interstorey drift have been checked with reference to the limit reported in the Eurocode 8. In particular the considered limit refers to buildings having non structural elements of brittle materials attached to the structure:

$$d_r\, \nu \leq 0.005\ \mathrm{h} \tag{59}$$

where d_r is the the maximum relative displacement between two consecutive storeys and h is the correspondinbg stprey height.

If this serviceability requirement is not verified the structural stiffness can be improved by increasing the beam sections or the ultimate design displacement. In fact, in both cases the final results will be a more rigid structure with respect to the one obtained in the worked example herein presented. In Table 13 the final results are reported.

Table 13: Limitation of interstorey drift

STOREY	d_s [mm]	d_r [mm]	υ	$d_r\ \upsilon$	0.005 h
5°	6.2448	0.8497		0.4248	1.5
4°	5.3950	1.2280		0.6140	1.5
3°	4.1669	1.5483	0.5	0.7741	1.5
2°	2.6186	1.6274		0.8137	1.5
1°	0.9911	0.9911		0.4955	1.5

To provide a more robust validation of the design methodology, non-linear incremental dynamic analyses have been developed with reference to the same structural model used for push-over analyses. Record-to-record variability has been accounted for considering 7 recorded accelerograms selected from PEER data base [41].

In Table 14 main features of the records (name, date, magnitude, ratio between PGA and gravity acceleration, length and step recording) are given. These earthquake records have been selected to approximately match the linear elastic design response spectrum of Eurocode 8, for type A soil. Moreover, in order to perform IDA analyses, each ground motion has been scaled to obtain the same value of the spectral acceleration $S_a(T_1)$ corresponding to the fundamental period of vibration T_1 of the structure ($T_1 = 0.54$). This is the seismic intensity measure (IM) adopted for IDA analyses where $S_a(T_1)$ values have been progressively increased.

Table 14: Accelerogram characteristics

Earthquake (record)	**Component**	**Date**	**PGA/g**	**Length**	**Step recording**
Kobe (Kakogawa)	KAK000	1995/01/16	0.251	40.95	0.01
Northridge (Stone Canyon)	SCR000	1994/01/17	0.252	39.99	0.01
Palm Springs (Soboba)	H08000	1986/07/08	0.250	26.00	0.005
Santa Barbara (Courthouse)	SBA132	1978/08/13	0.102	12.57	0.01
Spitak Armenia (Gukasian)	GUK000	1988/07/12	0.199	19.89	0.01
Duzce, Turkey (Lamont)	375-N	1999/11/12	0.97	41.50	0.01
Victoria Mexico (Chihuahua)	CHI102	1980/06/09	0.150	26.91	0.01

In Figure 9, the maximum interstorey drift ratio (MIDR) versus spectral acceleration curve is reported. It is important to underline that, for each record the obtained pattern of yelding has been monitored for increasing values of $S_a(T_1)$ by checking that plastic hinge development is always in perfect agreement with the global mechanism. In addition, the limit line of the ultimate plastic rotation provided by hinges (0.01 rad) is reported.

This results testifies the accuracy of the proposed design procedure even under actual seismic actions.

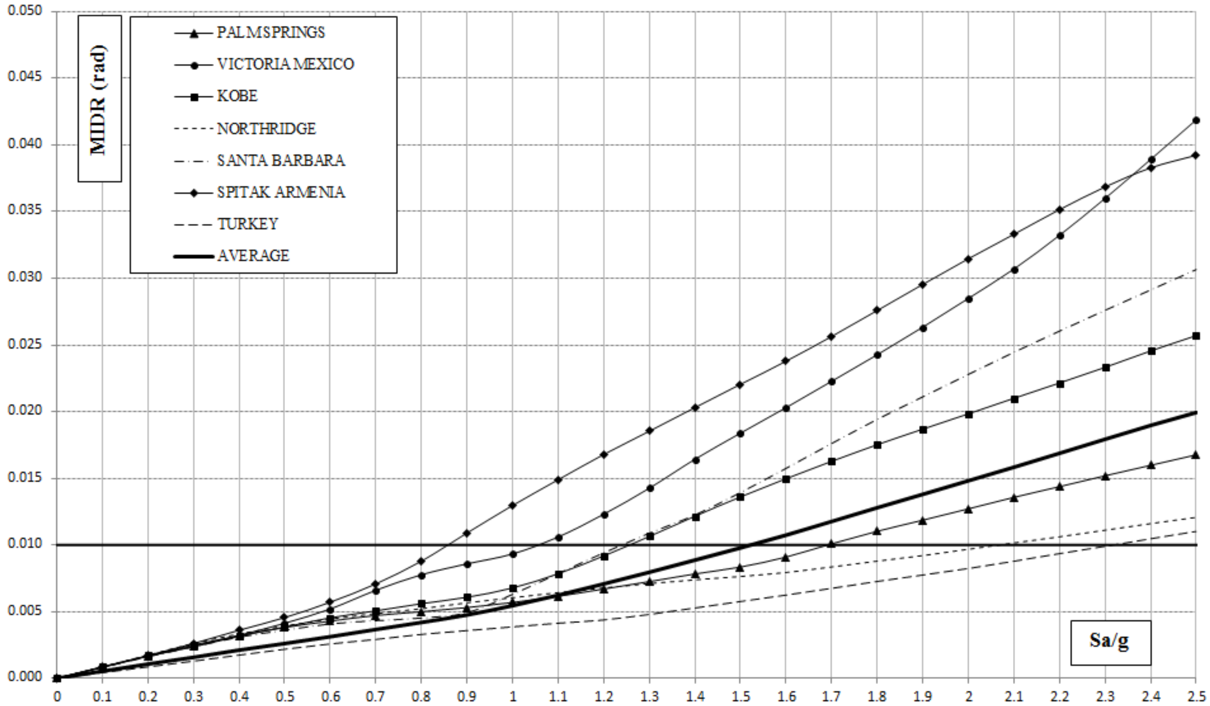

Figure 9: *Maximum interstorey drift ratio versus* $\boldsymbol{S_a(T_1)}$

As an example, Figure 10 provides the distribution of plastic hinges for increasing value of $S_a(T_1)$ with reference to Kobe earthquake record. As a consequence of the obtained design goal, the spectral acceleration values leading to collapse, given in Table 15, are very high and compatible with the adoption of the designed structure even in the case of destructive earthquakes. As depicted in Figure 10 the frames present some spurious hinges at the column ends of some storeys. Indeed these hinges are displayed by the SAP but their plastic rotation is very close to zero so that they do not partecipate in the development of the collapse mechanism. In addition the average value of $S_a(T_1)$ leading to collapse is near to 1.50g while the average PGA is about 0.70g.

Table 15: $S_a(T_1)$ *and PGA values corresponding to attainment of the structural collapse*

Earthquake (record)	Sa_c/g	PGA_c/g
Kobe (Kakogawa)	1.25	0.71
Northridge (Stone Canyon)	2.09	0.74
Palm Springs (Soboba)	1.70	1.44
Santa Barbara (Courthouse)	1.23	0.61
Spitak Armenia (Gukasian)	0.85	0.48
Duzce, Turkey (Lamont)	2.30	0.54
Victoria Mexico (Chihuahua)	1.05	0.39
Mean value	**1.50**	**0.70**

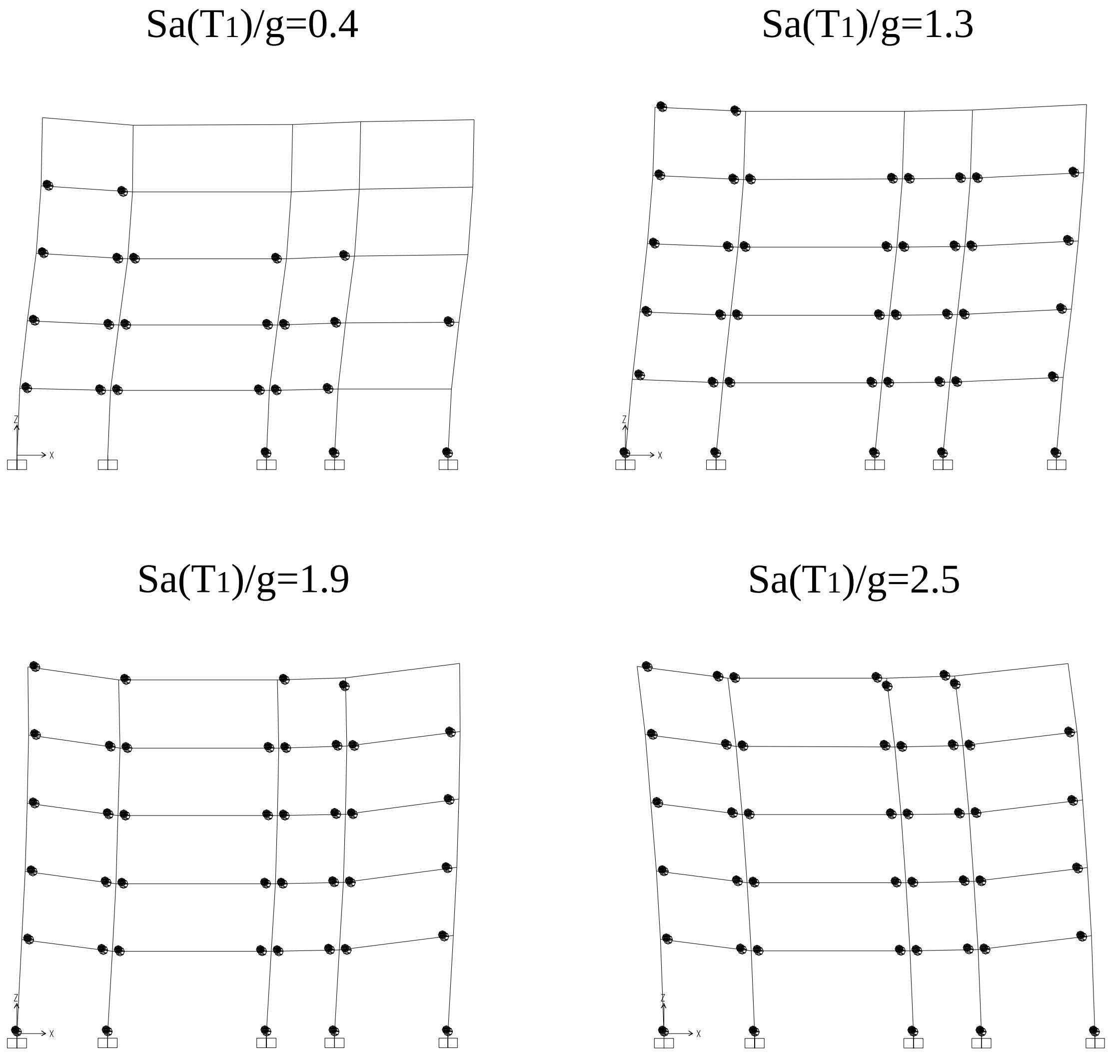

Figure 10: Pattern of yielding of the designed frame for increasing value of $S_a(T_1)$ with reference to Kobe earthquake record

5. CONCLUSIONS

In this paper a methodology called “Theory of Plastic Mechanism Control” for the design of reinforced concrete moment resisting frames has been presented. On the base of the extension of the kinematic theorem of plastic collapse to the concept of mechanism equilibrium curve, the Theory of Plastic Mechanism Control allows to evaluate the sum of plastic moments of the columns required at each storey in order to develope a collapse mechanism of global type.
The closed form solution of the design conditions makes the design procedure very easy to be applied even by means of hand calculations and, therefore, it could also be suggested for code purpose by definitely solving the problem of collapse mechanism control whose importance in seismic design is universally recognised. Beam-column hierarchy criterion, commonly suggested by seismic codes, appears only as a very rough approximation when compared to TPMC and its theoretical background. The reliability of the proposed design procedure has

been also demonstrated through its application to a four-bays, five-storeys frame, leading to the fulfilment of the design objective, i.e. the development of a collapse mechanism of global type, as it has been confirmed by the results of both push-over analysis and non-linear dynamic analyses. The proposed methodology can be considered as belonging to the Performance Based Seismic Design philosophy [42, 43]. In fact, in order to satisfy the limit states of “Life Safe” or “Near Collapse” the designer has to promote a dissipative collapse mechanism avoiding the so called “soft storey mechanism”. In addition, it is useful to underline that the proposed procedure constitutes a rigorous application of the capacity design principles. In fact, beams are designed in order to bear external loads, while columns are designed according to the maximum internal actions transmitted by the dissipative zones.
It is important to underline that the proposed procedure can be applied for MRFs characterized by a non-symmetric scheme and a non-symmetric reinforcement in each beam section.

REFERENCES

[1] Lee H.S.,
“Revised rule for concept of strong-column weak-girder design”, J. struct. eng. ASCE, 122, 359-364, 1996.

[2] Paulay T.,
“Seismic design of ductile moment resisting reinforced concrete frames, columns: evaluation of actions”, Bull. New Zealand Natl. Soc. Earthquake Eng., 10, 85-94, 1977.

[3] Paulay T.,
“Deterministic Design Procedure for Ductile Frames in Seismic Areas”, ACI Publication SP-63, American Concrete Institute, Detroit, pp. 357-381, 1980.

[4] EN 1998-1, “Eurocode 8: Design of Structures for Earthquake Resistance – Part 1: general Rules, Seismic Actions and Rules for Buildings”, CEN, 2004.

[5] New Zealand Standard Code of Practice for the Design of Concrete Structures, NZS 3101: Part 1; Commentary NZS 3101: Part 2; Standard Association of New Zealand, Wellington, New Zealand, 1982.

[6] Ahmad Heidari and Sadjad Gharehbaghi
Seismic performance improvement of Special Truss Moment Frames using damage and energy concepts - Earthquake Engineering & Structural Dynamics - Volume 44, Issue 7, June 2015, Pages: 1055–1073, - DOI: 10.1002/eqe.2499

[7] Godínez-Domínguez, E.A., Tena-Colunga, A.
Nonlinear behavior of code-designed reinforced concrete concentric braced frames under lateral loading Engineering Structures, 32 (4), pp. 944-963. doi: 10.1016/j.engstruct.2009.12.020

[8] Panelis G.G., Kappos A.J.,

"Earthquake-Resistant Concrete Structures", E&FN Spon, an imprint of Chapman & Hall, 1997.

[9] Bai J.L., Ou, J.P.
Realization of the global yield mechanism of RC frame structures by redesigning the columns using column tree method - Science China Technological Sciences Volume 58, Issue 10, 1 October 2015, Pages 1627-1637

[10] Leelataviwat S.a, Goel S.C.b, Stojadinović B.c
Energy-based seismic design of structures using yield mechanism and target drift Journal of Structural Engineering - Volume 128, Issue 8, August 2002, Pages 1046-1054

[11] Ye, L.-P., Ma, Q.-L., Miao, Z.-W. Study on weak beam-strong column design method of RC frame structures - Gongcheng Lixue/Engineering Mechanics - Volume 27, Issue 12, December 2010, Pages 102-113

[12] Yang P. , Tang J.
Analysis of failure mechanism of RC structures with specially shaped columns affected by the diaphragm - Tumu Jianzhu yu Huanjing Gongcheng/Journal of Civil, Architectural and Environmental Engineering - Volume 33, Issue 4, August 2011, Pages 35-41+59

[13] Mata P, .Barbat A.H., Oller S.,, Boroschek R.
Constitutive and geometric nonlinear models for the seismic analysis of RC structures with energy dissipators - Archives of Computational Methods in Engineering- Volume 15, Issue 4, December 2008, Pages 489-539

[14] Hwang, J.-S., Tsai, C.-H, Wang, S.-J., Huang, Y.-N.c
Applications of viscous dampers to RC structures with lightly reinforced concrete walls - Wind and Earthquake Engineering - Proceedings of the 10th East Asia-Pacific Conference on Structural Engineering and Construction, EASEC 2010 Volume 3, 2006, Pages 315-320

[15] Verderame, G.M., De Luca F. , Ricci P., Manfredi G.
Preliminary analysis of a soft-storey mechanism after the 2009 L'Aquila earthquake - Earthquake Engineering and Structural Dynamics Volume 40, Issue 8, 10 July 2011, Pages 925-944

[16] Hejazi, F., Zabihi, A., Jaafar, M.S.
Development of elasto-plastic viscous damper finite element model for reinforced concrete frames - Soil Dynamics and Earthquake Engineering, 65, pp. 284-293.doi: 10.1016/j.soildyn.2014.06.008

[17] Hejazi, F., Noorzaei, J., Jaafar, M.S., Abdullah, A.A.A.
Earthquake analysis of reinforce concrete framed structures with added viscous dampers (2009) World Academy of Science, Engineering and Technology, 38, pp. 205-210.

[18] İnel M., Ozmen H.B.

Effects of plastic hinge properties in nonlinear analysis of reinforced concrete buildings Engineering Structures, 28 (11), pp. 1494-1502. Cited 52 times. doi: 10.1016/j.engstruct.2006.01.017

[19] Oviedo A., J.A., Midorikawa, M., Asari, T.
Earthquake response of ten-story story-drift-controlled reinforced concrete frames with hysteretic dampers - Engineering Structures, 32 (6), pp. 1735-1746. doi: 10.1016/j.engstruct.2010.02.025

[20] Han J. , Li Y.-M., Ji S.-Y. Realization of rational failure mechanism for the RC frame structure - Harbin Gongye Daxue Xuebao/Journal of Harbin Institute of Technology Volume 42, Issue 12, December 2010, Pages 2003-2008

[21] Hejazi, F.ab , Shoaei, M.D.ab , Jaafar, M.S.ab , Rashid, R.S.B.M.ab
Effect of viscous dampers on yielding mechanisms of RC structures during earthquake - Earthquake and Structures - Volume 8, Issue 6, 2015, Pages 1499-1528

[22] Hejazi, F., Kojouri, S.J., Noorzaei, J., Jaafar, M.S., Thanoon, W.A., Abdullah, A.A.A.
Inelastic seismic response of RC building with control system (2011) Key Engineering Materials, 462-463, pp. 241-246 doi: 10.4028/www.scientific.net/KEM.462-463.241

[23] Christoph Mahrenholtz, Pao-Chun Lin, An-Chien Wu, Keh-Chyuan Tsai, Shyh-Jiann Hwang, Ruei-Yan Lin and Muhammad Y. Bhayusukma
Retrofit of reinforced concrete frames with buckling-restrained braces - Earthquake Engineering & Structural Dynamics - Volume 44, Issue 1, January 2015, Pages: 59–78, - DOI: 10.1002/eqe.2458

[24] D'Aniello, M., Della Corte, G., Mazzolani, F.M.
Seismic upgrading of RC buildings by steel eccentric braces: Experimental results vs numerical modeling - Proceedings of the 5th International Conference on Behaviour of Steel Structures in Seismic Areas - Stessa 200, Pages 809-814 Yokohama; Japan; 14- 17 August 2006.

[25] D'Aniello, M., Della Corte, G., Mazzolani, F.M.
Seismic upgrading of RC buildings by buckling restrained braces: Experimental results vs numerical modeling - Proceedings of the 5th International Conference on Behaviour of Steel Structures in Seismic Areas - Stessa 200, Pages 809-814 Yokohama; Japan; 14- 17 August 2006.

[26] Della Corte, G. , D'Aniello, M. , Landolfo, R.
2015 Field testing of all-steel buckling-restrained braces applied to a damaged reinforced concrete building - Journal of Structural Engineering (United States) Volume 141, Issue 1, 1 January

[27] Mistakidis, E.S. , De Matteis G., Formisano A.
Low yield metal shear panels as an alternative for the seismic upgrading of concrete structures - Advances in Engineering Software - Volume 38, Issue 8-9, August 2007, Pages 626-636

[28] Laura Eads, Eduardo Miranda, Helmut Krawinkler and Dimitrios G. Lignos,
An efficient method for estimating the collapse risk of structures in seismic regions - Earthquake Engineering & Structural Dynamics - Volume 42, Issue 1, January 2013, Pages: 25–41, DOI: 10.1002/eqe.2191

[29] M. Baradaran Shoraka, T. Y. Yang and K. J. Elwood
Seismic loss estimation of non-ductile reinforced concrete buildings - Earthquake Engineering & Structural Dynamics Volume 42, Issue 2, February 2013, Pages: 297–310, DOI: 10.1002/eqe.2213

[30] J. L. Domingues Costa, R. Bento, Vsevolod Levtchitch and M. P. Nielsen
Rigid-plastic seismic design of reinforced concrete structures - Earthquake Engineering & Structural Dynamics - Volume 36, Issue 1, January 2007, Pages: 55–76, DOI: 10.1002/eqe.617

[31] Xiao Lu, Xinzheng Lu, Hong Guan and Lieping Ye
Collapse simulation of reinforced concrete high-rise building induced by extreme earthquakes Earthquake Engineering & Structural Dynamics - Volume 42, Issue 5, 25 April 2013, Pages: 705–723, DOI: 10.1002/eqe.2240

[32] Mazzolani F.M:, Piluso V.,
"Plastic Design of Seismic Resistant Steel Frames", Earthquake Engineering and Structural Dynamics, Vol. 26, pp. 167-191, 1997.

[33] Longo A.; Montuori R.; Piluso V.,
Theory of Plastic Mechanism Control for MRF-CBF dual system and its validation. Bulletin of Earthquake Engineering (BEEE), 2014.

[34] Longo A., Montuori R., Piluso V.,
Failure Mode Control and Seismic Response of Dissipative Truss Moment Frames, Journal of Structural Engineering, Vol. 138, pp.1388-1397, 2012.

[35] Montuori R., Piluso V., Troisi M.,
Innovative structural details in MR-frames for free from damage structures Mechanics Research Communications – Vol. 58 pp. 146-156, 2014.

[36] Montuori R., Nastri E., Piluso V.,
"Advances in Theory of Plastic Mechanism Control: Closed Form Solution for Mr-Frames", Earthquake Engineering and Structural Dynamics, 2015.

[37] Montuori R., Nastri E., Piluso V.,
Theory of Plastic Mechanism Control for Eccentrically Braced Frames with inverted Y-scheme, Journal of Constructional Steel Research, Volume 92, pp. 122-135, 2014.

[38] Longo A., Montuori R., Piluso V.,
Moment Frames - Concentrically Braced Frames Dual Systems: Analysis of Different Design Criteria, Structure and infrastructure engineering, 2015.

[39] Montuori R., Muscati R.,

"Plastic Design of Seismic Resistant Concrete Frame", Earthquakes and Structures, Vol. 8, No. 1, pp. 205-224, January 2015.

[40] CSI 2007. SAP 2000: Integrated Finite Element Analysis and Design of Structures. Analysis Reference. Computer and Structure Inc. University of California, Berkeley.

[41] Pacific Earthquake Engineering Research Center, PEER Strong Motion Database, http://peer.berkeley.edu.

[42] Park R.,
"Ductile Design Approach for Reinforced Concrete Frames", Earthquake Spectra, EERI, 2(3), pp. 565-619, 1986.

[43] SEAOC. Vision 2000 - A Framework for Performance Based Design, Volumes I, II, III. Structural Engineers Association of California, Vision 2000 Committee. Sacramento, California, 1995.