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ADVANCES IN THEORY OF PLASTIC MECHANISM CONTROL: CLOSED FORM SOLUTION FOR MR-FRAMES

Rosario Montuori, Elide Nastri, Vincenzo Piluso

Department of Civil Engineering, University of Salerno

ABSTRACT

In this paper new advances in the application of "Theory of Plastic Mechanism Control" (TPMC) are presented. TPMC is aimed at the design of structures assuring a collapse mechanism of global type. The theory has been developed in the nineties with reference to moment-resisting frames (MRFs) and progressively extended to all the main structural typologies commonly adopted as seismic-resistant structural systems. In particular, the outcome of the theory is the sum of the plastic moments of the columns required, at each storey, to prevent undesired failure modes, i.e. partial mechanisms and soft-storey mechanisms. The theory is used to provide the design conditions to be satisfied, in the form of a set of inequalities where the unknowns are constituted by the column plastic moments. This set of inequalities was originally solved by means of an algorithm requiring an iterative procedure. The advances presented in this paper are constituted by the identification of a "closed form solution". This result is very important, because the practical application of TPMC can now be carried out even with very simple hand calculations. The practical application of TPMC is herein presented with reference to the design of a multi-storey frame whose pattern of yielding is validated by means of both push-over analysis and incremental dynamic analyses.

1. INTRODUCTION

A fundamental principle of capacity design of MR-Frames is that plastic hinge formation in columns during an earthquake should be avoided, in order to make sure that the seismic energy is dissipated by the beams only. Therefore, the optimisation of the energy dissipation capacity of structures is achieved when a collapse mechanism of global type is developed [1-3].

In order to decrease the probability of plastic hinge formation in columns, MR-Frames must be designed to have strong columns and weak beams. To this scope different simplified design criteria have been proposed [4-10] and the so-called beam-column hierarchy criterion has been introduced in Eurocode 8 [11].

Even though studies on this topic started several decades ago mainly with reference to reinforced concrete structures [9, 12-14] and, in particular, in New Zealand where the capacity design procedure found its codification since 1982 [15], codified design rules included in Eurocode 8 as well as similar procedures adopted by other codes cannot achieve the design goal, i.e. the development of a global type mechanism.

There are a number of reasons why the beam-column hierarchy criterion cannot achieve the above mentioned design goal and these have been widely discussed both with reference to reinforced concrete frames [16] and to steel frames [17].
Among the different reasons leading the beam-column hierarchy criterion to fail in the achievement of the design goal, probably the most important, and difficult to be accounted for in a simplified design approach, is the shifting of the contraflexure point in columns during the seismic excitation. This considerable shifting leads to a bending moment distribution substantially different from that resulting from code-prescribed design rules [18-19]. The shift of the contraflexure point is caused by the formation of hinges in beams adjacent to the column and even in part of the columns. All these factors alter the stiffness of beam-column subassemblage, hence the moment distribution.
The main reason why the above issue cannot be accounted for by means of a simplified design rule, such as the beam-column hierarchy criterion, is that the second principle of capacity design [20] cannot be easily applied in case of multiple resisting mechanisms not located in series. In fact, according to the second principle of capacity design, non-dissipative zones (i.e. the columns in case of MR-Frames) need to be designed considering the maximum internal actions which the dissipative zones (i.e. the beam ends in case of MR-Frames) are able to transmit at their ultimate conditions. The beam-column hierarchy criterion is based on the possibility to accurately evaluate, at any beam-to-column joint, the sum of the bending moments which the beams are able to transmit when ultimate conditions occur, but, conversely, because of the shifting of contraflexure point in columns during the seismic excitation, it is practically impossible to predict how the above sum is shared between the end sections of the top and bottom column converging in the joint [2-10]. For this reason, it is well known that the beam-column hierarchy criterion, being based on simple joint equilibrium, is only able to prevent "soft storey" mechanisms, but it does not allow the development of a collapse mechanism of global type.
For this reason, a rigorous design procedure, based on the kinematic theorem of plastic collapse, has been presented in 1997 [17], aiming to guarantee a collapse mechanism of global type where plastic hinges develop at the beam ends only, while all the columns remain in elastic range. Obviously, exception is made for base section of first storey columns, leading to a kinematic mechanism. Starting from this first work, the "Theory of Plastic Mechanism Control" (TPCM) has been outlined as a powerful tool for the seismic design of steel structures. It consists on the extension of the kinematic theorem of plastic collapse to the concept of mechanism equilibrium curve. In fact, for any given structural typology, the design conditions to be applied in order to prevent undesired collapse mechanisms can be derived by imposing that the mechanism equilibrium curve corresponding to the global mechanism has to be located below those corresponding to all the other undesired mechanisms up to a top sway displacement level compatible with the local ductility supply of dissipative zones.
This design approach was successively extended to MRFs with semi-rigid connections [21], MRFs with RBS connections [22], EB-Frames with horizontal links (i.e. split-K scheme and D-scheme) [23-24] or with inverted Y scheme [25-26], knee-braced frames [27], dissipative truss-moment frames DTMFs [28-29] and MRF-CBF dual systems [30].
Starting from the above background, in this paper new advances in the application of the "Theory of Plastic Mechanism Control" are reported. In particular, by means of new considerations regarding collapse mechanism typologies, a closed form solution has been found. The design conditions to be satisfied to prevent undesired collapse mechanisms can now be solved without any iterative

procedure, so that the unknown of the design problem, i.e. column sections at each storey, can now be directly derived. The extreme simplicity of the resulting design procedure will be emphasised by means of a worked example aiming to show its practical application which can now be carried out even by means of hand calculations. In addition, static inelastic analysis (push-over analysis) and incremental dynamic analyses are successively carried out to compare the actual inelastic behaviour of the designed frame with the design goal.

2. THEORY OF PLASTIC MECHANISM CONTROL

The theory of plastic mechanism control, originally proposed by Mazzolani and Piluso [17], is based on the upper bound theorem of plastic collapse extended to the concept of mechanism equilibrium curve. Before then, rigid-plastic analysis was used only for the computation of the collapse load multiplier of structures completely defined from the mechanical point of view, i.e. already designed structures whose load carrying capacity was under investigation. Conversely, to the best of authors knowledge, thanks to TPMC rigid-plastic analysis was for the first time recognised as a powerful tool for seismic design of structures.

In particular, TPMC allows the theoretical solution of the problem of designing a structure failing in global mode, i.e. assuring that plastic hinges develop only at beam ends while all the columns remain in elastic range with the only exception of base sections at first storey columns. The beam sections are assumed to be known quantities, because they are preliminarily designed to withstand vertical loads according to the non-seismic load combination, while the unknowns of the design problem are the column sections needed to assure the desired collapse mechanism, i.e. the global mechanism.

To this scope, TPMC is based on the kinematic or upper bound theorem of plastic collapse within the framework of limit analysis. According to the theory of limit analysis, the assumption of a rigid-plastic behaviour of the structure until the complete development of a collapse mechanism is made. It means that the attention is focused on the condition the structure exhibits in the collapse state by neglecting each intermediate condition. Given the above, it is possible to recognise three main collapse mechanism typologies the structure is able to exhibit. These mechanisms, depicted in Fig. 1, have to be considered undesired, because they do not involve all the dissipative zones. The global mechanism, representing the design goal, is a particular case of type-2 mechanism involving all the storeys.

However, the simple application of the kinematic theorem of plastic collapse is not sufficient to assure the desired collapse mechanism, because high horizontal displacements occur before the complete development of the kinematic mechanism. These displacements give rise to significant second order effects which cannot be neglected in the seismic design of structures, particularly in case of moment-resisting steel frames. Therefore, the basic principle of TPMC is essentially constituted by the extension of the kinematic theorem of plastic collapse to the concept of mechanism equilibrium curve.

Within the framework of a kinematic approach, for any given collapse mechanism, the mechanism equilibrium curve can be easily derived by equating the external work to the internal work due to the plastic hinges involved in the collapse mechanism, provided that the external second-order work due to vertical loads is also evaluated [17]. The resulting linearised mechanism equilibrium curve is given by:

$$\alpha = \alpha_0 - \gamma\delta \tag{1}$$

where α_0 is the kinematically admissible multiplier of horizontal forces according to first order rigid-plastic analysis and γ is the slope of the mechanism equilibrium curve [17].

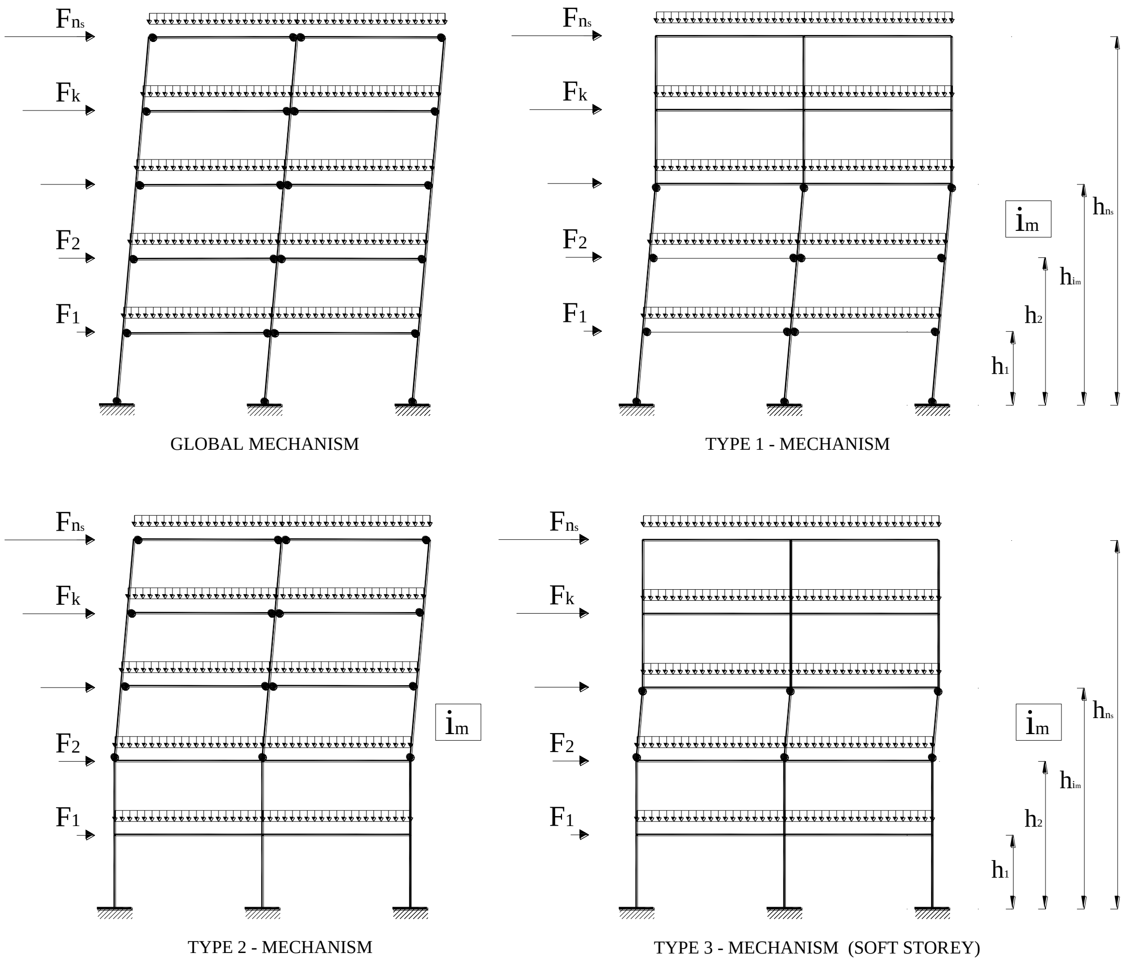

Figure 1: *Collapse mechanism of MR-Frames*

In the following, for sake of simplicity, reference is made to the case of uniform vertical loads acting on the beams satisfying the limitation [17]:

$$q_{jk} \leq \frac{4\, M_{b.jk}^2}{l_j^2} \tag{2}$$

where q_{jk} is the uniform vertical load applied to the beam of j-th bay and k-th storey, $M_{b.jk}$ is the corresponding beam plastic moment and l_j is the j-th bay span. Such limitation assures that beam plastic hinges develop at the beam ends. It can be demonstrated [17] that in case of vertical loads exceeding the above limit the second plastic hinge in the beam develops in an intermediate section, so that the external work due to the uniform vertical loads has also to be considered.

Under the above assumption, in the case of global type mechanism, as shown in Fig.1, the kinematically admissible multiplier of horizontal forces is given by:

$$\alpha_0^{(g)} = \frac{\sum_{i=1}^{n_c} M_{c.i.1} + 2\sum_{k=1}^{n_s}\sum_{j=1}^{n_b} M_{b.jk}}{\sum_{k=1}^{n_s} F_k h_k} \tag{3}$$

where F_k and h_k are, respectively, the seismic force applied at k-th storey and the k-th storey height with respect to the foundation level; $M_{c.i.k}$ is the plastic moment of i-th column of k-th storey

reduced due to the contemporary action of the axial force; n_c, n_b and n_s are the number of columns, bays and storeys, respectively.
Regarding the slope $\gamma^{(g)}$ of the mechanism equilibrium curve, it is given by [17]:

$$\gamma^{(g)} = \frac{1}{h_{ns}} \frac{\sum_{k=1}^{n_s} V_k h_k}{\sum_{k=1}^{n_s} F_k h_k} \tag{4}$$

where V_k is the total vertical load acting at k-th storey.

With reference to i_mth mechanism of type-1, the kinematically admissible multiplier of seismic horizontal forces is given by:

$$\alpha_{i_m}^{(1)} = \frac{\sum_{i=1}^{n_c} M_{c.i.1} + 2\sum_{k=1}^{i_m-1} \sum_{j=1}^{n_b} M_{b.jk} + \sum_{i=1}^{n_c} M_{c.i.i_m}}{\sum_{k=1}^{i_m} F_k h_k + h_{i_m} \sum_{k=i_m+1}^{n_s} F_k} \tag{5}$$

while the slope of the mechanism equilibrium curve is [17]:

$$\gamma_{i_m}^{(1)} = \frac{1}{h_{i_m}} \frac{\sum_{k=1}^{i_m} V_k h_k + h_{i_m} \sum_{k=i_m+1}^{n_s} V_k}{\sum_{k=1}^{i_m} F_k h_k + h_{i_m} \sum_{k=i_m+1}^{n_s} F_k} \tag{6}$$

With reference to i_mth mechanism of type-2, the kinematically admissible multiplier of seismic horizontal forces is given by:

$$\alpha_{i_m}^{(2)} = \frac{\sum_{i=1}^{n_c} M_{c.i.i_m} + 2\sum_{k=i_m}^{n_s} \sum_{j=1}^{n_b} M_{b.jk}}{\sum_{k=i_m}^{n_s} F_k (h_k - h_{i_m-1})} \tag{7}$$

while the slope of the mechanism equilibrium curve is [17]:

$$\gamma_{i_m}^{(2)} = \frac{1}{h_{n_s} - h_{i_m-1}} \frac{\sum_{k=i_m}^{n_s} V_k \left(h_k - h_{i_m-1}\right)}{\sum_{k=i_m}^{n_s} F_k (h_k - h_{i_m-1})} \tag{8}$$

It is useful to note that, for i_m=1 Eq. (7) and Eq. (8) are coincident with Eq. (3) and Eq. (4), respectively, because in such case the mechanism is coincident with the global one.
Finally, with reference to i_mth mechanism of type-3, the kinematically admissible multiplier of horizontal forces, for $i_m = 1$, is given by:

$$\alpha_1^{(3)} = \frac{2\sum_{i=1}^{n_c} M_{c.i.1}}{h_1 \sum_{k=1}^{n_s} F_k} \tag{9}$$

and, for $i_m > 1$, is given by:

$$\alpha_{i_m}^{(3)} = \frac{2\sum_{i=1}^{n_c} M_{c.i.i_m}}{\left(h_{i_m} - h_{i_m-1}\right) \sum_{k=i_m}^{n_s} F_k} \tag{10}$$

In addition, the corresponding slope of the mechanism equilibrium curve is given by [17]:

$$\gamma_{i_m}^{(3)} = \frac{1}{h_{i_m} - h_{i_m-1}} \frac{\sum_{k=i_m}^{n_s} V_k}{\sum_{k=i_m}^{n_s} F_k} \tag{11}$$

It is important to underline that, for any given geometry of the structural system, the slope of mechanism equilibrium curve attains its minimum value when the global type mechanism is developed [2].This issue assumes a paramount importance in TPMC exploiting the extension of the kinematic theorem of plastic collapse to the concept of mechanism equilibrium curve.

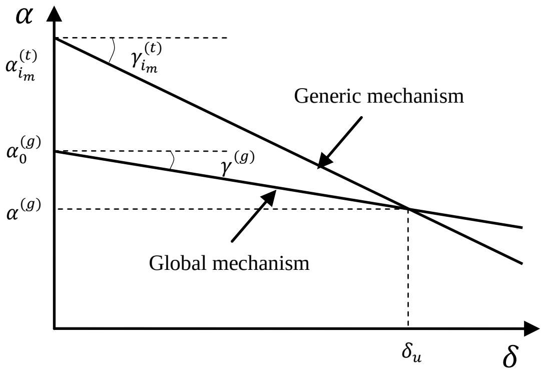

Figure 2: Design conditions

In fact, according to the kinematic theorem of plastic collapse, extended to the concept of mechanism equilibrium curve, the design conditions to be fulfilled in order to avoid all the undesired collapse mechanisms require that the mechanism equilibrium curve corresponding to the global mechanism has to be located below those corresponding to all the undesired mechanisms within a top sway displacement range, δ_u, compatible with the ductility supply of structural members (Figure 3):

$$\alpha_0^{(g)} - \gamma^{(g)}\delta_u \leq \alpha_{i_m}^{(t)} - \gamma_{i_m}^{(t)}\delta_u \quad for \quad i_m = 1,2,3,\dots,n_s \quad t = 1,2,3 \tag{12}$$

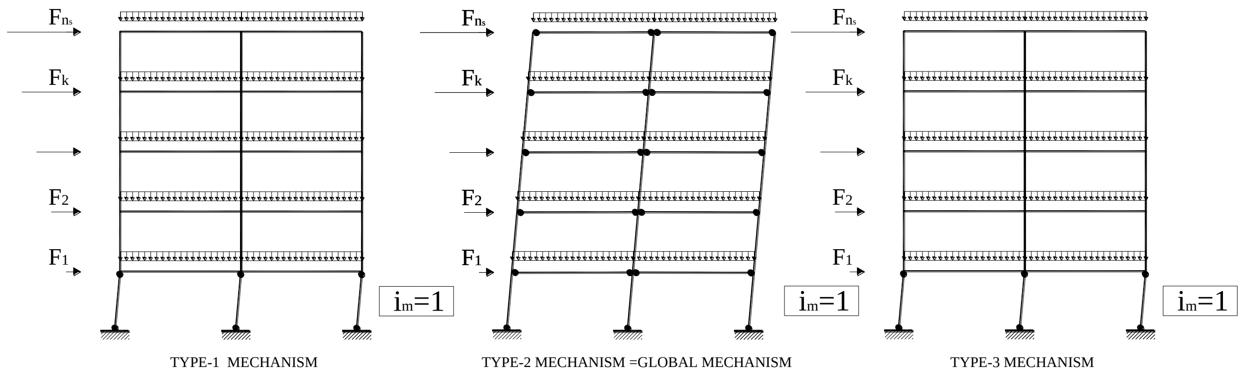

Figure 3: Collapse mechanism of MR-Frames for $i_m = 1$

Eq. (12) constitutes the statement of the theory of plastic mechanism control and it is valid independently of the structural typology. This is the reason why TPMC has been applied with success to MR-Frames, EB-Frames, knee braced frames, MRF-CBF dual systems and dissipative truss-moment frames. Therefore, TPMC really constitutes a general approach to the seismic design of structures aiming to the control of the collapse mechanism. The robustness of the theory is founded on the kinematic theorem of plastic collapse and on second-order rigid-plastic analysis.

Conversely, hierarchy criteria commonly suggested in modern seismic codes often do not exhibit any sound theoretical basis. As an example, the beam-column hierarchy criterion, suggested for the column design of MR-Frames, is merely based on the joint equilibrium occurring when the beam ends are yielded and strain-hardened up to their ultimate limit state, but no information can be theoretically derived about the distribution of bending moments between the columns converging in the joint. As a consequence, beam-column hierarchy criterion can only be an approximate application of the second principle of capacity design.

Within this context it cannot be forgotten that Leonardo Da Vinci stated “Study science before and then follow practice. Those who fall in love with practice without science are like the helmsman who enters ship without a rudder nor compass and has never certain where he is going”. The kinematic theorem of plastic collapse and its extension to the concept of mechanism equilibrium curve are the rudder and the compass of TPMC.

3. CLOSED FORM SOLUTION

As already stated, TPMC was originally developed in nineties, so that the design conditions given by Eq. (12) do not constitute any new. However, aiming to the solution of the set of inequalities, the original work was based on an iterative procedure, so that the application of TPMC required the development of specific computer programs. The advances presented in this paper are based on new observations leading to a closed form solution of Eq. (12). The resulting design procedure is now extremely simple and well suited even for hand calculations.

In particular, the solution is obtained according to the following steps:

a) Design of beam sections to withstand vertical loads acting in the non-seismic load combination.

b) Selection of a design top sway displacement δ_u compatible with the ductility supply of structural members. To this scope, in the following, the plastic rotation capacity of beams is assumed equal to 0.04 rad so that $\delta_u = 0.04\, h_{ns}$ where h_{ns} is the height of the structure.

c) Computation of the axial load acting in the columns at collapse state, i.e. when a collapse mechanism of global type is completely developed.

d) Computation of the slopes of mechanism equilibrium curves $\gamma_{i_m}^{(t)}$ by means of Eqs. (6), (8) and (11). The slope of the global mechanism equilibrium curve, $\gamma^{(g)}$, is provided by Eq. (4) and it is the minimum among the $\gamma_{i_m}^{(t)}$ values computed before.

e) Computation of the required sum of plastic moment of columns, reduced due to the contemporary action of the axial force, $\sum_{i=1}^{n_c} M_{c.i.1}$, for $i_m = 1$, i.e. at the first storey, by means of the following relation:

$$\sum_{i=1}^{n_c} M_{c.i.1} \geq \frac{2\sum_{k=1}^{n_s}\sum_{j=1}^{n_b} M_{b.jk} + \left(\gamma_1^{(3)} - \gamma^{(g)}\right)\delta_u \sum_{k=1}^{n_s} F_k h_k}{2\dfrac{\sum_{k=1}^{n_s} F_k h_k}{h_1 \sum_{k=1}^{n_s} F_k} - 1} \qquad (13)$$

Equation (13) is derived from design conditions (12) for $i_m = 1$ and $t = 1$ or $t = 3$, because for $i_m = 1$ type-1 mechanism and type-3 mechanism are coincident as depicted in Figure 3. Furthermore, it is important to underline that, for $i_m = 1$, type 2 mechanism is coincident with the global mechanism so that Eq. (12), for $i_m = 1$ and $t = 2$ becomes an identity. The above observations are of paramount importance from the practical point of view, because they allow to design first storey columns directly by means of Eq. (13) and to avoid any iterative procedure leading to a closed form solution easy to be applied by hand calculations.

f) The sum of the required plastic moments of columns at first storey is distributed among the columns proportionally to the axial load acting at the collapse state, so that, the design internal actions ($M_{c.i.1}, N_{c.i.1}$ for $i = 1,2,\dots,n_c$) are derived and the column sections at first storey can be designed. As column sections are selected from standard shapes, the obtained value of $\sum_{i=1}^{n_c} M_{c.i.1}$, namely $\sum_{i=1}^{n_c} M_{c.i.1}^{*}$ is generally greater than the required minimum value provided by Eq. (13). Therefore, the mechanism equilibrium curve $\alpha = \alpha_0^{(g)} - \gamma^{(g)}\delta$ has to be evaluated accordingly, i.e. by means of Eq. (3) by replacing the term $\sum_{i=1}^{n_c} M_{c.i.1}$, with the value $\sum_{i=1}^{n_c} M_{c.i.1}^{*}$resulting from standard shapes.

In addition, the multiplier of seismic horizontal forces corresponding to the ultimate design displacement can be computed as $\alpha^{(g)} = \alpha_0^{(g)} - \gamma^{(g)}\delta_u$ (Figure 3).

g) Computation of the required sum of plastic moment of columns, reduced due to the contemporary action of the axial force, $\sum_{i=1}^{n_c} M_{c.i.i_m}^{(t)}$, for $\mathrm{i_m} > 1$ and $t = 1,2,3$ by means of the following relations:

$$\sum_{i=1}^{n_c} M_{c.i.i_m}^{(1)} \geq \left(\alpha^{(g)} + \gamma_{i_m}^{(1)}\delta_u\right)\left(\sum_{k=1}^{i_m} F_k h_k + h_{i_m}\sum_{k=i_m+1}^{n_s} F_k\right) - \sum_{i=1}^{n_c} M_{c.i.1}^{*} - 2\sum_{k=1}^{i_m-1}\sum_{j=1}^{n_b} M_{b.jk} \qquad (14)$$

needed to avoid type-1 mechanisms;

$$\sum_{i=1}^{n_c} M_{c.i.i_m}^{(2)} \geq \left(\alpha^{(g)} + \gamma_{i_m}^{(2)}\delta_u\right)\sum_{k=i_m}^{n_s} F_k\left(h_k - h_{i_m-1}\right) - 2\sum_{k=i_m}^{n_s}\sum_{j=1}^{n_b} M_{b.jk} \qquad (15)$$

needed to avoid type-2 mechanisms;

$$\sum_{i=1}^{n_c} M_{c.i.i_m}^{(3)} \geq \left(\alpha^{(g)} + \gamma_{i_m}^{(3)}\delta_u\right)\frac{\left(h_{i_m} - h_{i_m-1}\right)}{2}\sum_{k=i_m}^{n_s} F_k \qquad (16)$$

needed to avoid type-3 mechanism.

Eq. (14), (15) and (16) have been directly derived from Eq. (12) for $i_m > 1$ and $t = 1$, $t = 2$ and $t = 3$, respectively.

h) Computation of the required sum of the reduced plastic moments of columns for each storey as the maximum value among those coming from the above design conditions:

$$\sum_{i=1}^{n_c} M_{c.i.i_m} = max\left\{\sum_{i=1}^{n_c} M_{c.i.i_m}^{(1)}, \sum_{i=1}^{n_c} M_{c.i.i_m}^{(2)}, \sum_{i=1}^{n_c} M_{c.i.i_m}^{(3)}\right\} \qquad (17)$$

i) The sum of the required plastic moment of columns at each storey, reduced for the contemporary action of the axial force, is distributed among all the storey columns, proportionally to the axial force acting at collapse state. The knowledge of these plastic moments $M_{c.i.i_m}$, coupled with the axial force $N_{c.i.i_m}$ at the collapse state, allows the design of column sections from standard shapes.

j) If necessary, a technological condition is imposed by requiring, starting from the base, that the column sections cannot increase along the building height. If this condition requires the change of column sections at first storey then the procedure needs to be repeated from point f). In fact, in this case, a new value of $\sum_{i=1}^{n_c} M_{c.i.1}^{*}$ is obtained and, as a consequence, the value of the sum of the required plastic moments of columns at each storey (Eq. (17)) changes. It is important to underline that the possibility of a revision of column sections is due to their selection from standard shapes while the theory provides a closed form solution. In order to avoid any revision of the column sections and to minimise the column sections at upper storeys, the use of dog-bones at the base of first storey columns can be suggested. This choice has two advantages, because, on one hand, it allows to fix the value of

$\sum_{i=1}^{n_c} M_{c.i.1}$ satisfying Eq. (13) by equality and, on the other hand, can promote the safeguard of column base connections making easier their design.

4. WORKED EXAMPLE

In order to present in detail the practical application of the design procedure based on TPMC, the seismic design of a six-bay five-storey moment resisting frame is shown. The inelastic behaviour of the designed structure is successively examined by means of both static and dynamic non-linear analyses, confirming the fulfilment of the design goal, i.e. the achievement of a collapse mechanism of global type and pointing out the excellent seismic performance obtained.

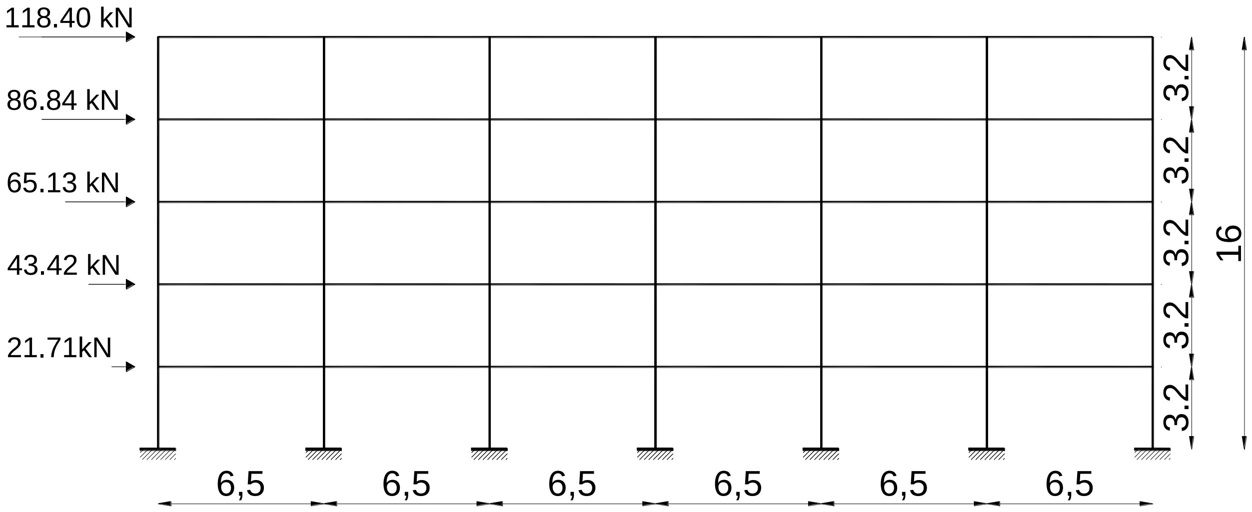

Figure 4: *Structural scheme of the example frame (dimension in m)*

The structural scheme of the frame to be designed is shown in Figure 4. The bay span is equal to 6.5 m; the interstorey height is equal to 3.20 m. The characteristic values of the vertical loads acting on the beams are equal to 15 and 10 kN/m for permanent (G_k) and live (Q_k) actions, respectively.
According to Eurocode 8 [11], the value of the period of vibration to be used for preliminary design is:

$$T = 0.085\, H^{3/4} = 0.085 \times 16^{3/4} \approx 0.68\, s \qquad (18)$$

where H is the total height of the frame.
With reference to the design spectrum for stiff soil conditions (soil class A of Eurocode 8) and by assuming a behaviour factor q equal to 6 the horizontal seismic forces are those depicted in Figure 4. The structural material adopted for the structure is steel grade S275 with a partial safety factor equal to 1.1.
In the following, the numerical development of the design steps for the structural scheme described above is provided.

a) Design of beam sections to withstand vertical loads

The combination of actions corresponding to the frame subjected to vertical loads only is:

$$q_v = 1.3\, G_k + 1.5\, Q_k = 34.5\, kN/m \qquad (19)$$

Therefore, in order to withstand such uniform vertical load an IPE330 section in adopted for the beams.

With reference to the seismic design situation, corresponding to the combination of actions, $G_k + \psi_2 Q_k + E_d$, the uniform vertical load acting on the beams is ($\psi_2 = 0.3$ for residential buildings):

$$q = 15 + 0.3 \times 10 = 18\ kN/m \quad (20)$$

The plastic design resistance of beams is:

$$M_{b.Rd} = \frac{W_{pl} f_{yk}}{\gamma_{M0}} = \frac{(804 \times 10^{-6})(275 \times 10^{3})}{1.10} = 201\ kN\ m \quad (21)$$

where W_{pl} is the plastic modulus, f_{yk} is the characteristic yield stress and γ_{M0} is the adopted partial safety factor. Therefore, the limit value of the uniform vertical load is:

$$q_{lim} = \frac{4M_{b.Rd}}{L^2} = \frac{4 \times 201}{(6.50)^2} \approx 19.03\ kN/m \quad (22)$$

With reference to the seismic design situation, this limit value is not exceeded; therefore plastic hinges develop at the beam ends.

b) Selection of the design top sway displacement

The selection of the maximum top sway displacement up to which the global mechanism has to be assured is a very important design issue, because the value of this displacement governs the magnitude of second order effects accounted for in the design procedure. In addition, the complete development of the collapse mechanism could be prevented by the occurrence of a plastic rotation demand exceeding the local ductility supply. Therefore, a good criterion to choose the design ultimate displacement δ_u is to relate it to the plastic rotation supply of beams or beam-to-column connections by assuming $\delta_u = \theta_u h_{n_s}$ (where θ_u is the plastic rotation supply, considered in this case equal to 0.04 rad). As a consequence, the design value of the top sway displacement has been assumed equal to:

$$\delta_u = 0.04\ h_{n_s} = 0.04 \cdot 16 = 0.64\ m \quad (23)$$

c) Computation of the axial load acting at collapse state in the columns

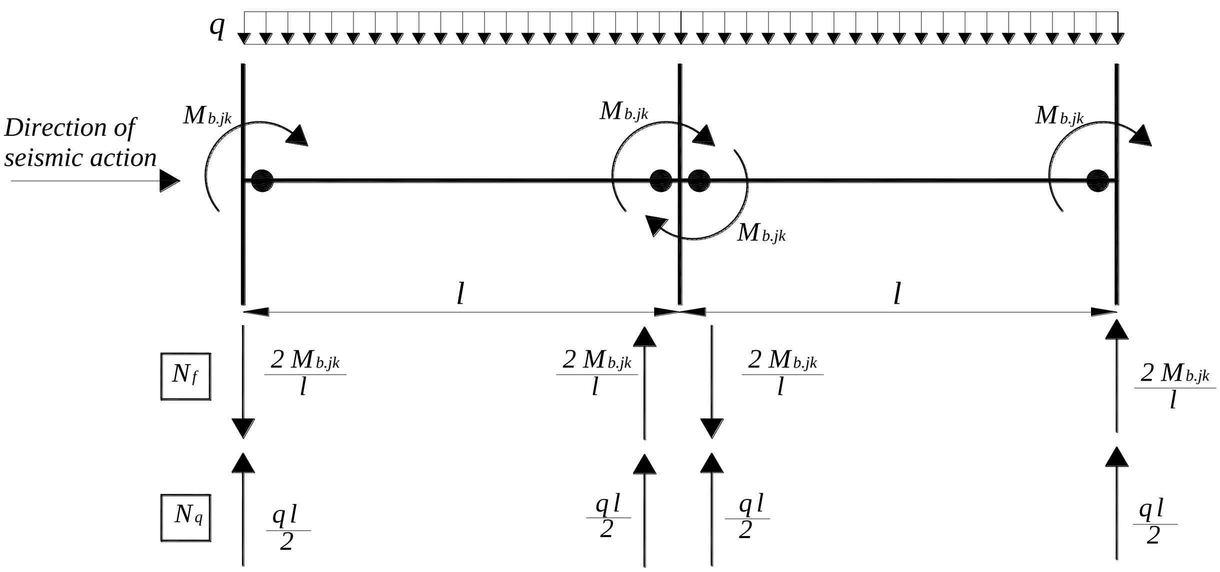

Figure 5: Loads transmitted by the beams to the columns at collapse state

In agreement with the global mechanism, axial forces in the columns at collapse state depend both on distributed loads acting on the beams and on the shear action coming from the development of

plastic hinges at the beam ends, as depicted in Figure 5. So that, the total load transmitted by the beams to the columns is the sum of two contributions. The first one, N_q, is related to the vertical loads acting in the seismic load combination (i.e. the sum of $ql/2$ type contributions). The second one, N_f, is related to the shear actions due to the plastic hinges developed at the beams ends (i.e. the sum of $2M_{b.jk}/l$ type contributions).
However, seismic actions can be acting either in the positive direction or in the negative direction, so that the maximum axial forces has to be considered.

Table 1: Axial forces acting at collapse state in the columns

STOREY	External columns			Internal columns		
i_m	N_q (kN)	N_f (kN)	N_{tot} (kN)	N_q (kN)	N_f (kN)	N_{tot} (kN)
1	292.500	309.346	601.846	585.000	0	585.000
2	234.000	247.477	481.477	468.000	0	468.000
3	175.500	185.608	361.108	351.000	0	351.000
4	117.000	123.738	240.738	234.000	0	234.000
5	58.500	61.869	120.369	117.000	0	117.000

In Table 1 the two contributions, N_q and N_f, and the total value, N_{tot}, of the axial force are reported for each storey both for internal columns and for external columns.

d) Computation of the slopes of mechanism equilibrium curve $\gamma_{i_m}^{(t)}$

By means of Eqs. (6), (8) and (11) the slopes of mechanism equilibrium curves are computed. These values are reported in Table 2.

Table 2: Slopes of mechanism equilibrium curves (cm^{-1})

STOREY i_m	$\gamma_{i_m}^{(1)}$	$\gamma_{i_m}^{(2)}$	$\gamma_{i_m}^{(3)}$
1	0.0327	0.0053	0.0327
2	0.0152	0.0060	0.0280
3	0.0095	0.0074	0.0243
4	0.0068	0.0102	0.0214
5	0.0053	0.0185	0.0185

The slope value corresponding to the global mechanism, $\gamma^{(g)}$ is the minimum among all the $\gamma_{i_m}^{(t)}$ values:

$$\gamma^{(g)} = 0.0053\ cm^{-1} \quad (24)$$

e) Computation of the required sum of plastic moment of columns at first storey, $\sum_{i=1}^{n_c} M_{c.i.1}$, *reduced due to the contemporary action of the axial load*

As previously pointed out, the required sum of plastic moment of columns at first storey is provided by Eq. (13). In the examined case, this sum is equal to 2969.824 kNm and has to be distributed among the columns proportionally to the total axial force acting at collapse state. Therefore, the required bending moment for each column, $M_{req.c.i.1}$, the required plastic modulus, $W_{pl.req}$, the obtained plastic modulus, $W_{pl.obt}$, the selected profile and the obtained bending resistance for internal and external column, $M_{obt.c.i.1}$, are reported in Table 3.

Table 3: Design of the column sections at first storey

	N_{tot} (kN)	$M_{req.c.i.1}$ (kN m)	$W_{pl.req}$ (cm^3)	$W_{pl.obt}$ (cm^3)	PROFILE	$M_{obt.c.i.1}$ (kN m)
External columns	601.8	432.9	1731.6	1869.0	**HE 300 B**	444.1
Internal columns	585.0	420.8	1683.2	1869.0	**HE 300 B**	446.5

f) Selection of the column sections at first storey

As reported in Table 3 the selected profile of first storey columns is HE300B so that, the obtained sum of column plastic moments at first storey, $\sum_{i=1}^{n_c} M^*_{c.i.1}$, is:

$$\sum_{i=1}^{n_c} M^*_{c.i.1} = 3120.5 \text{ kN m} \tag{25}$$

which is greater than the required one because of the column selection from the standard shapes.
In addition, the value of $\alpha^{(g)}$ is provided, by replacing $\sum_{i=1}^{n_c} M^*_{c.i1}$ with $\sum_{i=1}^{n_c} M_{c.i.1}$, by Eq. (3) and Eq. (4):

$$\alpha^{(g)} = \alpha_0^{(g)} - \gamma^{(g)}\delta_u = \frac{\sum_{i=1}^{n_c} M^*_{c.i.1} + 2\sum_{k=1}^{n_s}\sum_{j=1}^{n_b} M_{b.jk}}{\sum_{k=1}^{n_s} F_k h_k} - \gamma^{(g)}\delta_u = 3.4779 \tag{26}$$

so that, the multiplier of seismic horizontal forces corresponding to the ultimate displacement is now a known quantity.

g) Computation of the sum of plastic moment of columns, reduced due to the contemporary action of the axial load, $\sum_{i=1}^{n_c} M^{(t)}_{c.i.i_m}$, required at any storey to avoid undesired mechanisms.

Table 4: Sum of reduced plastic moment of columns required at each storey to avoid undesired mechanisms

STOREY i_m	$\sum_{i=1}^{n_c} M^{(1)}_{c.i.i_m}$ (kN m)	$\sum_{i=1}^{n_c} M^{(2)}_{c.i.i_m}$ (kN m)	$\sum_{i=1}^{n_c} M^{(3)}_{c.i.i_m}$ (kN m)
1	<u>3120.51</u>	0.00	3120.51
2	<u>3714.55</u>	1574.86	2644.71
3	<u>4086.06</u>	270.83	2178.45
4	<u>3732.72</u>	-549.96	1591.38
5	<u>2412.90</u>	-645.90	883.50

h) Computation of the maximum value of $\sum_{i=1}^{n_c} M_{c.i.i_m}$.

The sum of the plastic moment of columns governing the column design at each storey is given in Table 4 by the underlined values. It can be recognized that, in the examined case, the need to avoid type-1 mechanisms always governs the design of columns.

i) Design of column sections at each storey

The required sum of column plastic moments reduced due to the contemporary action of the axial load $M_{req.c.i.i_m}$, the required and the obtained plastic modulus, $W_{pl.req}$ and $W_{pl.obt}$, the selected standard shapes and the obtained plastic moment $M_{obt.c.i.i_m}$ are given in Table 5.

Table 5: Design of column sections at each storey

STOREY $\boldsymbol{i_m}$		$\boldsymbol{N_{tot}}$ (kN)	$\boldsymbol{M_{req.c.i.i_m}}$ (kN m)	$\boldsymbol{W_{pl.req}}$ (cm^3)	$\boldsymbol{W_{pl.obt}}$ (cm^3)	**PROFILE**	$\boldsymbol{M_{obt.c.i.i_m}}$ (kN m)
2°	**External columns**	481.5	541.4	2165.9	2408	**HE 340 B**	602.0
	Internal columns	468.0	526.3	2105.3	2408	**HE 340 B**	602.0
3°	**External columns**	361.1	595.6	2382.5	2408	**HE 340 B**	602.0
	Internal columns	351.0	578.9	2315.8	2408	**HE 340 B**	602.0
4°	**External columns**	240.7	544.1	2176.5	2408	**HE 340 B**	602.0
	Internal columns	234.0	528.9	2115.6	2149	**HE 320 B**	537.2
5°	**External columns**	120.4	351.7	1406.9	1534	**HE 280 B**	383.5
	Internal columns	117.0	341.9	1367.5	1534	**HE 280 B**	383.5

j) Checking of technological condition

As the column sections obtained at the first storey are smaller than those required at some storeys above, the technological condition occurs, so that the column sections at first storey are revised by using HE340B standard shapes. As a consequence, the value of $\sum_{i=1}^{n_c} M^*_{c.i.1}$ needs to be updated and the procedure needs to be repeated from step f). The new results are reported in Table 6 and Table 7.

Table 6: Sum of reduced plastic moment of column required at each storey to avoid undesired mechanisms

STOREY $\boldsymbol{i_m}$	$\sum_{i=1}^{n_c} \boldsymbol{M^{(1)}_{c.i.i_m}}$ (kN m)	$\sum_{i=1}^{n_c} \boldsymbol{M^{(2)}_{c.i.i_m}}$(kN m)	$\sum_{i=1}^{n_c} \boldsymbol{M^{(3)}_{c.i.i_m}}$(kN m)
1	<u>4139.74</u>	0.00	4139.74
2	<u>3227.60</u>	2319.05	2773.32
3	<u>3820.75</u>	757.78	2289.27
4	<u>3635.66</u>	-284.65	1675.50
5	<u>2412.90</u>	-548.84	932.03

Table 7: Design of column sections at each storey

STOREY $\boldsymbol{i_m}$		$\boldsymbol{N_{tot}}$ (kN)	$\boldsymbol{M_{req.c.i.i_m}}$ (kN m)	$\boldsymbol{W_{pl.req}}$ (cm^3)	$\boldsymbol{W_{pl.obt}}$ (cm^3)	PROFILE	$\boldsymbol{M_{obt.c.i.i_m}}$ (kN m)
1°	**External columns**	601.8	432.9	1731.7	2408	**HE 340 B**	589.5
	Internal columns	585.0	420.8	1683.2	2408	**HE 340 B**	592.2
2°	**External**	481.5	470.5	1881.96	2408	**HE 340 B**	602.0

	columns						
	Internal columns	468.0	457.3	1829.3	2408	**HE 340 B**	602.0
3°	External columns	361.1	556.9	2227.8	2408	**HE 340 B**	602.0
	Internal columns	351.0	541.4	2165.5	2408	**HE 340 B**	602.0
4°	External columns	240.7	530.0	2119.90	2149	**HE 320 B**	602.0
	Internal columns	234.0	515.1	2060.6	2149	**HE 320 B**	537.2
5°	External columns	120.4	351.7	1406.9	2408	**HE 280 B**	383.5
	Internal columns	117.0	341.9	1367.5	2408	**HE 280 B**	383.5

It is important to note that column sections at 4th storey are now different from those initially reported in Table 5. In particular, for external columns, HE320B in place of HE340B sections has been selected (Table 7). This apparently weird situation occurs because, by increasing the sum of column plastic moments at first storey, the sum of required column plastic moments for $i_m > 1$ is affected by the increase of the first right hand side term of Eq. (14) and the increase of the subtracting second term at right hand side of Eq. (14).

5. VALIDATION OF THE DESIGN PROCEDURE

In order to validate the design procedure, both static non linear analysis (push-over) and incremental dynamic non-linear analyses have been carried out, by means of SAP 2000 computer program [31], to investigate the actual seismic response of the designed frame. These analyses have the primary aim to confirm the development of the desired collapse mechanism typology and to evaluate the obtained energy dissipation capacity, testing the accuracy of the proposed design methodology.

Regarding the structural modelling, all the members are constituted by means of beam-column elements, whose mechanical non-linearities have been concentrated at their ends by means of plastic hinge elements. In particular, plastic hinges accounting for the interaction between axial force and bending moment have been defined both in case of columns and in case of beams. The constitutive law of such plastic hinge elements is provided by a rigid plastic moment-rotation curve. The push-over analysis has been led under displacement control taking into account both geometrical and mechanical non-linearities. The results of the push-over analysis are mainly constituted by the frame capacity curve which is depicted in Figure 6. In the same figure also two straight lines are given: the first one corresponding to linear elastic analysis and the second one corresponding to the linearised mechanism equilibrium curve whose expression, for the designed frame, is:

$$\alpha = 4.0729 - 0.005293\,\delta \tag{27}$$

Obviously, the base shear depicted in Figure 6 is, in this case, obtained by multiplying the value of α, given by Eq. (27), by the design base shear corresponding to $\alpha = 1$.

The comparison between the capacity curve and the above straights lines provides a first confirmation of the accuracy of the proposed design procedure. Under this point of view, it is useful to underline that for $E \rightarrow \infty$, being E the elastic modulus, the capacity curve tends to the bilinear curve given by the above straight lines.

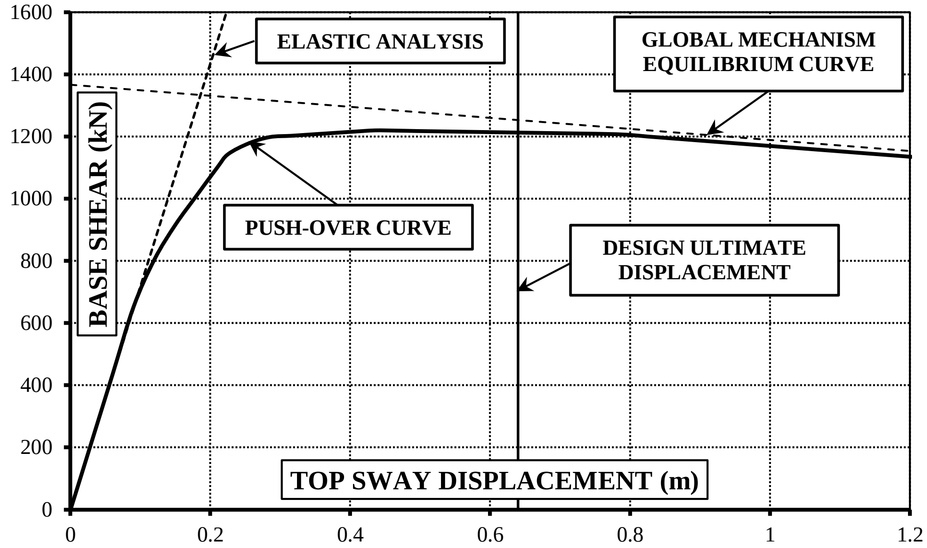

Figure 6: *Behavioural curve of the designed frame and comparison with the corresponding bilinear approximation*

A further confirmation, even the most important, of the fulfilment of the design objective is represented by the pattern of yielding developed at the occurrence of the design ultimate top sway displacement. In fact, developed plastic hinges are shown in Figure 7 and their pattern is in perfect agreement with the global mechanism. However, the complete development of the collapse mechanism does not occur, because plastic hinges at the first end of top storey beams are not still formed and column base sections are still in elastic range, and the structure remains stable even when the design ultimate displacement has been attained. For a displacement greater than the ultimate one, the global mechanism is completely developed according to the design goal.

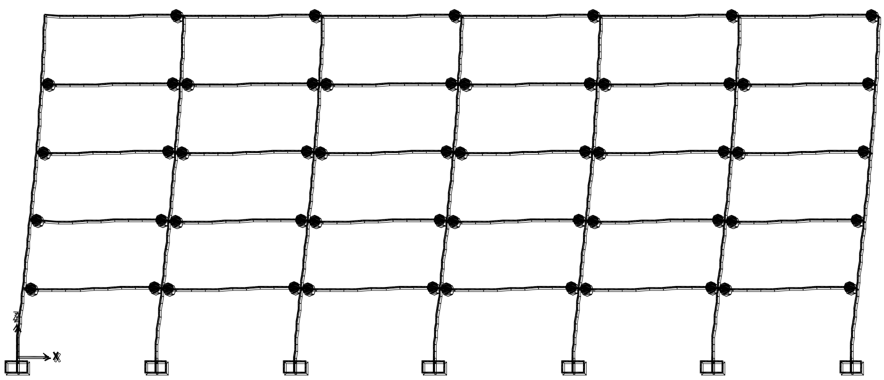

Figure 7: *Pattern of yielding of the designed frame at* $\delta = \delta_u$

In order to provide a more robust validation of the design methodology, non-linear incremental dynamic analyses have been carried out with reference to the same structural model used for push-over analyses and described above. In addition, 5% damping according to Rayleigh modelling has been assumed.

Record-to-record variability has been accounted for considering 7 recorded accelerograms selected from PEER [32]data base. In Table 8, the main features of the records (name, date, magnitude, ratio between PGA and gravity acceleration, length and step recording) are given. These earthquake records have been selected to approximately match the linear elastic design response spectrum of Eurocode 8, for type A soil. Moreover, in order to perform IDA analyses, each ground motion has been scaled to obtain the same value of the spectral acceleration $S_a(T_1)$ corresponding to the fundamental period of vibration T_1 of the structure (T_1=1.0 s). This is the seismic intensity measure (IM) adopted for IDA analyses where $S_a(T_1)$ values have been progressively increased until the occurrence of structural collapse, corresponding to anyone of the following ultimate limit states: column buckling, complete development of a collapse mechanism, attainment of the limit value of plastic rotation of beams or columns.

Table 8: *Accelerogram characteristics*

Earhquake (record)	Component	Date	PGA/g	Length	Step recording
Victoria, Mexico (Chihuahua)	CHI102	1980/06/09	0.150	26.91	0.01
Coalinga (Slack Canion)	H-SCN045	1985/05/02	0.166	29.99	0.01
Kobe (Kakogawa)	KAK000	1995/01/16	0.251	40.95	0.01
Northridge (Stone Canyon)	SCR000	1994/01/17	0.252	39.99	0.01
Imperial Valley (Agrarias)	H-AGR003	1979/10/15	0.370	28.35	0.01
Santa Barbara (Courthouse)	SBA132	1978/08/13	0.102	12.57	0.01
Friuli, Italy (Tolmezzo)	TMZ000	1976/05/06	0.351	36.35	0.005

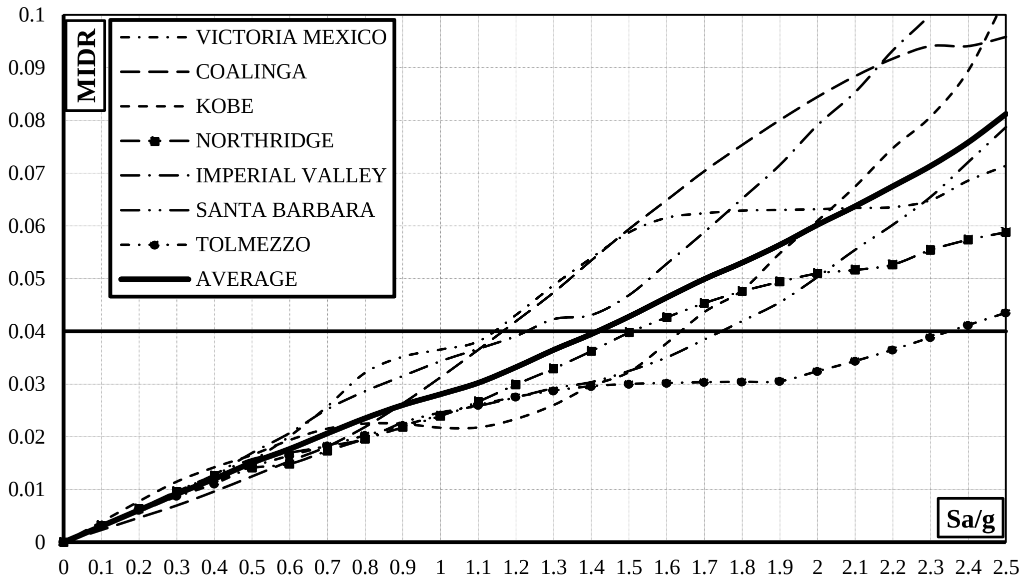

Figure 8: *Maximum interstorey drift ratio versus $S_a(T_1)$*

In Figure 8, the maximum interstorey drift ratio (MIDR) versus spectral acceleration curve is reported. MIDR curves appears quite regular and increasing without exhibiting dynamic instability. In addition, for each record the obtained pattern of yielding has been monitored for increasing values of $S_a(T_1)$ by checking that plastic hinge development is always in perfect agreement with the global mechanism. This results testifies the accuracy of the proposed design procedure even under actual seismic actions.

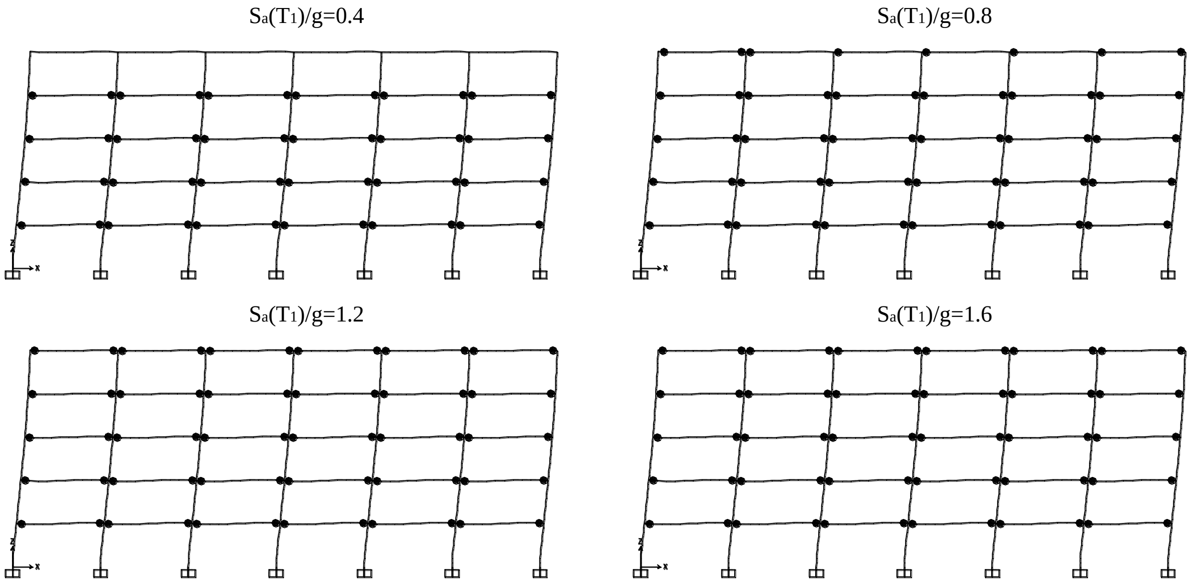

Figure 9: *Pattern of yielding of the designed frame for increasing value of $S_a(T_1)$ with reference to Kobe earthquake record*

As an example, Figure 9 provides the distribution of plastic hinges for increasing value of $S_a(T_1)$ with reference to Kobe earthquake record. As a consequence of the obtained design goal, the spectral acceleration values leading to collapse, given in Table 9, are very high and compatible with the adoption of the designed structure even in the case of destructive earthquakes. In particular, the average value of $S_a(T_1)$ leading to collapse is very close to 1.45 g while the average PGA is about 1.85 g. These very high values of spectral acceleration and PGA the structure is able to withstand testify the effectiveness of the design procedure to provide structures with excellent performances against life safety and collapse prevention limit states.

Table 9: $S_a(T_1)$ and PGA values corresponding to the attainment of the structural collapse

Earhquake (record)	**Sa_c/g**	**PGA_c/g**
Victoria, Mexico (Chihuahua)	1.15	0.82
Coalinga (Slack Canion)	1.15	0.79
Kobe (Kakogawa)	1.65	1.85
Northridge (Stone Canyon)	1.50	1.65
Imperial Valley (Agrarias)	1.25	2.42
Santa Barbara (Courthouse)	1.75	2.00
Friuli, Italy (Tolmezzo)	2.35	3.41
Mean value	**1.45**	**1.85**

6. CONCLUSIONS

In this paper new advances in the application of "Theory of Plastic Mechanism Control" have been presented pointing out how a closed form solution to the problem of assuring a collapse mechanism of global type can be achieved. This closed form solution constitutes a significant improvement compared to the original algorithm developed by Mazzolani and Piluso [17] in nineties which required an iterative solution. On the bases of the extension of the kinematic theorem of plastic collapse to the concept of mechanism equilibrium curve, the Theory of Plastic Mechanism Control allows to evaluate the sum of the plastic moments of columns required at each storey to obtain a

collapse mechanism of global type. This information has to be coupled with the computation of the axial forces occurring in the columns when the global mechanism is completely developed. The knowledge of bending moment M and axial force N allows the design of the column sections required to prevent any partial or storey mechanism.
The closed form solution of the design conditions makes now the design procedure very easy to be applied even by means of hand calculations and, therefore, it could also be suggested for code purposes by definitely solving the problem of collapse mechanism control whose importance in seismic design is universally recognised. Beam-column hierarchy criterion, commonly suggested by seismic codes, appears only as a very rough approximation when compared to TPMC and its theoretical background.
To show the accuracy of the proposed design procedure and its very simple practical application, a six-bay five-storey frame has been analysed leading to the fulfilment of the design goal, i.e. the development of a collapse mechanism of global type as testified by the results of both static non linear analysis and incremental dynamic analyses. These analyses have pointed out the excellent seismic performances obtained against life safety and collapse prevention limit states with an average value of the ultimate spectral acceleration equal to 1.45g which corresponds, for the analysed set of records, to an average value of PGA equal to 1.85 g.

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