

Manuscript Number: JCOMB-D-16-00613

Title: Novel "load adaptive algorithm based" procedure for 3D printing of lattice-based components showing parametric curved micro-beams

Article Type: SI: COMPOSITE LATTICE

Keywords: B. Directional orientation; B. Microstructure; C. Finite element analysis (FEA); D. Mechanical testing; 3D printing.

Corresponding Author: Dr. Francesco Naddeo, Ph.D.

Corresponding Author's Institution: University of Salerno

First Author: Francesco Naddeo, Ph.D.

Order of Authors: Francesco Naddeo, Ph.D.; Nicola Cappetti, Master Degree; Alessandro Naddeo, Master Degree

Abstract: This work deals with the development, the application and the experimental validation of a procedure based on an original algorithm, running in a finite element environment replacing the continuous mass of any solid with a cancellous bone-inspired lattice structure showing curved beams oriented on the basis of the external forces, sharing with it border and boundary conditions. For the validation of the new lattice structure a cubic representative volume element, showing curved micro-beams, was chosen, implementing periodic boundary conditions. At the end, the algorithm created a .stl file to be printed by a 3D printer using an appropriate polymer. The numerical results were compared with experimental results obtained by compression tests. Experimental/numerical correlation confirmed the validity of the FEM "beam element - based" lattice structure that could be applied to different solid shapes.

Suggested Reviewers: Roberto Guglielmo Citarella Master's degree Associate Professor, Department of Industrial Engineering, Department of Industrial Engineering, University of Salerno, Via Giovanni Paolo II, 132, 84084, Fisciano (SA), Italy  
rcitarella@unisa.it

Rocco Furferi  
Department of Industrial Engineering, University of Firenze, Via Santa Marta, 3  
rocco.furferi@unifi.it

Ernesto Reverchon  
Full Professor, Department of Industrial Engineering, University of Salerno, Via Giovanni Paolo II, 132, 84084 Fisciano, SA, Italy  
ereverchon@unisa.it

Ernesto Reverchon has developed strong skills in design and production of porous structures showing at the same time different levels of porosity to be used in the contest of computer aided tissue engineering.

Giovanni Francesco Solitro Ph.D.

Research Associate, Department of Orthopaedics, University of Illinois at Chicago, 835 South Wolcott Avenue. Chicago, United States of America  
solitrogf@hotmail.com

Giovanni Francesco Solitro has developed strong skills in the Biomechanics especially in the mechanical characterization of the bone tissue and in the parametric variational CAD modeling.

Alessandro Ruggiero

Associate Professor, Department of Industrial Engineering, University of Salerno, Via Giovanni Paolo II, 132, 84084 Fisciano, SA, Italy  
ruggiero@unisa.it

Opposed Reviewers:

Dear Editor

The paper entitled "*Novel "load adaptive algorithm based" procedure for 3D printing of lattice-based components showing parametric curved micro-beams*" by Francesco Naddeo, Nicola Cappetti and Alessandro Naddeo is an original article. The manuscript, including related data, figures and tables, has not been previously published and is not under consideration elsewhere.

This study deals with the development, the practical application and the experimental validation of a procedure based on an original algorithm (filed patent) that runs in a finite element environment; it operates inside a three-dimensional virtual solid of any shape, withstanding any kind of boundary conditions for both imposed displacements and loads (in the field of the static structures) replacing the continuous mass of the solid with an appropriate lattice structure, having cylindrical beams that are suitably curved and oriented according to the internal stresses induced by the boundary conditions, the new structure and the old one sharing the border. The user, in order to optimize the structural behavior, can manage several input parameters like average minimum distance between convergence nodes, target porosity, target average number of beams converging in each convergence node, curvature of the beams and beams' mechanical properties. By introducing parametric curved beams, this algorithm becomes a powerful tool capable to design a new porous material able to show a wide range of mechanical properties' values in output while maintaining a constant value of porosity i.e., a constant amount of material. A cubic representative volume element was appropriately designed and sized to conduct sensitivity analyzes in order to show the effect of changes in these parameters on the mechanical performances of the new porous material. The finite element results were compared with experimental results obtained by compression tests conducted on two polymeric 3D printed cubic specimen showing two different value of curvature characterizing the beams, the other input parameters being equal. The simulation results,

in terms of Young's modulus, confirmed the validity of the beam element - based structure that could be applied to different solid shapes and materials.

Possible applications could be the production of those porous materials for which it is required a set amount of material, a set external shape and at the same time heterogeneity of the mechanical properties at the micro and macro levels as for example in the field of the computational aided tissue engineering for production of scaffolds (bone-like structured foams) and in the field of ergonomics for production of highly structured foams.

I will be glad if you want to consider this paper for publication in "*COMPOSITES PART B: ENGINEERING: JCOMB*" - SPECIAL ISSUE ON "*COMPOSITE LATTICE MATERIALS AND STRUCTURES*".

Thank you for your consideration,

Sincerely

*Francesco Naddeo*

1 **Novel “load adaptive algorithm based” procedure for 3D printing of lattice-based**  
2  
3 **components showing parametric curved micro-beams**

4  
5 Francesco Naddeo<sup>a</sup>, Nicola Cappetti<sup>a</sup>, Alessandro Naddeo<sup>a</sup>

6  
7 <sup>a</sup>Department of Industrial Engineering, University of Salerno, Via Giovanni Paolo II, 132, 84084  
8  
9 Fisciano (SA) Italy.

10  
11 frnaddeo@unisa.it, ncappetti@unisa.it, anaddeo@unisa.it

12  
13 Corresponding author: Francesco Naddeo; frnaddeo@unisa.it

14  
15 **Abstract**

16  
17 This work deals with the development, the application and the experimental validation of a  
18  
19 procedure based on an original algorithm, running in a finite element environment replacing the  
20  
21 continuous mass of any solid with a cancellous bone-inspired lattice structure showing curved  
22  
23 beams oriented on the basis of the external forces, sharing with it border and boundary  
24  
25 conditions. For the validation of the new lattice structure a cubic representative volume element,  
26  
27 showing curved micro-beams, was chosen, implementing periodic boundary conditions. At the  
28  
29 end, the algorithm created a .stl file to be printed by a 3D printer using an appropriate polymer.  
30  
31 The numerical results were compared with experimental results obtained by compression tests.  
32  
33 Experimental/numerical correlation confirmed the validity of the FEM “beam element – based”  
34  
35 lattice structure that could be applied to different solid shapes.  
36

37  
38 **Keywords**

39  
40 B. Directional orientation; B. Microstructure; C. Finite element analysis (FEA); D. Mechanical  
41  
42 testing; 3D printing.

43  
44 **1. Introduction**

45  
46 In the past, researchers have always tried to create optimized structures for loads, with special  
47  
48 attention given to the amount of material used in order to reduce costs and lighten the  
49  
50 structures since the finite element method (FEM) was invented. Since 2000 [1] researchers  
51  
52 have developed, improved, and applied finite element-based topology optimization which  
53  
54 include a maturation of classical methods [2]. The most important types of topology optimization  
55  
56 are: density-based methods, which include the popular Solid Isotropic Material with Penalization  
57  
58 (SIMP) technique; hard-kill methods, including Evolutionary Structural Optimization (ESO);  
59

1 boundary variation methods; biologically inspired methods. In the area of biologically inspired  
2 engineering, we can note how living beings have developed biological tissues characterized by  
3 such optimized structures. For example, trabecular bone architecture undergoes adaptive  
4 changes that were first studied by Wolff [3], who described the trajectorial theory of trabecular  
5 orientation. Several mathematical models have been proposed for bone remodelling and  
6 various criticisms have been made on Wolff's law [4] about the one-to-one correspondence  
7 between the stress trajectories of continuous bodies and the discontinuous structure of the  
8 bone. The proposed models usually assumed bone tissue final configuration as a function of  
9 boundary conditions and bone to be a continuous, linearly elastic material. Anisotropic  
10 continuum-based models were used by Fernandes et al [5], Garcia et al [6] and Jacobs et al [7],  
11 while orthotropic continuum-based models were used by Cowin [8] and Huiskes and Hollister  
12 [9]. Some authors [10 - 11] have proposed Hierarchical multi-scale bone models, in which the  
13 micro/nanometric structure was taken into account. Tsubota et al [12] used large-scale voxel  
14 finite element models (FEMs) for simulating the trabecular bone remodelling. Optimization  
15 techniques were proposed by both Luo and An [13] and Boyle and Kim [14] while Hambli et al  
16 [11] introduced a coupled finite element and neural network computation method. Currently, as  
17 a non-destructive method in the field of mechanical behaviour research, finite element analysis  
18 is increasingly used as a way to realize new structures generally used for composite materials  
19 or porous materials not only to understand the internal stress distribution and the mechanical  
20 behaviour of already existing structures but also to design new structure. In this case, FEM  
21 results become input parameters managed by highly complex algorithms that, by means of  
22 looping procedures, are able to optimize virtual structures respect to a particular mechanical  
23 aspect.

24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

The aim of this work was to create a highly structured porous material biologically inspired through the three-dimensional printing which, in the same manner of biological tissues and, in the present case, of cancellous bone (on the base of Wolff's law), is able to adapt itself to external loading conditions, showing, at the same time, a lightweight structure with the possibility to parameterize the homogenized mechanical properties by simply varying the sensitive parameters that describe the geometry of the structure. In many fields of research on

1 the highly structured porous materials, the production is often bound to the use of particular  
2 materials, to respect a particular external geometrical shape and a particular porosity, i.e., a  
3 particular amount of material. By using the presented algorithm, once you set the material to be  
4 used (compatibly with the possibility of printing it) you can, for example, set the porosity at a  
5 certain level and have access to a wide range of mechanical properties by varying a few  
6 geometric parameters in input. These are much sought-after qualities in the field of Computer  
7 Aided Tissue Engineering (CATE) for the production of scaffolds. Scaffolding materials for bone  
8 regeneration, for example, should be osteoconductive and favour the adhesion and migration  
9 on the scaffolds of the osteoprogenitor cells that subsequently differentiate and form the new  
10 bone; for this reason the geometry of the porous material has to mimic that of natural biological  
11 tissue (that is generally naturally optimized to withstand the loads to which it is subjected during  
12 its life cycle). Furthermore in the design of such structures you must take into account the  
13 boundary conditions characterizing the period they will spend in bioreactors in which they will be  
14 subject to different types of mechanical stimuli both static and dynamic. In a previous study  
15 Baldino et al [15] have demonstrated that the curvature of the nanofiber is the parameter that  
16 most influences the weakness/stiffness of this kinds of porous material. For this reason we have  
17 developed a procedure based on an original algorithm [16], running in a finite element  
18 environment, operating inside a three-dimensional solid of any shape, withstanding any kind of  
19 boundary conditions for both imposed displacements and loads (in the field of the static  
20 structures) replacing the continuous mass of the solid with an appropriate lattice structure,  
21 having cylindrical beams directed according to the internal stresses induced by the boundary  
22 conditions, sharing with it border and boundary conditions. In this work, we have used  
23 parametric curved micro-beams in order to control the weakness of the whole structure after the  
24 optimization carried on by orienting the beams according to the boundary conditions. A cubic  
25 representative volume element (RVE) was appropriately selected and sized to conduct  
26 sensitivity analyzes to show the effect of changes in input parameters on the mechanical  
27 performances of the new porous material. The finite element results were compared with  
28 experimental results obtained by compression tests conducted on polymeric 3D printed cubic  
29 specimens. The simulation results in terms of Young's modulus confirmed the validity of the  
30

1 “beam element – based” lattice structure that could be applied to different solid shapes and  
2 materials.  
3

## 4 **2. Materials and Methods**

### 5 **2.1 Description of the algorithm**

6  
7 The authors created an algorithm running in Ansys FEM environment able to create a virtual  
8 beam network (beams of any length, of any type of section and aspect ratio) with beams  
9 connected to each other at the end points with internal fixed joints, inside a solid of any  
10 geometric shape. The network shares with the solid the borders and the boundary conditions.  
11 The algorithm remodels the beam network obtaining a structure having the beams mostly  
12 oriented along the principal stress directions recorded in the continuous mass. After appropriate  
13 sizing of the beams, the algorithm gives us a new structure able to efficiently withstand the  
14 same boundary conditions with a considerable lightening of the structure, and consequently a  
15 notable reduction in materials used. In this work, in order to have a direct control on the  
16 mechanical properties of the final structure, a parametric curvature has been introduced for  
17 each cylindrical beam on the bases of studies reported in a previous work [17].  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30

31 The algorithm consists of a single source code and has been written in ANSYS Parametric  
32 Design Language (APDL); during the calculation, the algorithm performs the following steps  
33 (Figure 1):  
34  
35  
36

- 37 1. Initially, it takes in input the three-dimensional CAD (Computer Aided Design) geometry  
38 of the solid taken in consideration;  
39
- 40 2. It creates a random distribution of keypoints (keypoint cloud) that is sufficiently uniformly  
41 distributed, and that occupies the volume of the solid;  
42  
43
- 44 3. for each generated keypoint, it defines a spherical space of selection (SSS) in order to  
45 intercept a specified number of keypoints [17] and generates line segments joining the  
46 initial keypoint, with each keypoint belonging to its SSS;  
47  
48
- 49 4. It activates a routine that conveniently eliminates both overlaps and intersections  
50 among line segments;  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65



- 1 5. It creates the mesh of beam elements in each line segment [17 - 18], obtaining a lattice  
2 structure occupying the volume of the solid that shows an isotropic mechanical  
3 behaviour;  
4  
5  
6
- 7 6. It applies to the FEM model linear material properties characterizing the real material  
8 behaviour;  
9
- 10 7. It applies boundary conditions to the solid that simulate the real conditions;  
11
- 12 8. It calculates the elastic solution due to the boundary conditions defined in step 7;  
13
- 14 9. In post-processing, it sorts, in an array, all the identification numbers of the created  
15 beams in an ascending manner according to an appropriate criterion based on the  
16 magnitude of the stresses to which each element is subjected during the FEM virtual  
17 test.  
18  
19  
20  
21  
22
- 23 10. It activates an iterative routine that, according to the array created in step 9, eliminates  
24 the beams starting from the first component of the array. The iteration stops at a  
25 specified number of beams such that the number of connections for each convergence  
26 node is equal to the desired average number of beams (ANB) converging on each  
27 node; simultaneously it activates an *ad hoc* routine able to guarantee structural  
28 continuity;  
29  
30  
31  
32  
33  
34
- 35 11. It activates a routine that eliminates the so-called “dead ends”; i.e., the branches of the  
36 structure made of beams that, having only a single point of connection with the rest of  
37 the structure, will not work [19];  
38  
39  
40
- 41 12. In order to reach the desired homogenized mechanical property of the final product, it  
42 activates a routine that introduces a parametric curvature to each beam and at the  
43 same time resize the beams (varying the cross section) to achieve the desired porosity  
44 (i.e., fraction of the volume of voids over the total volume of the inputted solid).  
45  
46  
47  
48
- 49 13. It generates in CAD environment the solid model of the new structure;  
50
- 51 14. It generates the file .stl for the following rapid prototyping phase in order to obtain the  
52 physical model.  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

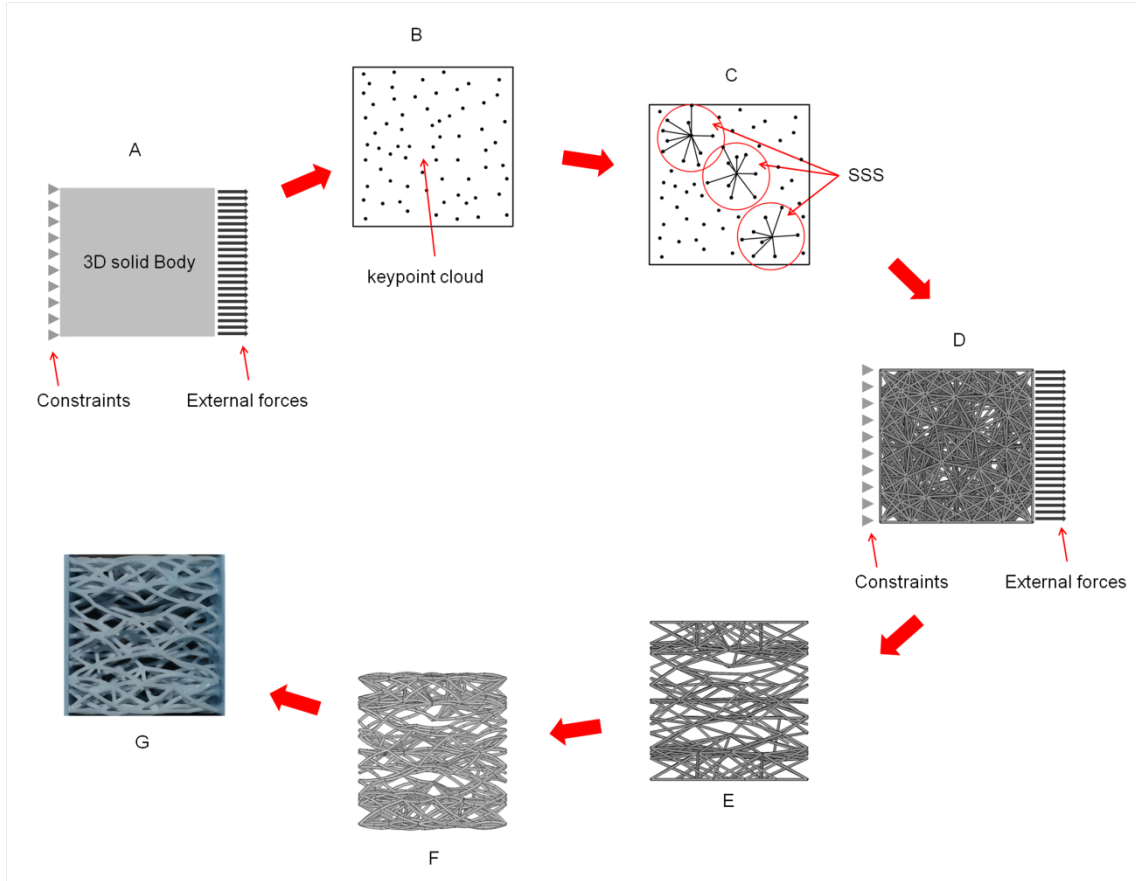


Figure 1. Algorithm fundamental steps for elaborating a generic cubic solid mass withstanding generic boundary conditions: A) step 1; B) step 2; C) steps 3 and 4; D) step 5 ; E) steps 6 - 11; F) steps 12 - 13; G) 3D printed polymeric specimen.

Previous studies [20] on the same topic, created three-dimensional beam-based structures by interpolating only the principal stress directions so that they did not select other elements that, being oriented not along isostatic lines may constitute real bracings and improve structural performance. In this work, the initial beam network (see step 5), that shows randomly oriented beams, allows for the most advantageous directions to be chosen to minimize the amount of material used, by eliminating selectively the less loaded beams until reaching a predefined number of converging beams for each convergence node. The final result consists of a new parametric curved beam network showing porosity set at a desired value, a desired ANB and that shares the border with the original solid. Beams are mostly oriented along the principal stress directions, whose convergence nodes have a desired average number of connections.

## 2.2 Input parameters

The algorithm's input parameters are the following:

- Average minimum distance between convergence nodes (*dist*) defined as a linear function of a characteristic size of the solid;
- Target porosity;
- Target ANB;
- Radius of the SSS, being a linear function of the previously defined "*dist*", that is chosen due to its appropriate ANB;
- Curvature of the beams;
- Mechanical properties.

## 2.3 FEM element type

In this work, we have implemented quadratic three-node straight beam elements in 3-D, showing circular cross section, which are suitable for analyzing slender to moderately stubby/thick beam structures. It takes account of both torsion-related shear stresses and flexure-related transverse-shear stresses, which henceforth will be generically referred to as "shear stresses" ( $\tau$ ), as the focus will be on their absolute maximum value and not on their causes. Six degrees of freedom occurred at each node for translations and rotations in directions  $X$ ,  $Y$  and  $Z$ .

## 2.4 FEM mechanical properties

The first application of the algorithm consisted of the realization of a sensitivity analysis campaign on input parameters realized on an appropriate cubic RVE with the aim to evaluate the effects of changes in sensitive input parameters, specified in section 2.2, on the mechanical performance of the new material in linear elastic conditions. For this reason, for the mechanical characterization of the beams, the elastic modulus was set to  $E = 1$  Pa while the Poisson's ratio was set to  $\nu = 0.3$  for ease of discussion and without any loss of generality. Later in the discussion, the chosen morphometric input data and the material properties of the polymer selected for printing and that have been introduced in the algorithm, will be reported.

## 2.5 RVE description and boundary conditions

1 In FEM analysis, the RVE should be large enough to contain all the intrinsic characteristics of  
2 the structure to be simulated, which, in our case, consists of geometrical characteristics and  
3 material properties. Cubic-shaped RVE (with a unitary volume for ease of discussion and  
4 without any loss of generality) was chosen to evaluate the mechanical performance of the  
5 algorithm; cubic-shaped RVE also allows an easy boundary conditioning.

6  
7 The target of this work was to optimize the structure of the chosen RVE orienting first the beams  
8 according to the loads transmitted internally by the boundary conditions and, subsequently,  
9 introducing a variable curvature to the beams to calibrate, as needed, the value of the  
10 homogenized mechanical properties. In this study, the RVE was subjected to a virtual tensile  
11 test along the X axis parallel to one of the edges of the cubic RVE to allow the algorithm to  
12 properly orient the beams along this axis.

### 23 **2.5.1 Periodic boundary conditions**

24 All the simulations, which have been realized to create and size the FEM model, were carried  
25 out by applying PBC to the RVE allowing to calculate more accurately the elastic constants of  
26 the simulated material [21 - 22]. For the specific study of this cubic RVE, an *ad hoc* written FEM  
27 routine was used; this routine can automatically create an identical surface distribution of nodes  
28 for the opposite sides [23]. Further, these nodes are opportunely selected on the basis of their  
29 location since the conditions on the displacements, in this case, have to couple the  
30 displacements at each point located on opposite sides of the cubic RVE. These conditions can  
31 be expressed in the following synthetic matrix form:

$$32 \quad u_2 - u_1 = \varepsilon(X_2 - X_1) \quad (1)$$

33 With  $\varepsilon$ , being the *strain tensor*,  $X_i$  ( $i = 1, 2$ ) being the position vector of the points belonging to  
34 opposite sides of the RVE, and  $u_i$  being the relative displacement.

### 48 **2.5.2 Homogenized modules**

49 The routine described in 2.5.1, for each simulation can automatically perform three static tensile  
50 tests and three static shear tests (with imposed average strains) in order to have an exhaustive  
51 representation of the mechanical behaviour of the structure. Imposing the overall strain to the  
52 RVE:  $\langle \varepsilon \rangle = [0 \dots \langle \varepsilon_i \rangle \dots 0]$ , and assuming elastic behaviour and small strains of the material, for

each i-th imposed strain, the routine calculates all the components of the corresponding i-th column of the stiffness matrix using the following formulation (elastic overall constitutive law):

$$\overline{C}_{hi} = \frac{\langle \sigma_h \rangle}{\langle \varepsilon_i \rangle} \forall h, i = 1, \dots, 6 \quad (2)$$

with  $\langle \sigma_h \rangle$  being the stress component calculated in the FEM post-processing environment, using the following relation:

$$\langle \sigma_h \rangle = \frac{\sum_{elem} \sigma_{h_{elem}} vol_{elem}}{vol_{RVE}} \quad (3)$$

$\sigma_{h_{elem}}$  being the h-th stress component of the single FEM element, due to the i-th imposed strain,  $vol_{elem}$  being the element volume, and  $vol_{RVE}$  the volume of the RVE. In this way, the routine calculates all the components of the stiffness matrix with Eq. 2 by means of a single FEM run, characterized by six sequential imposed strains. Finally, the routine automatically provides the stiffness and compliance matrices of the examined cubic RVE in output.

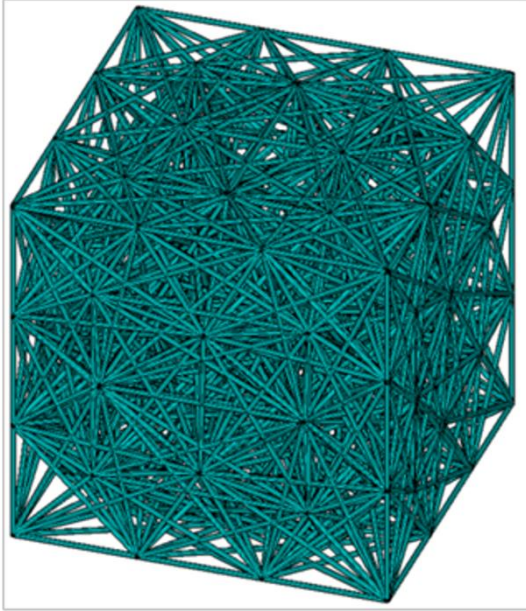
### 2.5.3 RVE sizing

As reported previously, the main algorithm, in step 5 (see section 2.1), creates a beam network that tends to show isotropic material behaviour (see Figure 2). In terms of statistics, the increasing of the RVE size (i.e., increasing the number of beams) causes an increase in the material's isotropic degree. For this reason, intrinsic statistical characteristics were taken into consideration when choosing the RVE size in order to obtain an acceptable starting isotropic degree [22]. An original isotropic criterion [23 -15] to optimize the size of the cubic RVE was implemented. It is based on the minimization of the following function:

$$|\Delta|^2 = \sum_{i,j=1}^6 (C_{i,j} - C_{i,j}^I(\lambda, G))^2 \quad (4)$$

with  $C_{i,j}$  being the i, j-th component of the stiffness matrix, calculated from the FEM results;  $C_{i,j}^I$  being the i, j-th component of the unknown isotropic stiffness matrix; and  $\lambda$  and  $G$  representing the independent parameters defining the isotropic stiffness matrix (Lamè constants). The minimization of Eq. 4 led to the determination of  $\lambda$  and  $G$ , i.e., the parameters defining the behaviour of the isotropic material closer to the behaviour of the material simulated by the FEM calculation. Consequently, an ad hoc written routine calculated the parameter  $\delta$ , which was defined as the ratio of the norm of the "difference tensor"  $\Delta = C - C^I$  to the norm of

1 the tensor  $C^I$ . The parameter  $\delta$  provided information about the isotropic degree of the cubic  
2 RVE, which was directly related to the previously defined parameter “*dist*” of the RVE.  
3  
4  
5  
6



7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28 Figure 2. Example of an initial isotropic cubic RVE. For readability purposes, fewer mesh wires  
29 than those used in the calculation are represented.  
30  
31  
32  
33

34 The isotropic criterion showed that the value of the parameter  $\delta$  tended towards a small  
35 constant value for  $dist = \frac{Lrve}{10}$ , with  $Lrve$  being the edge length of the cubic RVE. In this way, the  
36 RVE size was not too large from a computational point of view, but sufficiently large to make the  
37 beams substantially oriented in a random manner. This structure gave to the RVE a sufficiently  
38 isotropic mechanical behaviour prior the application of the boundary conditions designed to  
39 orient the beams (see section 2.1, step 7).  
40  
41  
42  
43  
44  
45  
46

## 47 **2.6 Convergence analysis of the FEM results using straight beams**

48 Increasing the number of elements for each beam did not cause an appreciable variation in the  
49 FEM results in terms of elastic moduli. For this reason, each beam was modelled by means of a  
50 single beam element [24, 25, 18].  
51  
52  
53  
54

## 55 **2.7 Beam sorting criteria**

In the post-processing environment (see section 2.1, step 10), the algorithm sorts, in an array, all the identification numbers of the created beams in an ascending order according to an appropriate criterion based on the magnitude of the stresses to which each element is subjected during the virtual test. The choice of the criterion can be based on considerations relating to both the real behaviour of the simulated material (material ductility and brittleness) and the boundary conditions. In the present study, three different output entities were considered for each beam element:

- The quantity of elastic strain energy stored during the calculation,

$$\phi = \int C_{ijhk} \varepsilon_{ij} d\varepsilon_{hk} \quad (5)$$

with  $C_{ijhk}$  being the generic component of the elastic tensor of fourth order, and  $\varepsilon_{ij}$  and  $\varepsilon_{hk}$ , the generic strain components;

- Maximum equivalent von Mises stress ( $\sigma_{\text{eq, Max (von Mises)}}$ );
- Maximum principal stress' absolute value ( $\sigma_{\text{principal, Max}}$ );

In the second criterion, a selection based on the amount of distortion energy stored by the beams during simulated tension/compression test (von Mises failure criterion for ductile material) is preferred. The third criterion, conversely, only considers the effect of the normal stresses stored in the beams (Coulomb and Mohr failure criteria for brittle materials). To achieve this, the main algorithm is equipped with specific routines designed to calculate/retrieve and sort the above variables for each element in the post-processing environment (see section 2.1, step 10).

## 2.8 introduction of curved beams

Once the RVE lattice structure was realized, the beams properly oriented and the porosity  $P$  set, the ratio of the average diameter to the length of the cylindrical beams is univocally determined. Some geometric aspects of the microstructure of porous materials can deeply affect their mechanical behaviour. For example aerogel is a very compliant material. Baldino et al [15] produced Poly-L-Lactid Acid aerogels for bone scaffolding whose nanometric structure was characterize by a network of curved fibres. In this work, it has been demonstrated, through a parametric FEM models, that the average curvature characterizing the fibres scaffolds, leads to a bending/buckling tendency, determining a higher weakness of the entire structure. This

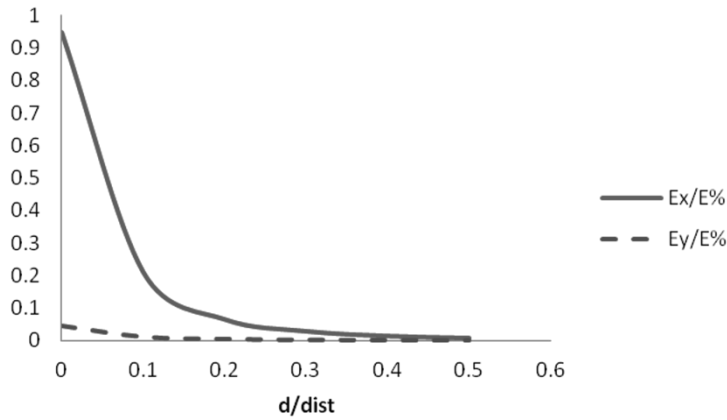
observation was supported by a previous study by Ma et al [19] that, by means of modelling,  
 tried to learn more about loop formation during the gel aggregation process and discovered that  
 bending was the major mode of deformation of the network. Also Pirard and Pirard [26]  
 proposed a mechanism based on the tendency of the network to buckle. In fact, in such lattice  
 structure, the straight beams are basically subject to tensile-compression stresses (within small  
 displacements), regardless of the type of boundary conditions applied to the nodes. It is  
 possible to demonstrate that by substituting the straight beams with curved beams a bending  
 moment appears for each beam, and that the bending moment contribution to the  
 displacements becomes more prominent with increasing of curvature, especially if the ratio of  
 the radius of curvature to beam thickness is greater than 10 as in the proposed FEM model [15].  
 These bending moments determine small local strains/stresses that, determine large deflections  
 of the extremes of each beam; i.e., in case of equal displacements, the curved beam lattice  
 structure will be subject to local strain/stress lower than the straight beam lattice structure. This  
 phenomenon implies a low homogenized Young's modulus which results in a high softness of  
 the whole structure. To advantage of this feature, we implemented a routine in the main  
 algorithm that substitutes the straight beams of the lattice structure with parametric cylindrical  
 curved beams based on spline curves (lying on a plane and having second order derivative at  
 the endpoints equal to zero), modelling the cylinder axis, interpolating three points, two of which  
 coincident with the endpoints of the beams and a third in a central position at a distance " $d$ "  
 from the straight segment linking the two endpoints; the plane of the spline curve being  
 randomly oriented around the straight segment linking the two endpoints. The volume occupied  
 by the beam network is:

$$V \cong \sum_{i=1}^n l_i(d)\Sigma \quad (6)$$

$n$  being the total number of beams,  $l_i$  the length of the  $i$ -th spline, which is a function of the  
 parameter  $d$ , and  $\Sigma$  the cross section of the beams that does not vary with  $i$ . This routine  
 produced an interpolating curve of output data points that describes the function  $\frac{E_i}{E} = \varphi(d)$ ,  $\forall i \in$   
 $[x, y, z]$ , with  $P$  being constant, by using the relationship (6),  $E_i$  being the homogenized Young's  
 modulus of the cubic RVE along the  $i$ -th axis and  $E$  being the input Young's modulus of the  
 curved Beams. Figure 3 shows an example of the range of the homogenized Young's modulus



1 values to which it is possible to access by varying  $d/dist$ , once set  $P = 0.8$ ,  $ANB = 6$ ,  $SSS =$   
2  
3 1.75.



21 Figure 3. Percentage ratio of  $E_x$  and  $E_y$  to  $E$ , in function of the beams' curvature represented by  
22 " $d/dist$ ", the porosity  $P$  being constant;  $E_z$  has not been introduced in the graph since it is quite  
23 superimposable to  $E_y$  one.  
24  
25  
26

27  
28  
29 If straight beams were implemented, once set the porosity, it could not be easily to vary the  
30 homogenized mechanical properties of the cubic RVE since the variation of the resistant section  
31 of the beams, for controlling these mechanical properties, would lead to a variation of the  
32 porosity. By introducing parametric curved beams this algorithm become a powerful tool  
33 capable to design a new porous material able to show a wide range of mechanical properties'  
34 values in output while maintaining a constant value of porosity i.e., a constant amount of  
35 material.  
36  
37  
38  
39  
40  
41  
42

### 43 2.8.1 Convergence analysis of the FEM results using curved beams

44  
45 A convergence analysis of the results (in terms of Young's Modulus) was performed choosing  
46 an average element size of the FEM model equal to  $M = L/12$ ,  $L$  being the length of the curved  
47 beam, with a corresponding percentage error of 0.055 compared to the asymptotic value.  
48  
49  
50

### 51 2.9. Sensitivity analysis campaign

52  
53 A sensitivity analysis campaign was realized on an appropriate cubic RVE, after the orientation  
54 of the beams, with the aim to evaluate the effects of changes in sensitive input parameters,  
55 specified in paragraph 2.2, on the mechanical performance of the new material in linear elastic  
56  
57  
58  
59  
60  
61

1 behaviour range. We have used a simple “one factor at a time” approach, estimating the  
2 influence of some geometric parameters on the response by independently varying the  
3 parameters over a certain range. However, this approach is unable to clearly reveal possible  
4 interactions between parameters [27] but it gives us information about the potentiality in  
5 managing of such an algorithm.  
6  
7  
8  
9

### 10 **2.9.1 Variation of the parameter SSS**

11 The SSS size must ensure an initial ANB as to have available a fairly large number of  
12 orientations to be selected. An excessive large SSS size, however, determines computational  
13 errors caused by possible intersections between curved beams, difficult to model in CAD  
14 environment, with subsequent inability to create the .stl file for 3D printing. For this reason three  
15 values of SSS radius ( $1.5 \times dist$ ,  $1.75 \times dist$  and  $2 \times dist$ ) were considered, the other input  
16 parameters being constant. The authors have judged the value of SSS radius equal to  $1.75 \times$   
17  $dist$  a fair compromise between fairly high value of the homogenized Young’s modulus and the  
18 risk of possible intersections between curved beams (if the curved beams remain, on average,  
19 of the same length, the probability that they intersect each other is greatly reduced).  
20  
21 Furthermore, in this way there is a better accuracy in modelling the boundary of the solid when  
22 it presents curvatures with a radius of the same order of magnitude as the average minimum  
23 distance between the convergence nodes.  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36

### 37 **2.9.2 Variation of the parameter ANB**

38 Subsequently, once set the SSS size, ANB was made to vary from a minimum of 3 to a  
39 maximum of 20. It was found that the homogenized Young’s modulus of the cubic RVE, in the  
40 loading direction ( $E_x$ ), tends to decrease with increasing of ANB, while the other two Young’s  
41 moduli ( $E_y$  and  $E_z$ ) tend to increase, thus tending to the same value for ANB greater than 12.  
42  
43 Thinking of a future production of bone scaffolds that have to mimic, among other things, also  
44 the geometric distribution of the trabeculae, ANB equal to 6 was chosen also taking account of  
45 the average number of converging trabeculae characterizing the transversely isotropic  
46 cancellous bone [18].  
47  
48  
49  
50  
51  
52  
53  
54

## 55 **3. Three dimensional printing and mechanical tests**

Two virtual models, characterized by the morphometric input data collected in the Table 1, were created using the algorithm described in this work that oriented the beams along the  $X$  axis. The aim was to print three-dimensional specimens to be tested in order to study the experimental/numerical correlation. The target of this phase of the study was to validate the FEM model characterized by a one-dimensional element (beams) based network. In this case, for ease of discussion and without any loss of generality, the parameters " $L_{rve}$ " (length of the cubic RVE) and " $dist$ " have been sized taking into account the characteristics of the 3D printer and of the available testing machine. The two models differ only in the curvature of the beams. The algorithm has provided, in output, two .iges files containing a set of solids representing the curved beams (inflected cylinders on the basis of spline generated by the algorithm) and the convergence nodes (spheres whose radius is equal to the inflected cylinders' radius).

Table 1 Morphometric input parameters characterizing the two cubic RVEs to be printed.

|                   | <b>RVE1</b> | <b>RVE2</b> |
|-------------------|-------------|-------------|
| <b>Lrve</b>       | 2cm         | 2cm         |
| <b>dist</b>       | $L_{rve}/5$ | $L_{rve}/5$ |
| <b>SSS radius</b> | $dist*1.75$ | $dist*1.75$ |
| <b>Porosity</b>   | 0.95        | 0.95        |
| <b>d/dist</b>     | 0.1         | 0.25        |
| <b>ANB</b>        | 4           | 4           |

These files have been treated with a special routine in CAD simulation environment to fuse the various solids together and create the .stl files to be supplied to the 3D printer. For printing we have chosen a 3D printer (DWS 020D produced by DWS srl - Via Lago di Levico, 3 - 36010 Zanè (VI) Italy); the specimens were equipped with rigid bases perpendicular to the axes of compression in such a way they also acted as a supporting material during the printing phase (see Figure 4). Compression mechanical properties of the printed material were measured using a CMT4000 Series (by SANS, China). The cubic specimens were compressed at a cross-head speed of 1mm/min along  $X$ ,  $Y$  and  $Z$  directions. Five specimens were tested for each configuration.

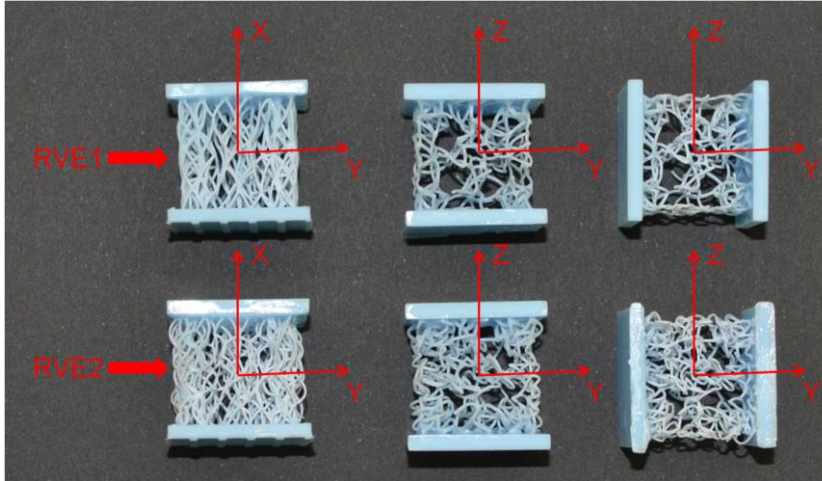


Figure 4. Printed specimens: in the first row, the three configurations of the RVE1 specimen are shown; in the second row, the three configurations of the RVE2 specimen are shown.

#### 4. Experiments

The algorithm was optimized to process the cubic RVE described in the section 2.5 and characterized by the parameters listed in the Table 1. The boundary conditions (see section 2.1, step 7) used to select the more loaded beams (oriented along the  $X$  axis) consisted of an overall deformation  $\langle \varepsilon \rangle = [\langle \varepsilon_x \rangle \dots 0]$ , with  $\langle \varepsilon_x \rangle$  being the strain component along the  $X$  axis set to  $10^{-4}$  for the sake of simplicity. The algorithm, once introduced the experimental mechanical properties and calculated the elastic solution (see section 2.1, step 8), selected the beams that were directed according to the internal stress by deleting the least stressed beams while maintaining structural continuity (see section 2.1, steps 9 - 11). At the end of the routine completed by the orientation of the beams (see section 2.1, steps 10 and 11), the algorithm, on the basis of the pseudocode listed in Section 2.1, introduced a parametric curvature to each beam and, at the same time, resized the beams (varying the circular cross section) to achieve the desired porosity (see section 2.1, step 12). Figure 5 shows the effect of the algorithm elaboration on the cubic RVE (a, b and c) and the 3D printed polymeric specimen (d). At the end, (see section 2.1, steps 13 and 14) two physical models characterized by the morphometric parameters listed in table 1 were realized by rapid prototyping technique with the purpose of subjecting them to compression test; the three homogenized Young's moduli have been evaluated in the three mutually perpendicular directions  $X$ ,  $Y$  and  $Z$  of the two cubic specimens. Once built the two virtual RVEs described in section 3, boundary conditions,

approximating the compression tests, have been applied in Ansys environment, assuming absence of sliding of the zones in contact with the rigid bases characterizing the polymeric specimen. Then the three Young's moduli have been evaluated in the three directions X, Y and Z of the cubic RVE.

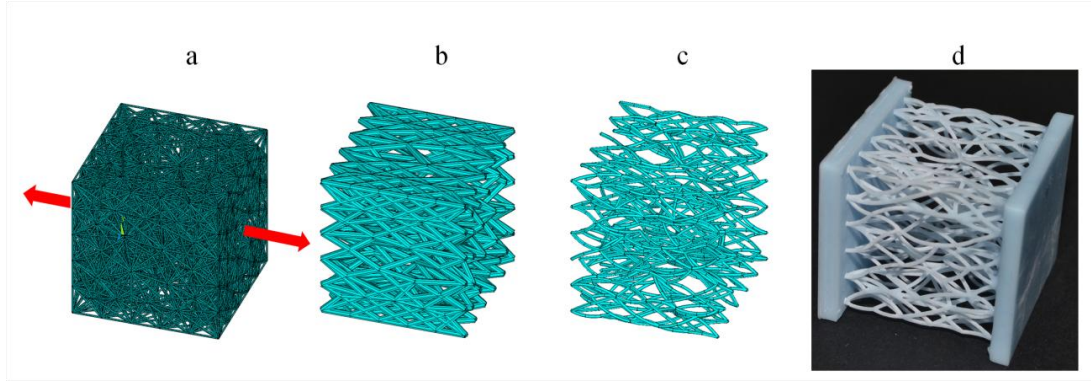


Figure 5. Effect of the algorithm elaboration on the cubic RVE: a) isotropic cubic lattice structure showing the direction of the imposed displacements ( $X$  axis); b) cubic lattice structure showing resized beams oriented along the  $X$  axis; c) cubic lattice structure showing curved beams; d) 3D printed polymeric specimen.

## 5. Results

### 5.1 Beam sorting criteria results

The choice of beam sorting criteria was based on a comparison of the FEM simulation results of the final cubic RVE showing the beams oriented along the  $X$  axis. For the sake of simplicity, an input Young's modulus for beams equal to  $E_i = 1Pa$  and with an overall deformation  $\langle \varepsilon_x \rangle = 10^{-4}$  were introduced as input parameters. The results used for comparison were maximum values ( $Maxv$ ) recorded in the whole RVE for von Mises equivalent stress, principal stress (absolute value), and shear stress (absolute value). To understand the average behaviour of the RVE structure and to avoid the possibility that abnormal maximum values could affect the evaluation, it was also considered the arithmetic mean value ( $Amv$ ) of the maximum values recorded in each beam of the same items that was calculated by the following formula:

$$Amv = \frac{1}{n} \sum_{i=1}^n X_{Max,i} \quad (7)$$

with  $n$  being the number of beams and  $X_{Max,i}$  representing, alternately, the maximum values of von Mises equivalent stress, principal stress (absolute value) and shear stress (absolute value) in the  $i$ -th beam.

Finally, a comparison was also made between the homogenized Young's moduli of the RVE, calculated in the preferred direction of the beams ( $X$ -axis) using the three different criteria.

Table 2 show the used criteria and the comparison among them.

Table 2 Comparison of the criteria.

| CRITERIA                            | Elastic strain Energy |            | $\sigma_{eq}$ (Von Mises) |            | $\sigma_{principal, Max}$ |            |
|-------------------------------------|-----------------------|------------|---------------------------|------------|---------------------------|------------|
|                                     | <i>Maxv</i>           | <i>Amv</i> | <i>Maxv</i>               | <i>Amv</i> | <i>Maxv</i>               | <i>Amv</i> |
| $\sigma_{eq, (Von Mises)}$ [Pa]     | 3.2032E-04            | 1.2202E-04 | 3.1591E-04                | 9.2860E-05 | 3.1591E-04                | 1.3007E-04 |
| $Abs(\sigma_{principal, Max})$ [Pa] | 3.2022E-04            | 1.1862E-04 | 3.1568E-04                | 9.1430E-05 | 3.1568E-04                | 1.2867E-04 |
| $Abs(\tau)$ [Pa]                    | 5.1530E-05            | 5.9400E-06 | 4.7010E-05                | 7.6400E-06 | 4.7010E-05                | 6.1900E-06 |
| RVE Young's Modulus ( $E_x$ ) [Pa]  | 1.6522E-01            |            | 1.6515E-01                |            | 1.6412E-01                |            |

As can be seen in the Table 2, regardless of the criterion used, *Maxv* for  $\sigma_{eq, (von Mises)}$  and  $Abs(\sigma_{principal, Max})$  assumes about the same value, while *Maxv* for  $Abs(\tau)$  assumes a value of about one order of magnitude lower. The criterion that sorts/eliminates the beams according to  $\sigma_{eq}$  (von Mises) allows to have in output a high homogenized Young's modulus value for the RVE, and, at the same time, to register relatively low *Maxv* for  $\sigma_{eq, (Von Mises)}$  and  $Abs(\sigma_{principal, Max})$  and moreover a low *Maxv* for  $Abs(\tau)$ . Using this criterion, we also have the lowest *Amv* for the same output results, with the exception of shear stresses that, however, remain over one order of magnitude lower compared to normal stresses. From the structural point of view, in the context of small deformations, the target was the optimization of the structural stiffness. Having to choose an internal configuration of the RVE lattice structure, boundary conditions and amount of material (porosity) being equal, the author chose equivalent von Mises stress criterion for subsequent FEM calculations. This criterion gave a beam configuration that showed a high Young's modulus and recorded, inside, the less burdensome stress state.

## 5.2 FEM results

1 A photopolymer named RD 096 blue (Digital Impression Resin, produced by DWS srl - Via Lago  
 2 di Levico, 3 - 36010 Zanè (VI), Italy) was chosen for the 3D printing; this polymer, after UV post-  
 3 curing, is characterized by the following elastic material properties:  $E = 3.120\text{GPa}$  and  $\nu = 0.3$ .  
 4  
 5 These properties have been introduced as input parameters in the described algorithm for the  
 6  
 7 linear elastic modelling of the curved beams. In order to take account of the real testing  
 8  
 9 conditions all the degree of freedom of the nodes belonging to the contact plane between  
 10  
 11 specimen and the polymeric bases, have been set equal to zero in order to simulate the  
 12  
 13 constraint effect of the base. The calculation of Young's modulus was carried out by imposing to  
 14  
 15 the free side of the RVE perpendicular to the compression axis, an arbitrary displacement and  
 16  
 17 obtaining its reaction force as the sum of the nodal forces. Initial section ( $Lrve^2$ ) has been  
 18  
 19 considered as reference in the calculation. The results of the simulations, in terms of Young's  
 20  
 21 modulus, conducted on the cubic RVEs are collected in Table 3:  
 22  
 23  
 24  
 25  
 26

27 Table 3 FEM results.

|             | RVE1   | RVE2  |
|-------------|--------|-------|
| $E_x$ (MPa) | 13.596 | 1.585 |
| $E_y$ (MPa) | 0.122  | 0.038 |
| $E_z$ (MPa) | 0.108  | 0.034 |

### 36 5.3 Experimental results

37  
 38 The results of the uniaxial compression tests in terms of Young's modulus, conducted on the  
 39  
 40 cubic specimens are collected in Table 4. The reported results are the arithmetic mean value of  
 41  
 42 the five specimens printed for each direction (X, Y and Z):  
 43  
 44  
 45  
 46

47 Table 4 Experimental results.

|             | RVE1   | RVE2  |
|-------------|--------|-------|
| $E_x$ (MPa) | 12.334 | 1.594 |
| $E_y$ (MPa) | 0.124  | 0.038 |
| $E_z$ (MPa) | 0.097  | 0.036 |

### 56 5.4 Comparisons between experimental and FEM results

The histograms reported in Figure 6 shows the comparison of experimental and numerical results in terms of Young's modulus. The percentage relative errors in the calculation of Young's moduli  $E_x$ ,  $E_y$  and  $E_z$  of RVE1 were respectively equal to:  $Err_x = -10.22\%$ ,  $Err_y = 1.62\%$ ,  $Err_z = -1.52\%$ ; the percentage relative errors in the calculation of Young's moduli  $E_x$ ,  $E_y$  and  $E_z$  of RVE2 were respectively equal to:  $Err_x = 0.55\%$ ,  $Err_y = -0.98\%$ ,  $Err_z = 6.69\%$ .

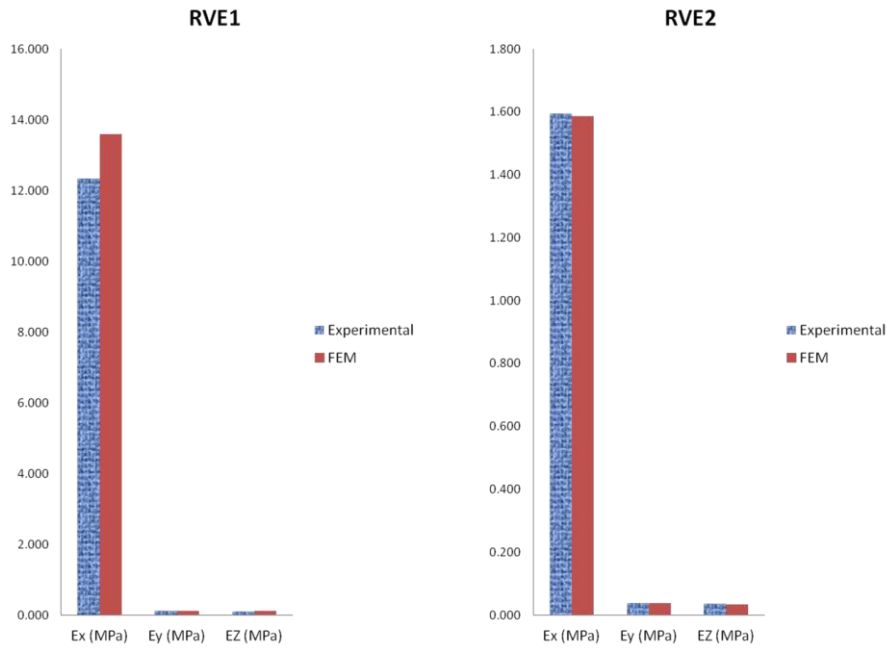


Figure 6. Experimental/numerical correlation in terms of Young's modulus.

## 6. Discussion, conclusions and perspectives

This work deals with the development, the application and the experimental validation of a procedure based on an original algorithm [16], running in a finite element environment, operating inside a three-dimensional solid of any shape replacing the continuous mass of the solid with an appropriate lattice structure showing curved cylindrical micro-beams, sharing with it border and boundary conditions. The algorithm input parameters are: average minimum distance between convergence nodes, target porosity, target ANB, radius of the SSS, average beam curvature and beams' mechanical properties. At the beginning, the algorithm generates an appropriate isotropic network of randomly oriented beams; then it adapts the network to the loads induced by the boundary conditions according to an appropriate criterion; at the end, a routine introduces a variable curvature at each beam in order to control the homogenized



1 Young's modulus of the porous structure. To show a practical application of the algorithm and  
2 for the validation of the final FEM model characterized by a one-dimensional element (beams)  
3 based network, a cubic representative volume element (RVE), properly sized, was chosen to  
4 facilitate the analysis of mechanical behaviours. By implementing PBC, the algorithm was able  
5 to retrieve the whole stiffness matrix of the cubic RVE. In this way, once properly curved and  
6 oriented the beams along a particular direction, the relationship between the compact material  
7 Young's modulus in input and the resulting homogenized Young's modulus of the porous RVE,  
8 along the main beam direction, was estimated. In this case study, once sized the beams' cross  
9 sections according to the desired porosity, the algorithm created two solid CAD models that  
10 were converted in .stl files and printed by a 3D printer using an appropriate polymer (the two  
11 models differ only in the curvature of the beams). The FEM results were compared with  
12 experimental ones obtained by compression tests conducted on the printed cubic RVE. The  
13 FEM and experimental results were very close with a mean relative error of 3.60% (see figure  
14 6). The worst correspondence between experimental and predicted elastic modulus has been  
15 observed for the sample RVE1 along the  $X$  axis. A three-dimensional element based network  
16 usually gives us results more close to reality. In this case, it would have been possible to build a  
17 three-dimensional element based FEM model after the realization of the CAD solid model (see  
18 section 2.1, step 13). Such a model would have required a very high computational burden  
19 significantly impairing the computing speed that is one of the peculiar characteristics of a  
20 parametric variational analysis. However, in this case the closeness between FEM and  
21 experimental results confirmed the validity of the "beam element – based" network in terms of  
22 Young's modulus. This Experimental/numerical correlation confirms the effectiveness of the  
23 procedure in this phase of design characterized by virtual realization of the parametric model,  
24 simulations of the elastic mechanical behaviour and three-dimensional printing. Possible  
25 applications could be the production of those porous tissues for which it is required a set value  
26 of porosity a set external shape, a low weight and heterogeneity of the mechanical properties at  
27 the micro and macro levels as for example in the field of research of the CATE for production of  
28 scaffolds (bone-like structured foams) and in the field of research of ergonomics for production  
29 of highly structured foams. The algorithm, subjected to further validation, could possibly be

1 applied to other areas of engineering. It can be easily modified in order to be able to control, for  
2  
3 example, the average resistant section of the curved beams, or, even better, the resistant  
4  
5 section of each beam according to the boundary conditions. Moreover, possible future  
6  
7 developments of FEM modelling may be the implementation of new material properties to cover  
8  
9 the field of geometrical and material nonlinearity, the evaluation of the effect of large  
10  
11 deformations and the analysis of the influence of the mutual contacts among curved beams.  
12

### 13 **Acknowledgements**

14  
15 We acknowledge the support of Giovanni Marinaccio and Angelo Salamini (Sintesi Sud S.r.l.  
16  
17 Corso Vittorio Emanuele, 233 83031 Ariano Irpino (AV), Italy) for the three dimensional printing  
18  
19 service, Prof. Loredana Incarnato and Eng. Emilia Garofalo, Ph.D (Department of Industrial  
20  
21 Engineering, University of Salerno, Via Giovanni Paolo II, 132, 84084 Fisciano, SA, Italy), for  
22  
23 the experimental testing phase.  
24

### 25 **References**

- 26  
27 [1] Deaton JD, Grandhi RV. A survey of structural and multidisciplinary continuum topology  
28  
29 optimization: post 2000. *Struct Multidiscip Optim* 2014; 49(1): 1–38  
30  
31 [2] Rozvany GIN. Aims, scope, methods, history and unified terminology of computer-aided  
32  
33 topology optimization in structural mechanics. *Struct Multidiscip Optim* 2001; 21: 90–108.  
34  
35 [3] Wolff J. *The law of bone remodelling*. Berlin, Germany: Springer-Verlag, 1986.  
36  
37 [4] Cowin SC. *Bone Mechanics Handbook*, second ed. USA: CRC Press, 2001.  
38  
39 [5] Fernandes P, Rodrigues H, Jacobs C. A model of bone adaptation using a global  
40  
41 optimisation criterion based on the trajectorial theory of Wolff. *Comput Methods Biomech*  
42  
43 *Biomed Eng* 1999; 2 (2): 125–138.  
44  
45 [6] Garcia JM, Doblaré M, Cegonino J. Bone remodelling simulation: a tool for implant design.  
46  
47 *Comput Mater Sci* 2002; 25: 100–114.  
48  
49 [7] Jacobs CR, Simo JC, Beaupre, GS, Carter, DR. Adaptive bone remodeling incorporating  
50  
51 simultaneous density and anisotropy considerations. *J Biomech* 1997; 30 (6): 603–613.  
52  
53 [8] Cowin SC. Bone stress adaptation models. *ASME J Biomech Eng* 1993; 115: 528 - 533.  
54  
55 [9] Huiskes R, Hollister SJ. From Structure to process, from organ to cell: recent developments  
56  
57 of FE-analysis in orthopaedic biomechanics. *ASME J Biomech Eng* 1993; 115: 520–527.  
58  
59  
60  
61  
62

- 1 [10] Coelho PG, Fernandes PR, Rodrigues HC, Cardoso JB, Guedes JM. Numerical modeling  
2 of bone tissue adaptation – a hierarchical approach for bone apparent density and trabecular  
3 structure. *J Biomech* 2009; 42 (7): 830–837.  
4  
5  
6  
7 [11] Hambli R, Katerchi H, Benhamou CL. Multiscale methodology for bone remodelling  
8 simulation using coupled finite element and neural network computation. *Biomech Model*  
9 *Mechanobiol* 2011; 10: 133–145.  
10  
11 [12] Tsubota K, Suzuki Y, Yamada T, Hojo M, Makinouchi A, Adachi T. Computer simulation of  
12 trabecular remodelling in human proximal femur using large-scale voxel FE models: approach  
13 to understanding Wolff's law. *J Biomech* 2009; 8: 1088–1094.  
14  
15 [13] Luo ZP, An KN. A theoretical model to predict distribution of the fabric tensor and apparent  
16 density in cancellous bone. *J Math Biol* 1998; 36 (6): 557–568.  
17  
18 [14] Boyle C, Kim IY. Three-dimensional micro-level computational study of Wolff's law via  
19 trabecular bone remodelling in the human proximal femur using design space topology  
20 optimization. *J Biomech* 2011; 44 (5): 935–942.  
21  
22 [15] Baldino I, Naddeo F, Cardea S, Naddeo A, Reverchon E. FEM modeling of the  
23 reinforcement mechanism of Hydroxyapatite in PLLA scaffolds produced by supercritical drying,  
24 for Tissue Engineering applications. *J Mech Behav Biomed Mater* 2015; 51: 225–236.  
25  
26 [16] Naddeo F. 2016 Metodologia per la realizzazione ottimizzata rispetto ai carichi di scaffold  
27 tissutali, Patent 102016000047650.  
28  
29 [17] Naddeo F, Baldino L, Cardea S, Naddeo A, Reverchon E. Optimization of an ad hoc  
30 realized lattice structure structured RVE for FEM modeling of nanoporous biopolymeric  
31 scaffolds obtained by supercritical fluids assisted process. *Chem Eng Trans* 2016; 49: 169-174.  
32  
33 [18] Lakatos É, Bojtár I. Trabecular bone adaptation in a finite element frame model using load  
34 dependent fabric tensors. *Mech Mater* 2012; 44: 130–138.  
35  
36 [19] Ma HS, Pre´vost, JH, Scherer GW. Elasticity of DLCA model gels with loops. *Int J Solids*  
37 *Struct* 2002; 39: 4605–4614.  
38  
39 [20] Rainer A, Giannitelli SM, Accoto D, Porcellinis S, Guglielmelli E, Trombetta M. Load-  
40 Adaptive Scaffold Architecturing: A Bioinspired Approach to the Design of Porous Additively  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

1 Manufactured Scaffolds with Optimized Mechanical Properties. *Ann Biomed Eng* 2012; 40(4):  
2 966-75.  
3

4  
5 [21] Hori M, Nemat-Nasser S. On two micromechanics theories for determining micro - macro  
6 relations in heterogeneous solids. *Mech Mater* 1999; 31: 667–682.  
7

8  
9 [22] Cricrì G, Garofalo E, Naddeo F, Incarnato L. Stiffness constants prediction of  
10 nanocomposites using a periodic 3D-FEM model. *J Polym Sci Part B Polym Phys* 2012; 50:  
11 207–220.  
12  
13

14  
15 [23] Naddeo F, Cappetti N, Naddeo A. Automatic versatile parametric procedure for a complete  
16 FEM structural analysis of composites having cylinder-shaped reinforcing fibres. *Comput Mater*  
17 *Sci* 2014; 81: 239–245.  
18  
19

20  
21 [24] Pothuaud L, van Rietbergen B, Charlot C, Ozhinsky E, Majumdar S. A new computational  
22 efficient approach for trabecular bone analysis using beam models generated with skeletonized  
23 graph technique. *Comput Methods Biomech Biomed Eng* 2004; 7 (4): 205–213.  
24  
25

26  
27 [25] van Lenthe GH, Stauber M, Müller R. Specimen-specific beam models for fast and accurate  
28 prediction of human trabecular bone mechanical properties. *Bone* 2006; 39: 1182–1189.  
29  
30

31  
32 [26] Pirard R, Pirard J-P. Aerogel compression theoretical analysis. *J Non-Cryst Solids* 1997;  
33 212: 262-267.  
34

35  
36 [27] Cappetti N, Naddeo A, Naddeo F, Solitro GF. Finite elements/Taguchi method based  
37 procedure for the identification of the geometrical parameters significantly affecting the  
38 biomechanical behavior of a lumbar disc. *Comput Methods Biomech Biomed Engin* 2016;  
39 19(12): 1278-1285.  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

1 **Figure Captions**

2  
3 Figure 1. Algorithm fundamental steps for elaborating a generic cubic solid mass withstanding  
4 generic boundary conditions: A) step 1; B) step 2; C) steps 3 and 4; D) step 5 ; E) steps 6 - 11;  
5  
6  
7 F) steps 12 - 13; G) 3D printed polymeric specimen.  
8  
9

10  
11 Figure 2. Example of an initial isotropic cubic RVE. For readability purposes, fewer mesh wires  
12 than those used in the calculation are represented.  
13  
14

15  
16  
17 Figure 3. Percentage ratio of  $E_x$  and  $E_y$  to  $E$ , in function of the beams' curvature represented by  
18 " $d/dist$ ", the porosity  $P$  being constant;  $E_z$  has not been introduced in the graph since it is quite  
19 superimposable to  $E_y$  one.  
20  
21  
22  
23  
24

25  
26 Figure 4. Printed specimens: in the first row, the three configurations of the RVE1 specimen are  
27 shown; in the second row, the three configurations of the RVE2 specimen are shown.  
28  
29  
30

31  
32 Figure 5. Effect of the algorithm elaboration on the cubic RVE: a) isotropic cubic lattice structure  
33 showing the direction of the imposed displacements ( $X$  axis); b) cubic lattice structure showing  
34 resized beams oriented along the  $X$  axis; c) cubic lattice structure showing curved beams; d) 3D  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

66  
67  
68  
69  
70  
71  
72  
73  
74  
75  
76  
77  
78  
79  
80  
81  
82  
83  
84  
85  
86  
87  
88  
89  
90  
91  
92  
93  
94  
95  
96  
97  
98  
99  
100  
101  
102  
103  
104  
105  
106  
107  
108  
109  
110  
111  
112  
113  
114  
115  
116  
117  
118  
119  
120  
121  
122  
123  
124  
125  
126  
127  
128  
129  
130  
131  
132  
133  
134  
135  
136  
137  
138  
139  
140  
141  
142  
143  
144  
145  
146  
147  
148  
149  
150  
151  
152  
153  
154  
155  
156  
157  
158  
159  
160  
161  
162  
163  
164  
165  
166  
167  
168  
169  
170  
171  
172  
173  
174  
175  
176  
177  
178  
179  
180  
181  
182  
183  
184  
185  
186  
187  
188  
189  
190  
191  
192  
193  
194  
195  
196  
197  
198  
199  
200  
201  
202  
203  
204  
205  
206  
207  
208  
209  
210  
211  
212  
213  
214  
215  
216  
217  
218  
219  
220  
221  
222  
223  
224  
225  
226  
227  
228  
229  
230  
231  
232  
233  
234  
235  
236  
237  
238  
239  
240  
241  
242  
243  
244  
245  
246  
247  
248  
249  
250  
251  
252  
253  
254  
255  
256  
257  
258  
259  
260  
261  
262  
263  
264  
265  
266  
267  
268  
269  
270  
271  
272  
273  
274  
275  
276  
277  
278  
279  
280  
281  
282  
283  
284  
285  
286  
287  
288  
289  
290  
291  
292  
293  
294  
295  
296  
297  
298  
299  
300  
301  
302  
303  
304  
305  
306  
307  
308  
309  
310  
311  
312  
313  
314  
315  
316  
317  
318  
319  
320  
321  
322  
323  
324  
325  
326  
327  
328  
329  
330  
331  
332  
333  
334  
335  
336  
337  
338  
339  
340  
341  
342  
343  
344  
345  
346  
347  
348  
349  
350  
351  
352  
353  
354  
355  
356  
357  
358  
359  
360  
361  
362  
363  
364  
365  
366  
367  
368  
369  
370  
371  
372  
373  
374  
375  
376  
377  
378  
379  
380  
381  
382  
383  
384  
385  
386  
387  
388  
389  
390  
391  
392  
393  
394  
395  
396  
397  
398  
399  
400  
401  
402  
403  
404  
405  
406  
407  
408  
409  
410  
411  
412  
413  
414  
415  
416  
417  
418  
419  
420  
421  
422  
423  
424  
425  
426  
427  
428  
429  
430  
431  
432  
433  
434  
435  
436  
437  
438  
439  
440  
441  
442  
443  
444  
445  
446  
447  
448  
449  
450  
451  
452  
453  
454  
455  
456  
457  
458  
459  
460  
461  
462  
463  
464  
465  
466  
467  
468  
469  
470  
471  
472  
473  
474  
475  
476  
477  
478  
479  
480  
481  
482  
483  
484  
485  
486  
487  
488  
489  
490  
491  
492  
493  
494  
495  
496  
497  
498  
499  
500  
501  
502  
503  
504  
505  
506  
507  
508  
509  
510  
511  
512  
513  
514  
515  
516  
517  
518  
519  
520  
521  
522  
523  
524  
525  
526  
527  
528  
529  
530  
531  
532  
533  
534  
535  
536  
537  
538  
539  
540  
541  
542  
543  
544  
545  
546  
547  
548  
549  
550  
551  
552  
553  
554  
555  
556  
557  
558  
559  
560  
561  
562  
563  
564  
565  
566  
567  
568  
569  
570  
571  
572  
573  
574  
575  
576  
577  
578  
579  
580  
581  
582  
583  
584  
585  
586  
587  
588  
589  
590  
591  
592  
593  
594  
595  
596  
597  
598  
599  
600  
601  
602  
603  
604  
605  
606  
607  
608  
609  
610  
611  
612  
613  
614  
615  
616  
617  
618  
619  
620  
621  
622  
623  
624  
625  
626  
627  
628  
629  
630  
631  
632  
633  
634  
635  
636  
637  
638  
639  
640  
641  
642  
643  
644  
645  
646  
647  
648  
649  
650  
651  
652  
653  
654  
655  
656  
657  
658  
659  
660  
661  
662  
663  
664  
665  
666  
667  
668  
669  
670  
671  
672  
673  
674  
675  
676  
677  
678  
679  
680  
681  
682  
683  
684  
685  
686  
687  
688  
689  
690  
691  
692  
693  
694  
695  
696  
697  
698  
699  
700  
701  
702  
703  
704  
705  
706  
707  
708  
709  
710  
711  
712  
713  
714  
715  
716  
717  
718  
719  
720  
721  
722  
723  
724  
725  
726  
727  
728  
729  
730  
731  
732  
733  
734  
735  
736  
737  
738  
739  
740  
741  
742  
743  
744  
745  
746  
747  
748  
749  
750  
751  
752  
753  
754  
755  
756  
757  
758  
759  
760  
761  
762  
763  
764  
765  
766  
767  
768  
769  
770  
771  
772  
773  
774  
775  
776  
777  
778  
779  
780  
781  
782  
783  
784  
785  
786  
787  
788  
789  
790  
791  
792  
793  
794  
795  
796  
797  
798  
799  
800  
801  
802  
803  
804  
805  
806  
807  
808  
809  
810  
811  
812  
813  
814  
815  
816  
817  
818  
819  
820  
821  
822  
823  
824  
825  
826  
827  
828  
829  
830  
831  
832  
833  
834  
835  
836  
837  
838  
839  
840  
841  
842  
843  
844  
845  
846  
847  
848  
849  
850  
851  
852  
853  
854  
855  
856  
857  
858  
859  
860  
861  
862  
863  
864  
865  
866  
867  
868  
869  
870  
871  
872  
873  
874  
875  
876  
877  
878  
879  
880  
881  
882  
883  
884  
885  
886  
887  
888  
889  
890  
891  
892  
893  
894  
895  
896  
897  
898  
899  
900  
901  
902  
903  
904  
905  
906  
907  
908  
909  
910  
911  
912  
913  
914  
915  
916  
917  
918  
919  
920  
921  
922  
923  
924  
925  
926  
927  
928  
929  
930  
931  
932  
933  
934  
935  
936  
937  
938  
939  
940  
941  
942  
943  
944  
945  
946  
947  
948  
949  
950  
951  
952  
953  
954  
955  
956  
957  
958  
959  
960  
961  
962  
963  
964  
965  
966  
967  
968  
969  
970  
971  
972  
973  
974  
975  
976  
977  
978  
979  
980  
981  
982  
983  
984  
985  
986  
987  
988  
989  
990  
991  
992  
993  
994  
995  
996  
997  
998  
999  
1000

**Tables**

Table 1 Morphometric input parameters characterizing the two cubic RVEs to be printed.

|                   | RVE1      | RVE2      |
|-------------------|-----------|-----------|
| <b>Lrve</b>       | 2cm       | 2cm       |
| <b>dist</b>       | Lrve/5    | Lrve/5    |
| <b>SSS radius</b> | dist*1.75 | dist*1.75 |
| <b>Porosity</b>   | 0.95      | 0.95      |
| <b>d/dist</b>     | 0.1       | 0.25      |
| <b>ANB</b>        | 4         | 4         |

Table 2 Comparison of the criteria.

| CRITERIA<br>OUTPUT                    | Elastic strain Energy |            | $\sigma_{eq}$ (Von Mises) |            | $\sigma_{principal, Max}$ |            |
|---------------------------------------|-----------------------|------------|---------------------------|------------|---------------------------|------------|
|                                       | Maxv                  | Amv        | Maxv                      | Amv        | Maxv                      | Amv        |
| $\sigma_{eq, (Von Mises)}$ [Pa]       | 3.2032E-04            | 1.2202E-04 | 3.1591E-04                | 9.2860E-05 | 3.1591E-04                | 1.3007E-04 |
| Abs( $\sigma_{principal, Max}$ ) [Pa] | 3.2022E-04            | 1.1862E-04 | 3.1568E-04                | 9.1430E-05 | 3.1568E-04                | 1.2867E-04 |
| Abs( $\tau$ ) [Pa]                    | 5.1530E-05            | 5.9400E-06 | 4.7010E-05                | 7.6400E-06 | 4.7010E-05                | 6.1900E-06 |
| RVE Young's Modulus ( $E_x$ ) [Pa]    | 1.6522E-01            |            | 1.6515E-01                |            | 1.6412E-01                |            |

Table 3 FEM results.

|             | RVE1   | RVE2  |
|-------------|--------|-------|
| $E_x$ (MPa) | 13.596 | 1.585 |
| $E_y$ (MPa) | 0.122  | 0.038 |
| $E_z$ (MPa) | 0.108  | 0.034 |

Table 4 Experimental results.

|             | RVE1   | RVE2  |
|-------------|--------|-------|
| $E_x$ (MPa) | 12.334 | 1.594 |
| $E_y$ (MPa) | 0.124  | 0.038 |
| $E_z$ (MPa) | 0.097  | 0.036 |

## Tables

Table 1 Morphometric input parameters characterizing the two cubic RVEs to be printed.

|                   | RVE1      | RVE2      |
|-------------------|-----------|-----------|
| <b>Lrve</b>       | 2cm       | 2cm       |
| <b>dist</b>       | Lrve/5    | Lrve/5    |
| <b>SSS radius</b> | dist*1.75 | dist*1.75 |
| <b>Porosity</b>   | 0.95      | 0.95      |
| <b>d/dist</b>     | 0.1       | 0.25      |
| <b>ANB</b>        | 4         | 4         |

Table 2 Comparison of the criteria.

| CRITERIA<br>OUTPUT                    | Elastic strain Energy |            | $\sigma_{eq}$ (Von Mises) |            | $\sigma_{principal, Max}$ |            |
|---------------------------------------|-----------------------|------------|---------------------------|------------|---------------------------|------------|
|                                       | Maxv                  | Amv        | Maxv                      | Amv        | Maxv                      | Amv        |
| $\sigma_{eq, (Von Mises)}$ [Pa]       | 3.2032E-04            | 1.2202E-04 | 3.1591E-04                | 9.2860E-05 | 3.1591E-04                | 1.3007E-04 |
| Abs( $\sigma_{principal, Max}$ ) [Pa] | 3.2022E-04            | 1.1862E-04 | 3.1568E-04                | 9.1430E-05 | 3.1568E-04                | 1.2867E-04 |
| Abs( $\tau$ ) [Pa]                    | 5.1530E-05            | 5.9400E-06 | 4.7010E-05                | 7.6400E-06 | 4.7010E-05                | 6.1900E-06 |
| RVE Young's Modulus ( $E_x$ ) [Pa]    | 1.6522E-01            |            | 1.6515E-01                |            | 1.6412E-01                |            |

Table 3 FEM results.

|             | RVE1   | RVE2  |
|-------------|--------|-------|
| $E_x$ (MPa) | 13.596 | 1.585 |
| $E_y$ (MPa) | 0.122  | 0.038 |
| $E_z$ (MPa) | 0.108  | 0.034 |

Table 4 Experimental results.

|             | RVE1   | RVE2  |
|-------------|--------|-------|
| $E_x$ (MPa) | 12.334 | 1.594 |
| $E_y$ (MPa) | 0.124  | 0.038 |
| $E_z$ (MPa) | 0.097  | 0.036 |

Figure 1  
[Click here to download high resolution image](#)

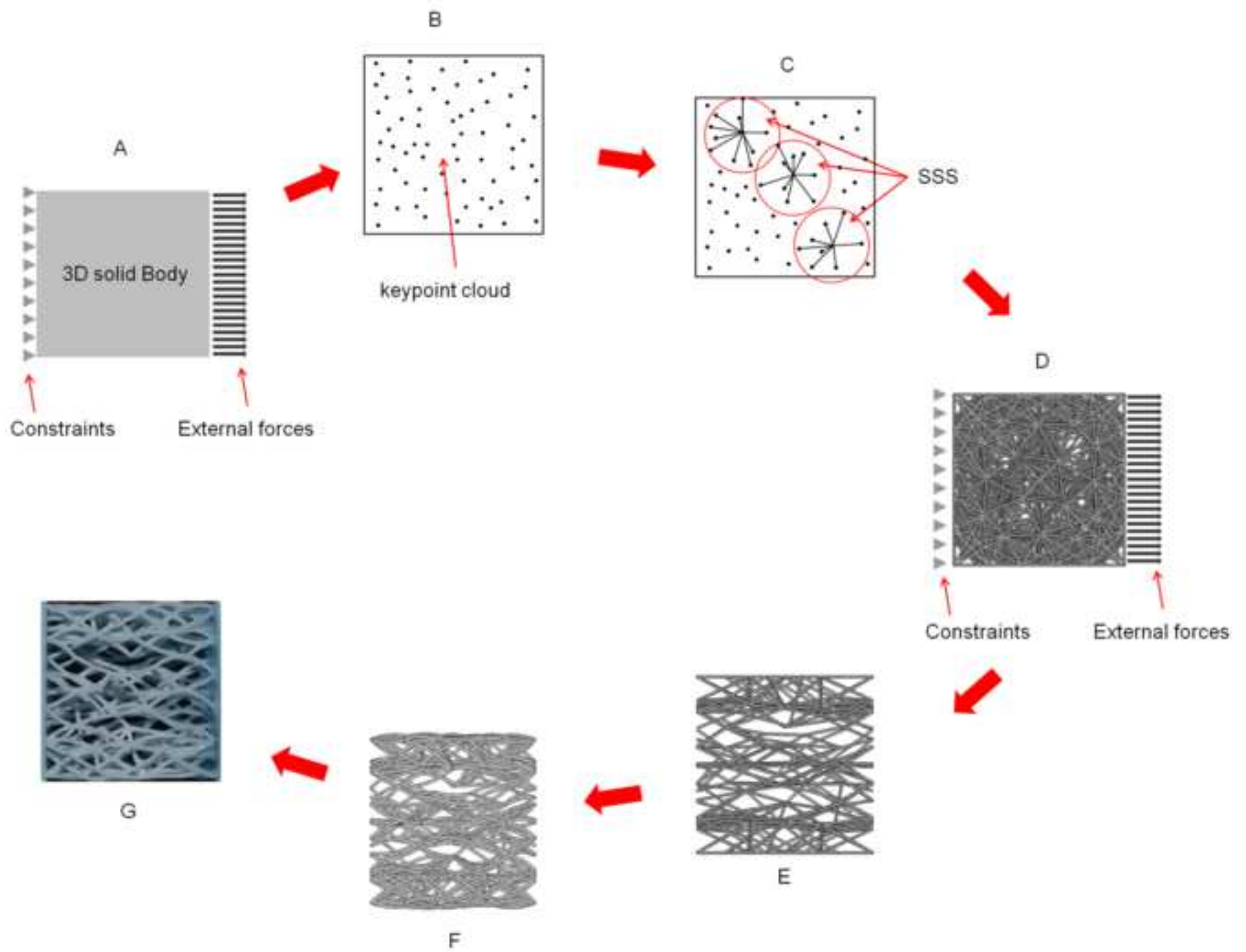




Figure 2  
[Click here to download high resolution image](#)

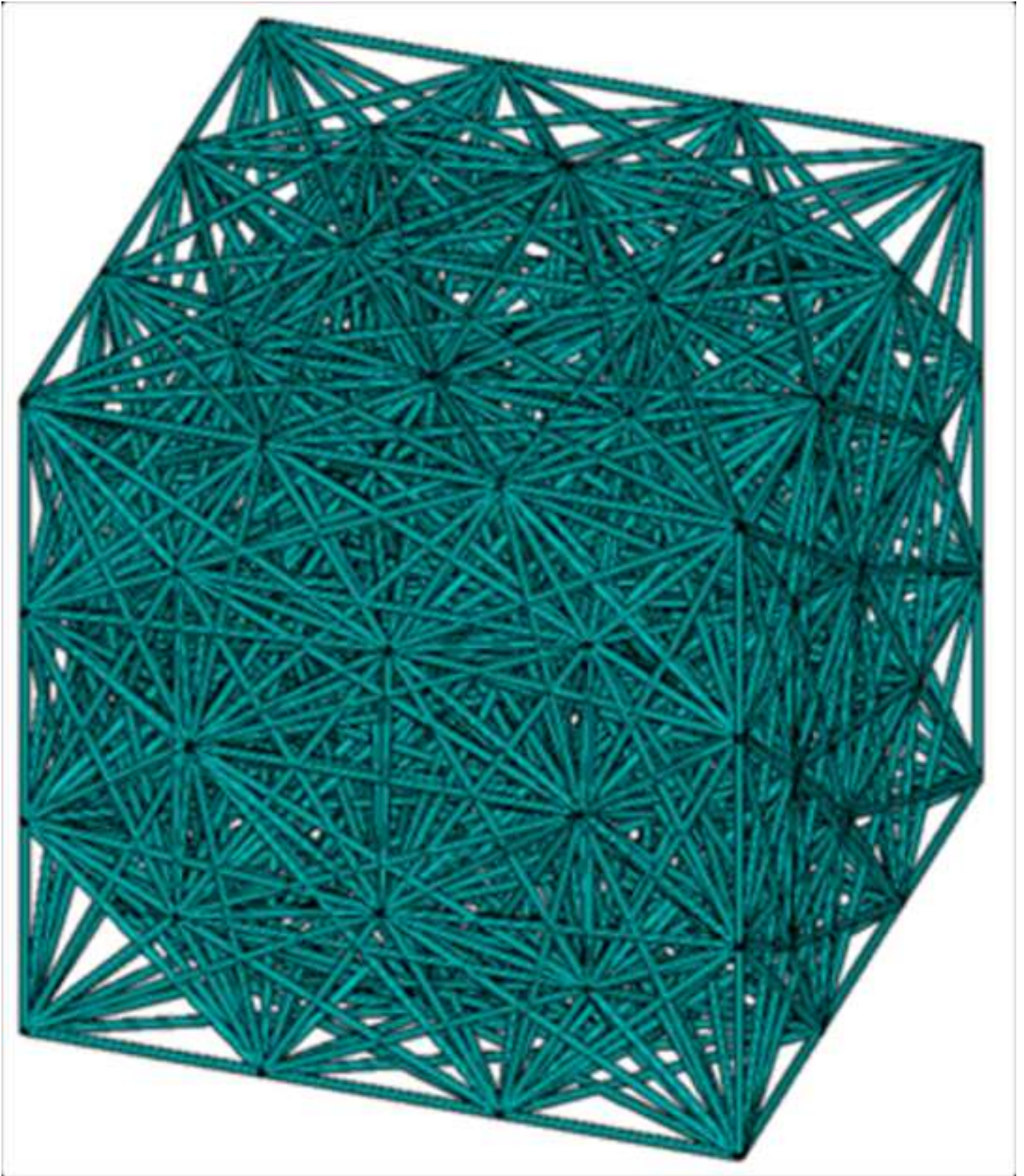


Figure 3  
[Click here to download high resolution image](#)

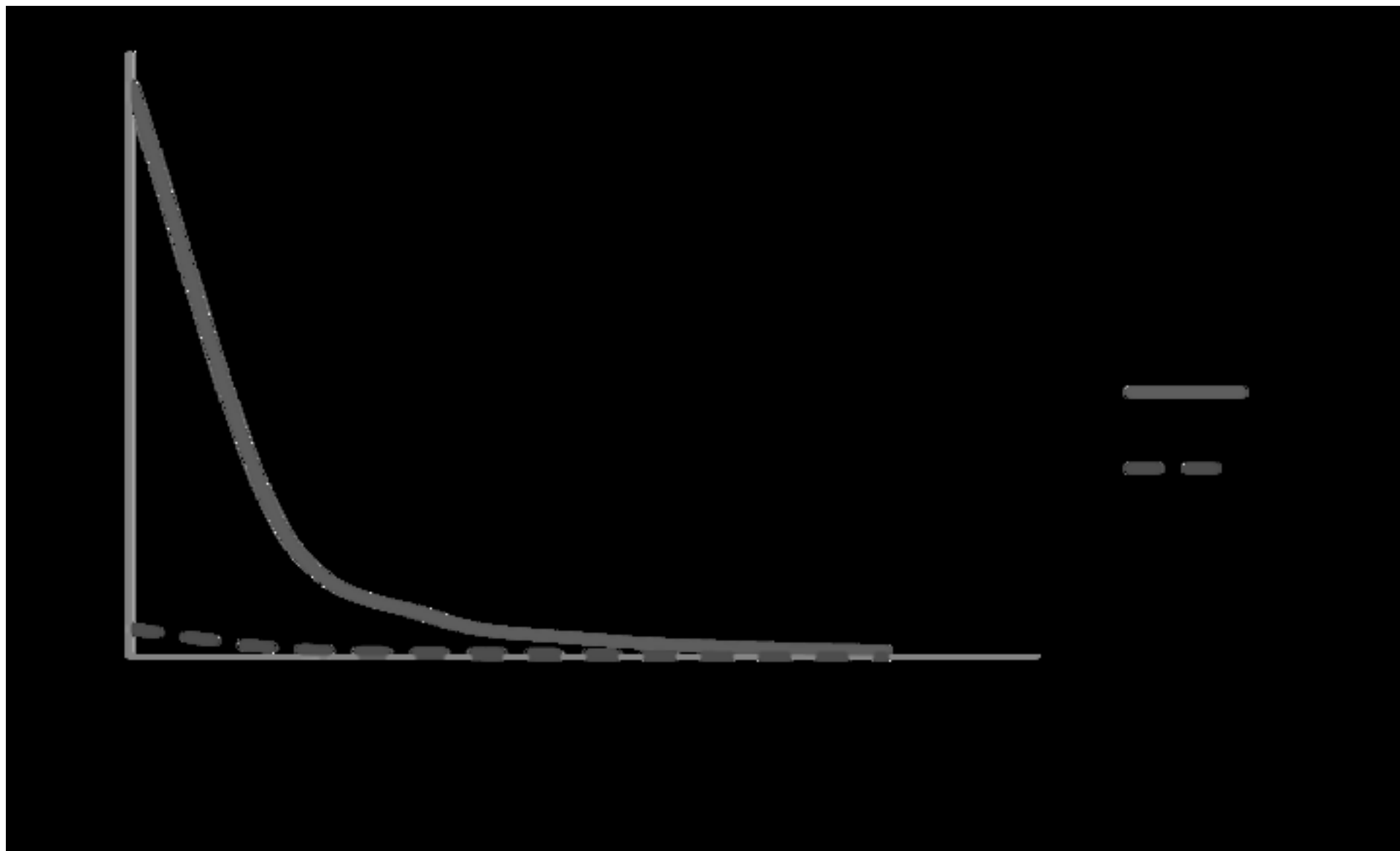




Figure 4  
[Click here to download high resolution image](#)

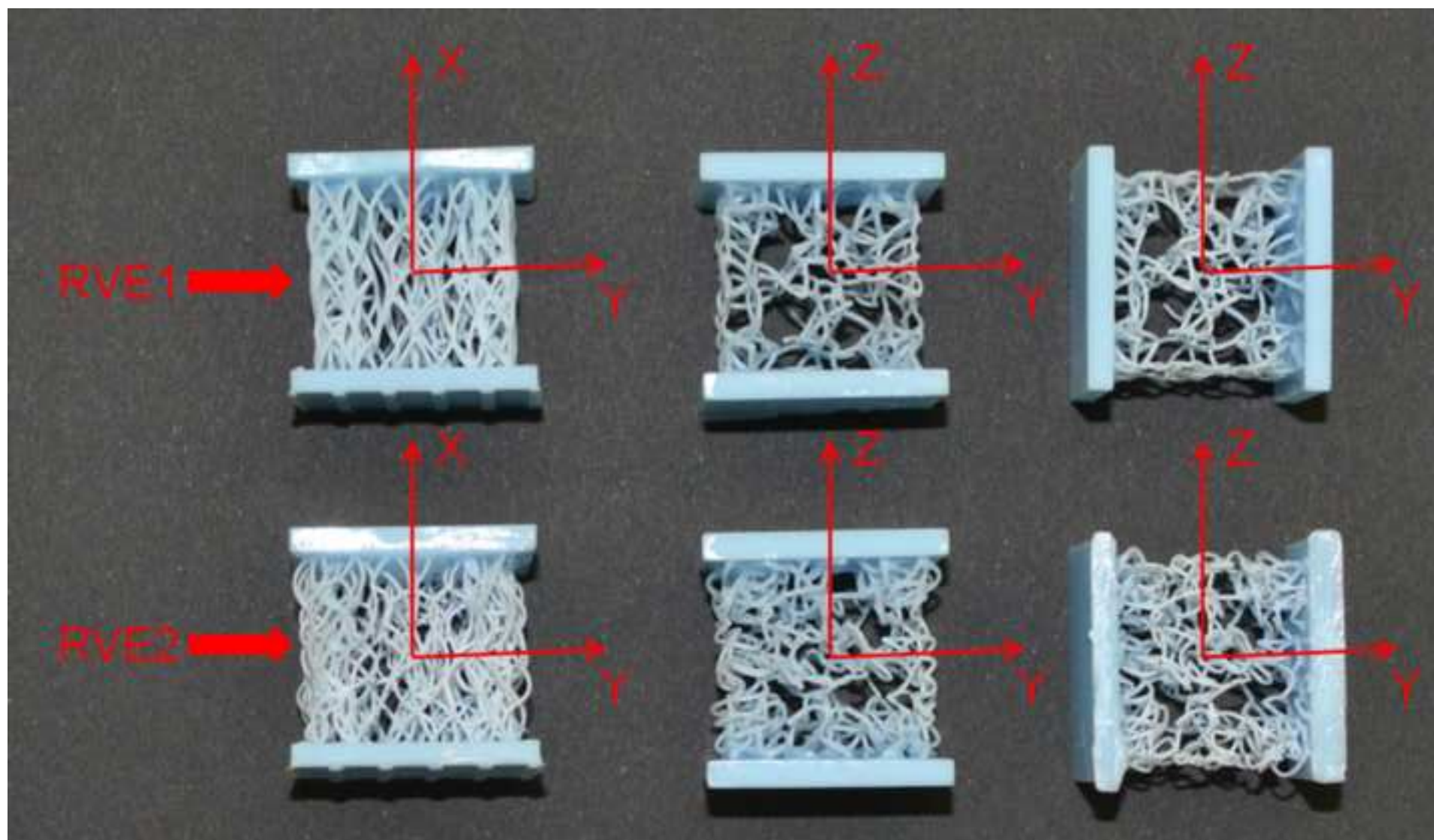


Figure 5  
[Click here to download high resolution image](#)

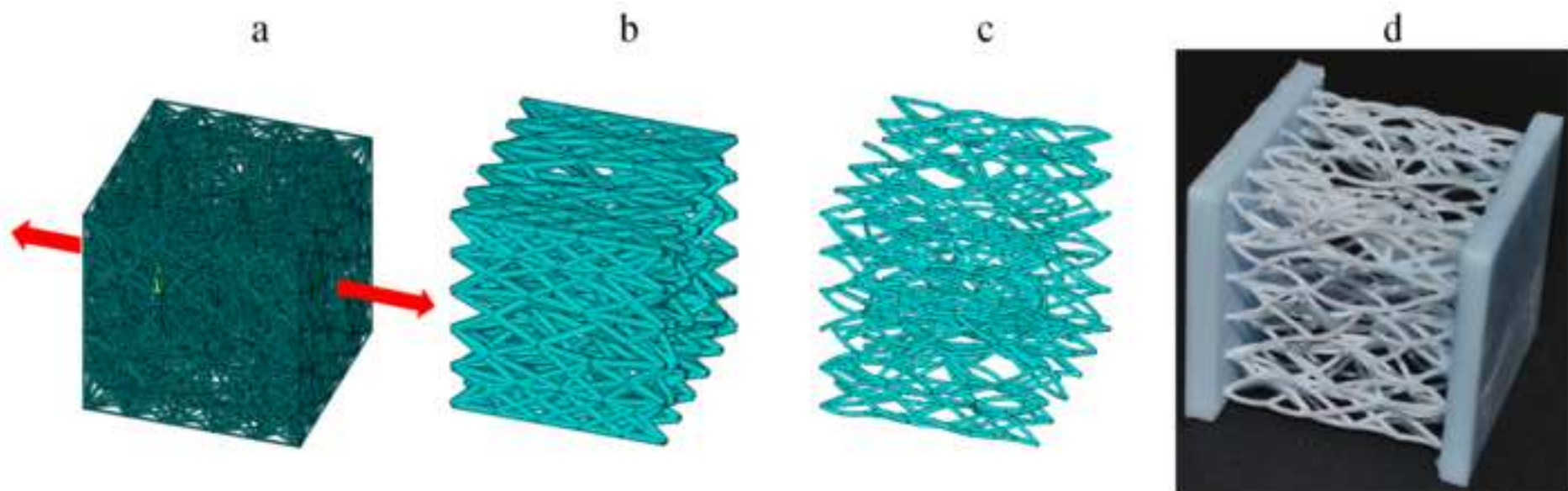


Figure 6  
[Click here to download high resolution image](#)

