Generation Rescheduling and Load Shedding in Distribution Systems under Imprecise Information

Vito Calderaro, *Member, IEEE,* Vincenzo Galdi, *Member, IEEE,* Giuseppe Graber, *Student Member, IEEE,* and Antonio Piccolo, *Member, IEEE*

***Abstract*—Microgrids represent a smart solution to increase power system reliability through the development of self-supplied islands that integrate distributed power generation and other smart technologies. In this paper we propose a solution method for an optimal generation rescheduling and load shedding problem in microgrids in order to determine a stable equilibrium state following outages. To address this problem, focused on MV distribution systems, a new solution methodology based on the use of fuzzy numbers is proposed. The approach allows representing the sources of uncertainty in the data or approximations made during the computation and considering many *possible* scenarios in case of outages. In order to demonstrate the performance and the effectiveness of the proposed method, several simulations have been carried out on a 69-bus radial distribution system and results have been compared with those obtained by using a stochastic optimization approach. The encouraging results are presented and discussed.**

***Index Terms*—microgrid, fuzzy numbers, rescheduling, load shedding.**

I. Introduction

Usually, as consequence of the growth in power demand and the requirements of higher power quality levels, power systems are operated in more and more stressful conditions. In case of congestion or overloads in one or more distribution lines, due to unexpected outages of generation, sudden increase in demand, tripping of lines, or failures of other equipment, the resulting power imbalance could lead to instability and security analysis and contingency plans must be made at regular intervals [1]. Nevertheless, load shedding, demand side management (DMS) and distributed generation control can be an opportunity to better manage distribution systems. In fact, nowadays some distributed energy resource (DER) units can be programmable (dispatchable) in contrast to units intuitively not-programmable, (such as wind, solar etc.) and information and communication technology (ICT) tools allow implementing smart strategies to manage the power demand [2]-[4].

In order to take advantage of the new technology opportunities, the distribution network requires advanced management policies as those based on the concept of microgrids. In fact, in case of faults, sudden dramatic load changes, and insufficient generation that can create power mismatch between generation and power demand, it is possible to define operative microgrid strategies based on an optimal use of available dispatchable DERs, which ensure a rapid and effective service recovery incrementing the power system reliability: the microgrid can isolate itself via an utility branch circuit and coordinate generators and load in order to guarantee power supply.

V. Calderaro, V. Galdi, G. Graber, and A. Piccolo are with the Department of Industrial Engineering, University of Salerno, Fisciano (SA), 84084 Italy e-mail: vcalderaro@unisa.it, vgaldi@unisa.it, ggraber@unisa.it, apiccolo@unisa.it

In the literature, few studies deal simultaneously with generation rescheduling and load shedding (GRLS) problems. In [5],[6] sensitivity-based optimum generation rescheduling and/or load shedding schemes to alleviate overloading of transmission lines are reported. Optimization techniques are used for GRLS in the reliability evaluation of conventional power systems [7]-[9]. More recent works are in [10]-[12]: a particle swarm optimization method is used to solve a GRLS multiobjective problem in order to alleviate overload and minimize the operation cost [10], a decision tree-based preventive and corrective method such as GRLS schemes to enhance the security of power system is proposed in [11]; Wang et al. propose a risk-based method for coordinating GRLS to enhance overall transient stability of power systems [12]. Other papers deal with congestion management by focusing on load shedding scheme [13]-[15] and generation rescheduling [16]-[19], separately.

The bibliographic survey gives three suggestions: i) the possibility to apply GRLS management strategies in microgrids that could improve the reliability level of the distribution systems in the presence of DERs; ii) the necessity to formulate GRLS problem in microgrids considering the uncertainty due to not-programmable DERs and no well-known load demand profiles; iii) to develop a complete strategy to support the continuity of service in case of faults by completing the only rescheduling scheme already proposed in [19]. The three previous points lay the foundations for this paper and represent also its main contributions. In fact, we formulate and solve a GRLS problem for microgrids in order to determine a stable equilibrium state following outage for increasing continuity of service. The solution takes into account the uncertainty due to the fact that i) lower and upper limits of some of the dominant constraints are not sharp but rather soft, ii) the load profiles in MV distribution system are not deterministic, and iii) a part of distributed renewable generation introduces uncertainty in power production. Possibility theory is well suited for treating such situations where there are inaccurate, nested, or few data to describe adequately the problem. In possibility theory, information is modeled by possibility distributions, which in same cases are analogous to probability distributions. In fact, possibility distribution should be used as mathematical description of the event only when information are inaccurate (uncertain) [20]-[22]; if information are precise, then possibility distribution must not be used because it is a poor information. In the literature probability and possibility theory have been largely used in power system in order to take into account ”uncertain information”. The first approach appears in the 1970s when a probabilistic load flow were proposed [23]-[24]. The second approach based on fuzzy set theory is developed in [25]-[28] when uncertainty is supposed to be originated by a vague or inaccurate concept, which is not the case of the probabilistic models highly related to the statistical behavior of a phenomenon. More recent ideas is in [29] that includes a representation of the uncertainties associated with renewable resources and loads based on fuzzy intervals.

Here, in order to illustrate the proposed approaches, an optimization problem based on the possibility theory is formulated, the solution method is tested on a 69-bus distribution network, and the results are compared with a classical stochastic optimization approach.

The paper is laid out as follows: Section II briefly presents possibility theory and fuzzy set theory. In Section III the mathematical formulation of the problem is illustrated. In Section IV, the solution algorithm is shown and in Section V the methodology is applied to case studies. Sections VI and VII discuss and conclude the work.

II. Uncertainty by means of possibility theory: fundamental concepts

In power systems, problems that treat the uncertainty with the classical probabilistic approach can be critical because in many cases we do not have enough data to build reliable probabilistic distributions, and practical application suffer the lack of information. These considerations led to formulate a theory of possibility based on a fuzzy set approach [30]. In fact, often it is necessary to take into account of both aleatoric and epistemic effects, so that the uncertainty can be accounted by the model by possibility distributions. Here, we recall the fundamental concepts of the possibility theory and fuzzy sets.

A. Possibilistic Theory and Fuzzy sets

Since Zadeh introduced the concept of possibility, the fuzziness has been handled by possibility distributions, and a fuzzy variable with its membership function is related to a possibility distribution in the same manner as the corresponding probability distribution of the random variable [31]. Let A a fuzzy set, a subset of a universal set U, represented by an ordered pair composed by a generic element and its membership value:

$$A = \{(x, \pi_A(x)) | x \in U\} \tag{1}$$

A possibility distribution π_A on U is a mapping from U to the unit interval [0,1] attached to the single-valued variable A. The function π_A represents a flexible restriction, which constrains the possible values of A according to the available information, with the following conventions: $\pi_A(x)$=0 means that A=x is definitely impossible; $\pi_A(x)$=1 means that absolutely nothing prevents that A=x. Intermediary levels of plausibility about the possible values of A are modeled by letting $\pi_A(x)$ between 0 and 1 for some values x. The quantity $\pi_A(x)$ thus represents the degree of possibility of the assignment A=x. Then one can acknowledge the fact that some values of x are more possible than others, according to available information.

In order to deep the concept of possibility distribution, we introduce the α-cut of a fuzzy set A, a classical set A_α that contains all the elements in U with a membership value in A greater or equal than α, that is,

$$A_\alpha = \{(x \in U | \pi_A(x) \geq \alpha, \alpha \in [0, 1]\} \tag{2}$$

A possibility distribution can be seen as a sequence of nested confidence intervals, coincident with the α-cuts of a fuzzy set A. The confidence level regarding the truth of a sentence, like ”the value of the uncertain measurement belongs to an interval A_α”, represents the necessity (nec), which is maximum for α=0 and it decreases as α increases: nec (A_α)=1-α. The possibility (pos) is related to necessity through the following:

$$pos(S) = 1 - nec(S^c) \tag{3}$$

where S is a subset of A, and S^c is the complementary subset.

For each possible event A=x its possibility $\pi_A(x)$ is the greatest possible probability of the event [32]. The necessity, instead, is the minimal but certain probability value of the event occurrence. If the possibility value $\pi_A(x)$ and the necessity $\eta_A(x)$ value are known then the event A=x occurs with probability at least equal to the necessity but not exceeding the possibility. The possibility and necessity measures of an event occurrence in the solution of real problems correspond, respectively, to the extremely optimistic and to extremely pessimistic approach to the problem.

B. Fuzzy numbers representation

A fuzzy number $\tilde{a}$ is a fuzzy subset in U that fulfills the following conditions: i) $\tilde{a}$ is normal, ii) $\tilde{a}$ is convex, iii) $\tilde{a}$ has a bounded support, and iv) every α-cut of $\tilde{a}$ is a closed interval in U. Here, we use fuzzy numbers with triangular shape. The membership function is defined as

$$\pi_a(x) = \pi_a(x, a_1, a_2, a_3) = \\ = \begin{cases} \frac{(x-a_1)}{(a_2-a_1)} & \text{if } a_1 \leq x \leq a_2 \\ \frac{(a_3-x)}{(a_3-a_2)} & \text{if } a_2 \leq x \leq a_3 \\ 0 & \text{if } x > a_3 \vee x < a_1 \end{cases} \tag{4}$$

The α-cut representation is achieved building α dependent function for left and right edges of π_A. Thus, for the fuzzy number $\tilde{a}$ we define:

$$a_\alpha = [a_\alpha^-, a_\alpha^+]$$

where

$$a_\alpha^- = (a_2 - a_1)\alpha + a_1 \tag{5}$$

$$a_\alpha^+ = (a_2 - a_3)\alpha + a_3 \tag{6}$$

C. Sum, absolute value and scalar multiplication

Let $\tilde{a}$ and $\tilde{b}$ be two fuzzy numbers and $a_\alpha = [a_\alpha^-, a_\alpha^+]$ and $b_\alpha = [b_\alpha^-, b_\alpha^+]$ their α-cuts, respectively. The sum of $\tilde{a}$ with $\tilde{b}$ is a fuzzy number $\tilde{c}$ with the following α-cut:

$$c_\alpha = (a+b)_\alpha = [a_\alpha^- + b_\alpha^-, a_\alpha^+ + b_\alpha^+] = [c_\alpha^-, c_\alpha^+] \quad (7)$$

Thus, the fuzzy number is well defined and if we consider the corresponding membership function, we have the following:

$$\pi_{a+b}(x) = \pi_{a+b}(x, a_1 + b_1, a_2 + b_2, a_3 + b_3) \quad (8)$$

For the absolute value the membership function is:

$$\pi_{|a|}(x) = \pi_{|a|}(x, |a_1|, |a_2|, |a_3|) \quad (9)$$

Here, we define only the scalar multiplication between a fuzzy number $\tilde{a}$ with a scalar k. In particular, the product is a fuzzy number with the following membership function:

$$\pi_{ka}(x) = \pi_{ka}(x, ka_1, ka_2, ka_3) \quad (10)$$

D. Comparison of fuzzy numbers

In the literature, quite a few fuzzy ranking methods for comparing fuzzy numbers are present, but they are able to guarantee the comparison only in particular conditions [33]. Since the method used here is closely related to the addressed problem, we use the ranking method for comparing fuzzy numbers introduced in [19,34]. For our purpose three cases are considered: inequality between a fuzzy number and a real number; equality between two fuzzy numbers and inequality between two fuzzy numbers. In particular, i) a fuzzy number is less than a real number if each element of the set of the associate membership function is less than the real number; ii) two fuzzy numbers are equals if the correspondent elements of their set are equal to each other; iii) a fuzzy number is greater than of a fuzzy number if each element of the first set is greater than of each element of the second set.

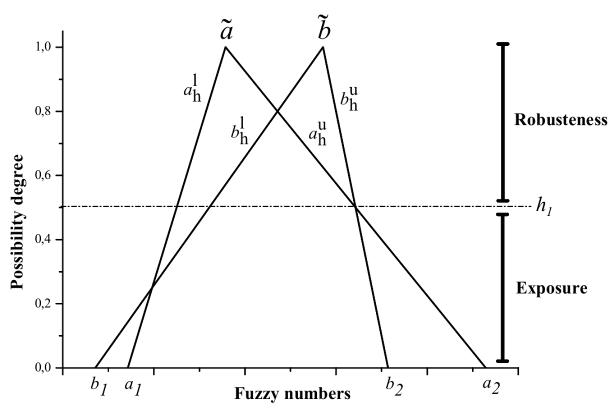

Fig. 1. Comparison between fuzzy numbers

E. Fuzzy constraints

The fuzzy constraints can be satisfied in different ways. In this paper we consider the Soyster's criterion. Let consider two fuzzy numbers $\tilde{a}$ and $\tilde{b}$ with triangular shapes as in Fig. 1, the inequality $\tilde{a} \tilde{\leq} \tilde{b}$ is satisfied up to the level h_1 if $\tilde{a} \tilde{\leq}_{h_1} \tilde{b}$, that is the left-hand side satisfies $a_h^l \leq b_h^l$ and the right-hand side satisfied for any α-cut $h \in [h_1, 1]$. The level h_1 (β-level) is a measurement of the corresponding risk accepted by the planner and is called *exposure*. Instead, the value 1-β is a measurement of *robustness* of the planning solution.

III. Load and distributed generation modeling

Uncertainty in planning studies stems from several sources both internal and external to the distribution power system. The most relevant uncertainty sources are:

- price of electricity based on competing energy sources;
- lower and upper limits of power production of some of the dominant constraints, which are not sharp but rather soft;
- weather conditions;
- improvements on the energy end use.

Here, we face a planning study based on the solution of a GRLS problem. It is modeled by using fuzzy variables that represent the possibility distributions of the demand values and power production in the presence of distributed generation units with aleatory sources. Forecasting these variations involves uncertainty, which could be significant especially in the medium and long term. In the following the uncertainty associated to power demand and power production will be modeled by triangular fuzzy numbers. In particular, a linguistic declaration about the absorption of power as "power load may occur between P_a and P_c MW but it is likely P_b" is modeled by the fuzzy number $\pi_P(x, P_a, P_b, P_c)$ sketched as in Fig. 2; then we distinguish between programmable and not-programmable DERs. In particular, we assume that not-programmable DER is connected at the same bus of programmable DER, so that the produced power is affected from an uncertainty degree.

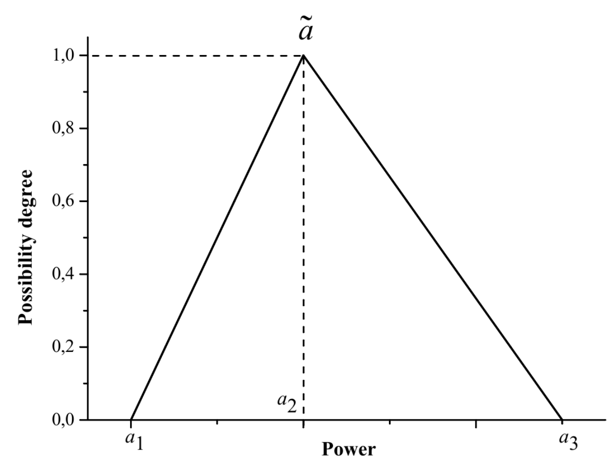

Fig. 2. Example of triangular fuzzy number

IV. Mathematical formulation of the generation rescheduling and load shedding problem

In this paper, GRLS problem deals with the determination of a new stable equilibrium state (steady state analysis) following an outage in a distribution network hosting DERs. The objective of the problem is to reschedule power generation as close as possible to the power generated in order to minimize production costs and, if necessary, to reduce the load, classified with different priorities, in order to guarantee the maximum continuity of service. The problem formulation is divided in two parts: the first one describes the rescheduling problem, the second one formalizes the load shedding problem.

A. Rescheduling problem

Rescheduling problem is formulated as an optimization fuzzy problem based on an AC power flow, subject to operating constraints. The mathematical model can be expressed as

$$\min_{\tilde{\mathbf{P}}_G \in \mathcal{F}^{N_g}} \tilde{F}_1(\tilde{\mathbf{P}}_G) \quad \forall \alpha \in [0,1] \quad s.t. \quad \tilde{x}(\tilde{\mathbf{P}}_G) \in \mathbf{X} \tag{11}$$

where $\mathcal{F}$ is the subset of fuzzy variables, $\tilde{\mathbf{P}}_G$ is the set of the fuzzy active power variables produced by the DER units, N_g is the number of the programmable DERs, and

$$\tilde{F}_1 = \sum_{i=1}^{N_g} \tilde{P}_{G_i} - \tilde{P}^L_{TOT} \tag{12}$$

In (12) $\tilde{P}_{G_i}$ represents the active power of the DER at the bus i; $\tilde{P}^L_{TOT}$ represents the overall load active power. The uncertainty on produced active power is due to the presence of not-programmable DERs connected to the same bus of the programmable DERs.

The electrical and operational constraints are summarized by

$$\mathbf{X} = \{\tilde{\mathbf{r}}(\tilde{\mathbf{P}}_G) | \tilde{\mathbf{r}}(\tilde{\mathbf{P}}_G) \leq 0\} \tag{13}$$

where the vector function $\tilde{\mathbf{r}}(\tilde{\mathbf{P}}_G)$ describes both the equality constraints (i.e. load flow equations) and the constraints for a correct system operation as

$$\begin{cases} \tilde{P}_i^{SP} = \tilde{V}_i \sum_{i=1}^{N_b} \tilde{V}_j Y_{ij} \cos(\tilde{\delta}_i - \tilde{\delta}_j - \theta_{ij}) \quad i \in nP \\ \tilde{Q}_j^{SP} = \tilde{V}_j \sum_{k=1}^{N_b} \tilde{V}_k Y_{jk} \sin(\tilde{\delta}_j - \tilde{\delta}_k - \theta_{jk}) \quad j \in nQ \\ |\tilde{I}_h| \leq I_h^{max} \quad h = 1...N_l \\ P_{Gi}^{min} \leq \tilde{P}_{Gi} \leq P_{Gi}^{max} \quad i = 1...N_g \\ Q_{Gi}^{min} \leq \tilde{Q}_{Gi} \leq Q_{Gi}^{max} \quad i = 1...N_g \\ V_{Gi}^{min} \leq \tilde{V}_{Gi} \leq V_{Gi}^{max} \quad i = 1...N_g \end{cases} \tag{14}$$

where N_b is the number of load buses, nP and nQ are the list of the buses in which the active and reactive power are specified, respectively. $\tilde{P}_i^{SP}$ and $\tilde{Q}_i^{SP}$ are the real and reactive power specified at i-th and j-th bus (load and generation); $\tilde{V}_i$ and $\tilde{\delta}_i$ are the i-th bus voltage (magnitude and angle) and Y_{ij} and θ_{ij} are the ij-th element of the bus admittance matrix (magnitude and angle). $\tilde{I}_h$ is the current on the line h. The terms with max and min indicate the limits of the corresponding electrical quantities: V_{Gi} and Q_{Gi} describe the voltage and the reactive power at the generation i bus. The terms with the symbol $\sim$ are fuzzy numbers.

B. Load Shedding Problem

In the load shedding problem we assume that each bus is associated with an aggregate load with a priority degree, so that all buses can be clustered in priority classes. This assumption introduces a constraint in the order of the load reduction. For this reason a multilevel optimization problem is formulated. Multilevel optimization recognizes that there is a hierarchy of decision makers with decision made at different levels within the hierarchy [36]. The multilevel optimization was introduced in 1952 by von Stackelberg, who proposed a two level strategy for systems where policy makers at the top level influence the decision of private individuals and companies. According to this strategy, the problem of load shedding must take into account the load shedding solutions obtained at lower levels to guarantee the correct order of load reduction based on the priority of the loads. Analytically, the optimization problem is formulated as

$$\begin{aligned} &\min_{\tilde{\mathbf{P}}_L^L \in \mathcal{F}^{N_b}} \tilde{F}_1(\tilde{\mathbf{P}}_L^H, \tilde{\mathbf{P}}_L^M, \tilde{\mathbf{P}}_L^L) \quad \forall \alpha \in [0,1] \\ &s.t. \quad \tilde{x}(\tilde{\mathbf{P}}_L^H, \tilde{\mathbf{P}}_L^M, \tilde{\mathbf{P}}_L^L) \in \mathbf{X} \end{aligned} \tag{15}$$

where P_L^M solves

$$\begin{aligned} &\min_{\tilde{\mathbf{P}}_L^M \in \mathcal{F}^{N_b}} \tilde{F}_2(\tilde{\mathbf{P}}_L^H, \tilde{\mathbf{P}}_L^M) \quad \forall \alpha \in [0,1] \\ &s.t. \quad \tilde{x}(\tilde{\mathbf{P}}_L^H, \tilde{\mathbf{P}}_L^M) \in \mathbf{X} \end{aligned} \tag{16}$$

where P_L^H solves

$$\begin{aligned} &\min_{\tilde{\mathbf{P}}_L^H \in \mathcal{F}^{N_b}} \tilde{F}_3(\tilde{\mathbf{P}}_L^H), \quad \forall \alpha \in [0,1] \\ &s.t. \quad \tilde{x}(\tilde{\mathbf{P}}_L^H) \in \mathbf{X} \end{aligned} \tag{17}$$

in (15)-(18) $\tilde{\mathbf{P}}_L^H, \tilde{\mathbf{P}}_L^M, \tilde{\mathbf{P}}_L^L$ are the sets of the high, medium and low priority load, respectively. Furthermore,

$$\begin{cases} \tilde{F}_1 = \sum_{i=1}^{N_g} \tilde{P}_{Gi} - \sum_{j=1}^{N_b} (\tilde{P}_{Lj}^H + \tilde{P}_{Lj}^M + \tilde{P}_{Lj}^L) \\ \tilde{F}_2 = \sum_{i=1}^{N_g} \tilde{P}_{Gi} - \sum_{j=1}^{N_b} (\tilde{P}_{Lj}^H + \tilde{P}_{Lj}^M) \\ \tilde{F}_3 = \sum_{i=1}^{N_g} \tilde{P}_{Gi} - \sum_{j=1}^{N_b} \tilde{P}_{Lj}^H \end{cases} \tag{18}$$

where N_b is the number of load buses. Obviously, in (18), we sum only the demand powers with a priority p, even though the sum is extended to all buses; we consider null the demand power at the bus associated to a load with a priority different from p. The electrical and operational constraints in (15)-(18) are summarized by

$$\mathbf{X} = \{\tilde{\mathbf{r}}(\tilde{\mathbf{P}}_L) | \tilde{\mathbf{r}}(\tilde{\mathbf{P}}_L) \leq 0\} \tag{19}$$

where $\tilde{\mathbf{P}}_L = \{\tilde{\mathbf{P}}_L^H, \tilde{\mathbf{P}}_L^M, \tilde{\mathbf{P}}_L^L\}$ and the vector $\tilde{\mathbf{r}}(\tilde{\mathbf{P}}_L)$ describe both the equality constraints (i.e. load flow equations) and the inequality econstraints for a correct system operation as

$$\begin{cases} \tilde{P}_i^{SP} = \tilde{V}_i \sum_{i=1}^{N_b} \tilde{V}_j Y_{ij} \cos(\tilde{\delta}_i - \tilde{\delta}_j - \theta_{ij}) & i \in nP \\ \tilde{Q}_j^{SP} = \tilde{V}_j \sum_{k=1}^{N_b} \tilde{V}_k Y_{jk} \sin(\tilde{\delta}_j - \tilde{\delta}_k - \theta_{jk}) & j \in nQ \\ |\tilde{I}_h| \leq I_h^{max} \quad h = 1...N_l \\ V_i^{min} \leq \tilde{V}_i \leq V_i^{max} \quad i = 1...N_l \\ \tilde{P}_G^{sh} + (\Delta f_{min})D \leq \Delta \tilde{P}_L \leq \tilde{P}_G^{sh} + (\Delta f_{max})D \end{cases} \tag{20}$$

where D is the damping load constant, $\Delta \tilde{P}_L$ is the active load power to disconnect for the load-shedding. $\tilde{P}_L^{sh}$ is the active power lost after the outage and $\Delta f_{min(max)}$ is the frequency deviation between the rated value and the minimum (maximum) standard limit. The last inequality allows steady state frequency to maintain within a permissible range in steady state analysis. It has been obtained ignoring generator droops [37].

V. Solution Algorithm

The solution method is based on the algorithm whose flow chart is shown in Fig. 3. When an outage occurs, and microgrid is formed, first, the algorithm tries to perform a rescheduling of the dispatchable generators: this operation can lead to an optimal rescheduling valid for each α or a feasible rescheduling solution valid only for particular values of α. If the total generated power is less than the power absorbed by loads, it will be impossible to perform the rescheduling procedure, thus a load shedding routine starts, followed by a power rescheduling in order to find an optimal generators operating point.

In details, the solution of the rescheduling optimization problem is obtained in two steps: in the first one the fuzzy optimization problem (11) is translated to a real number-based minimization problem by applying α-cuts (α=1), which corresponds to a deterministic classical problem for rated values. It is a constrained non-linear optimization problem that can be solved by using a Hessian, computed by a quasi-Newton approximation. In particular, the power flow is solved assuming that one of the microgrid generator takes the slack node role and limiting its active and reactive power according to power limits. In the second step the rescheduling algorithm allows obtaining the solution in terms of fuzzy numbers in order to verify the possibility degree. The procedure is based on the following sub-steps:

- translate the deterministic solution to a fuzzy number solution by applying the same maximum uncertainty width interval (α=0), as defined before the rescheduling to DER buses (PV buses) and load buses (PQ buses). In fact, the imprecise information depends on the not-programmable sources and it is independent from the obtained solution;
- define an interval for all load flow input variables by setting a value of α;
- carry out Monte Carlo simulations by using, for all power flow input variables, values selected in the interval;
- check that the power flow solutions are feasible according to technical constraints;
- repeat the last two steps in order to obtain a fuzzy number solution for each α-cut included in A=[0,0.2,0.4,0.6,0.8,1] (*Monte Carlo fuzzyfication* block).

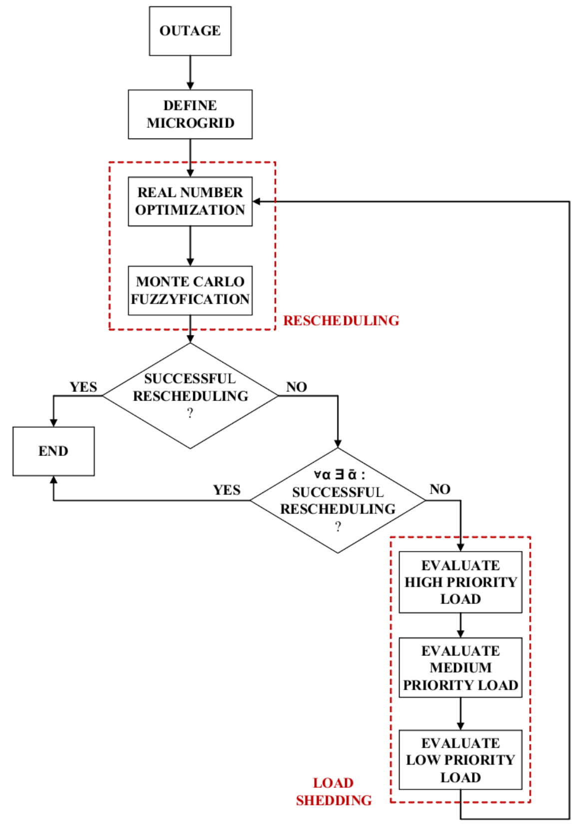

Fig. 3. Solution algorithm flow chart

The load shedding problem, instead, is traced back to a knapsack problem - given a backpack that can withstand a certain weight and N objects characterized by a value, the problem aims to choose which of these items to put in the backpack to get the most value without exceeding the sustainable weight of the backpack itself. Each load has a priority value: high, medium, or low. Firstly, the algorithm minimizes high priority loads to disconnect from the microgrid according to the generators maximum power (17), then repeats the same procedure for the lower priority loads (16)-(15). If two or more loads have the same priority, load shedding routine disconnects from microgrid loads with higher power in order to minimize total disconnected loads. The solution algorithm is coded in Matlab™.

VI. Simulations and Results

In order to show the effectiveness of the proposed methodology, a study is presented. The test is performed on a 69 branch, 9 lateral test grid derived from a portion of the *PG&E* distribution network (Fig. 4) [38]. We perform a comparison between the proposed fuzzy method and an approach well known in literature, as the stochastic optimization. Total distribution network load is 3802.19 kW for α=1 in the fuzzy approach and corresponds to the maximum value of the probability density function (pdf) for the stochastic optimization. We consider 9 buses with not-programmable

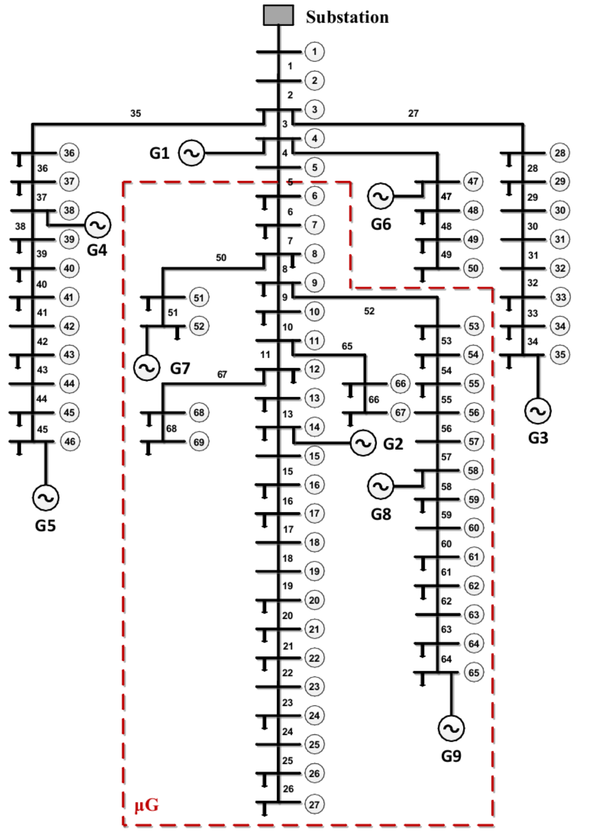

Fig. 4. C69-bus test network

and programmable DER units, in correspondence of the buses 4, 14, 35, 38, 46, 47, 52, 58 and 65, able to guarantee the total active power generation equal to 3950 kW, which is the maximum value of the possibility function (α=1) and the maximum value of the pdf. For each DER unit we assume an uncertainty of $\pm$ 5% described by using fuzzy numbers in the proposed approach, and by a probability density function in the stochastic approach. To better compare the results obtained by applying the two methods, we suppose the fuzzy membership function and the pdf similar in shape (symmetrical triangular). In order to implement the two methods, Monte Carlo simulations are carried out on 10000 different values where input variables, in the assumed input bounds, are randomly selected.

In the case study, following to the outage of the line 5, the main part of the network is isolated, so the big microgrid (μG) is formed and joined results of rescheduling and load shedding are discussed. The total rated generation power and the total load are equal to 2.232 MW and 2.676 MW, respectively. We assign to the load buses connected to the μG one of the following priority value: LOW, MEDIUM or HIGH.

In Fig. 5 we show the fuzzy membership functions (fuzzy approach) and the normalized histograms (stochastic approach) after the outage of the total generation active power and total demand. If we apply the rescheduling routines by both methods, it is not possible to obtain a feasible solution because the maximum total generation power is lower than the total demand, so that the load shedding procedure is run.

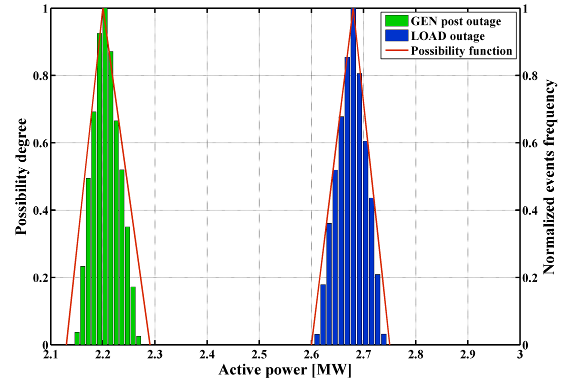

Fig. 5. Total generators and loads active power post outage

Fig. 6 shows the results obtained by load shedding routines: the algorithm in order to maximize the supplied total load in the μG, disconnects only one big load (1.244 MW - bus 61) with LOW priority value. The two approach allow obtaining similar results. In particular, the fuzzy arithmetic lead to a more conservative solution since the membership functions, for α=0, are slightly larger than the normalized histograms amplitude. It is worth noting that the fuzzy load shedding procedure implements only arithmetic fuzzy operations without Monte Carlo simulations, so that we have a feasible solution with a low computational time-consuming but an increasing amplitude of the solution interval.

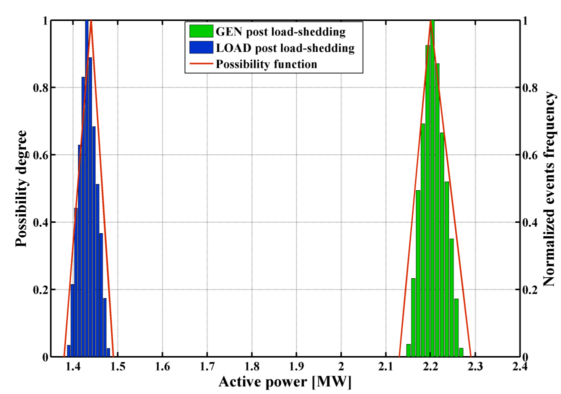

Fig. 6. Total generators and loads active power post load-shedding

The total generators active power, after the performed load shedding routine, is much greater (2.232 MW) than total load active power (1.433 MW), as shown in Fig. 6, thus by applying the rescheduling algorithm, the new total generation active power matches the total power demand, according to the values shown in Fig. 7. In the Table I, we list the variation of the power generation after the rescheduling; in particular, the second and the fourth columns show the minimum and maximum value for α=0, the third column the more *possible* value for α=1, whereas the fifth column shows the active

TABLE I
RESCHEDULING RESULTS

Gen number	P_{inf} $[MW]$	P $[MW]$	P_{sup} $[MW]$	ΔP $[MW]$
14	0.108	0.124	0.125	+0.024
52	0.456	0.491	0.510	-0.309
58	0.317	0.344	0.356	-0.156
65	0.488	0.525	0.556	-0.275
Tot	1.372	1.484	1.535	-

TABLE II
IMPACT OF THE SLACK NODE ON RESCHEDULING

Slack number	ΔP_{14} $[MW]$	ΔP_{52} $[MW]$	ΔP_{58} $[MW]$	ΔP_{65} $[MW]$	$Loss$ [%]
14	+0.024	-0.309	-0.156	-0.275	3.4
52	-0.003	-0.463	+0.072	-0.185	11.6
58	+0.031	-0.168	-0.288	-0.221	7.0
65	+0.005	-0.082	-0.004	-0.613	4.9

power variation regarding the rated power pre-outage for α=1.

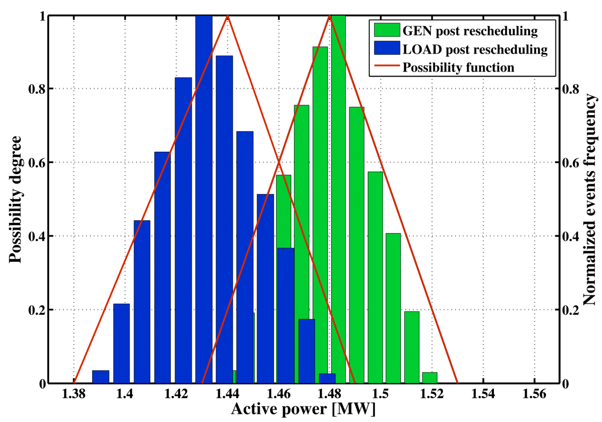

Fig. 7. Total generators and loads active power post rescheduling

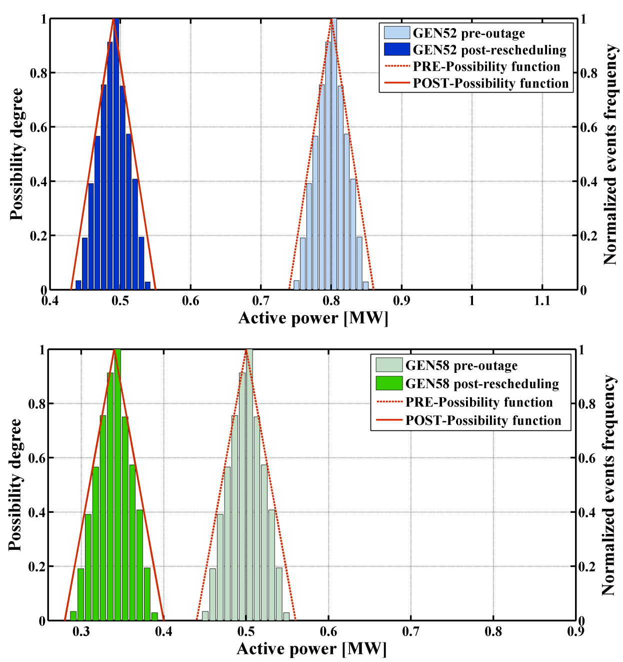

Fig. 8. Active power production of the generators G7 and G8

The rescheduling results in terms of generators active power at the buses 52, and 58 are shown in Fig. 8: we plot in stretched red line the membership function after the rescheduling procedure and in dotted red line the pre-outage ones. The solution shows a decrease of the active power of both generators in order to match the total load. The results obtained by using the stochastic optimization method are similar in comparison with those achieved by the proposed fuzzy approach.

The losses of the system are illustrated in Table II by varying the position of the slack bus. In particular, in order to implement the rescheduling algorithm we obtain four rescheduling solutions changing the slack bus. The best rescheduling solution in terms of losses (3.4 %) is achieved assuming the generator 14 as the slack bus.

VII. CONCLUSION

The paper proposes a new methodology based on fuzzy numbers in order to solve an optimization rescheduling and load shedding problems in microgrids with imprecise information. The integration of fuzzy possibility theory and Monte Carlo simulations allow obtaining a large band solution corresponding to power system scenarios with high or low possibility. Furthermore, the interpretation of the solutions is easy because the uncertainty characterizing both load and power generation is modeled on human's intuition. The main benefits of the proposed approach are due to the fact to have real rescheduling and load shedding solutions with different degree of *possibility*, so that technicians and operators can accept the proposed solutions with an aware risk level. The optimization procedure is based on a classical solution method using as input data the central values of fuzzy numbers. The effectiveness and intuitiveness of the approach is highlighted by tests run on a 69 bus distribution system and the results demonstrate that the proposed method is well suited for the assessment of uncertainty propagation in rescheduling and load shedding problems. Finally, the comparison between the proposed fuzzy method and stochastic optimization shows that the first approach is computationally efficient due to fuzzy arithmetic and allows obtaining a more conservative estimation of the solution.

REFERENCES

[1] J. Tang, J. Liu, F. Ponci, and A. Monti, "Adaptive load shedding based on combined frequency and voltage stability assessment using synchrophasor measurements," *IEEE Transactions on Power Systems*, vol.28, no.2, pp.2035-2047, 2013.

[2] R. Argiento, R. Faranda, A. Pievatolo, and E. Tironi, "Distributed Interruptible Load Shedding and Micro-Generator Dispatching to Benefit System Operations," *IEEE Transactions on Power Systems*, vol.28, no.2, pp.840-848, 2012.

[3] A. Borghetti, M. Bosetti, S. Grillo, S. Massucco, C.A. Nucci, M. Paolone, and F. Silvestro, "Short-Term Scheduling and Control of Active Distribution Systems With High Penetration of Renewable Resources," *IEEE Systems Journal*, vol.4, no.3, pp.313-322, 2010.

[4] D. Menniti, N. Scordino, and N. Sorrentino, "Secure and economic management of a power system in the presence of wind generation," *Electric Power System Research*, vol. 80, no. 11, pp. 379-388, 2010.

[5] K.R.C. Mamandur, and G.J. Berg, "Economic shift in electric power generation with line flow constraints," *IEEE Trans. Power App. Syst.*, vol. PAS-97, no. 7, pp. 16181626, 1978.

[6] T.K.P. Medicherla, R. Billinton, and M.S. Sachdev, "Generation rescheduling and load shedding to alleviate line overloadAnalysis," *IEEE Trans. Power App. Syst.*, vol. PAS-98, no. 6, pp. 18761884, 1979.

[7] A.C.G. Meio, J.C.O. Mello, and S. Granville, "The effects of voltage collapse problems in the reliability evaluation of composite systems," *EEE Trans. Power Syst.*, vol. 12, no. 1, pp. 480488, 1997.

[8] C. Dornellas, M. Schilling, A. Melo, J.C.S. Souza, and M.B. Do Coutto Filho, "Combining local and optimized power flow remedial measures in bulk reliability assessment," *IEE Proc. Gener. Transm. Distrib.*, vol. 150, no. 5, pp. 629634, 2003.

[9] A. Shandilya, H. Gupta, and J. Sharma, "Method for generation rescheduling and load shedding to alleviate line overloads using local optimisation," *IEE Proc. Gener. Transm. Distrib.*, vol.140, no.5, pp.337,342, 1993.

[10] J. Hazra, and A.K. Sinha, "Congestion Management Using Multiobjective Particle Swarm Optimization," *IEEE Transactions on Power Systems*, vol. 22, no. 4, pp. 17261734, 2007.

[11] I. Genc, R. Diao, V. Vittal, S. Kolluri, and S. Mandal, "Decision Tree-Based Preventive and Corrective Control Applications for Dynamic Security Enhancement in Power Systems," *IEEE Transactions on Power Systems*, vol. 25, no. 3, pp. 1611-1619, 2010.

[12] Z. Wang, X. Song, H. Xin, D. Gan, and K.P. Wong, "Risk-Based Coordination of Generation Rescheduling and Load Shedding for Transient Stability Enhancement," *IEEE Transactions on Power Systems*, vol. 28, no .4, pp. 4674-4682, 2013.

[13] E.E. Aponte, and J.K. Nelson, "Time Optimal Load Shedding for Distributed Power Systems," *IEEE Transactions on Power Systems*, vol. 21, no. 1, pp. 269277, 2006.

[14] Y. Tofis, Y. Yiasemi, and E. Kyriakides, "A Plug-and-Play Selective Load Shedding Scheme for Power Systems," to be published on *IEEE Systems Journal*, [On-line]. Available: http://ieeexplore.ieee.org.

[15] V. Calderaro, V. Galdi, V. Lattarulo, and P. Siano, "A new algorithm for steady state load-shedding strategy," in *Proc. IEEE OPTIM Int. Conf.*, 2010, pp.48-53.

[16] A. Borghetti, M. Bosetti, S. Grillo, S. Massucco, C.A. Nucci, M. Paolone and F. Silvestri, "Short-term Scheduling and Control of Active Distribution Systems with High Penetration of Renewable Resources," *IEEE Systems Journal*, vol. 4, n. 3, pp. 313-322, 2010.

[17] L. Lenoir, I. Kamwa, and L.A. Dessaint, "Overload Alleviation With Preventive-Corrective Static Security Using Fuzzy Logic," *IEEE Transactions on Power Systems*, vol. 24, no. 1, pp.134-145, 2009.

[18] Y. Wu, V.K.N. Lau, D.H.K. Tsang, Q. Li Ping, and M. Limin, "Optimal Energy Scheduling for Residential Smart Grid with Centralized Renewable Energy Source", *IEEE Systems Journal*, vol.8, no. 2, pp. 562-576, 2014.

[19] V. Calderaro, V. Galdi, G. Graber, and A. Piccolo, "Optimal Generation Rescheduling in Micrograds under Uncertainty," *International Review of Electrical Engineering*, vol. 8, n. 5, 2013.

[20] D. Dubois, and H. Prade, "Possibility Theory. An Approach to Computerized Processing of Uncertainty," Plenum Press, New York and London, 1988.

[21] G.J. Klir, and T.A. Folger, "Fuzzy Sets, Uncertainty and Information," Prentice-Hall International Inc., Englewood Cliffs, New Jersey, 1988.

[22] H.J. Zimmermann, "Fuzzy Set Theory and its Applications," Kluwer Academic Publishers, Boston/Dordrecht/London, 1996.

[23] B. Borkowska, "Probabilistic Load Flow," *IEEE Trans.Power App. Syst.*, vol. PAS-93, pp. 752759, May/Jun. 1974.

[24] R.N. Allan, and C.H. Grigg, "Probabilistic analysis of power flow," *Proc. Inst. Elect. Eng. C, Gen., Transm., Distrib.*, vol. 121, no. 12, pp. 15511556, Dec. 1974.

[25] J. Tome Saraiva, V. Miranda, L.M.V.G. Pinto, "Impact on some planning decisions from a fuzzy modelling of power systems," *IEEE Transactions on Power Systems*, vol.9, no.2, pp.819-825, 1994.

[26] P.R. Bijwe, M. Hanmandlu, and V.N. Pande, "Fuzzy power flow solutions with reactive limits and multiple uncertainties," *Elect. Power Syst. Res.*, vol. 76, pp. 145152, 2005.

[27] M. Cortes, R. Palma, and G. Jimenez, "Fuzzy load flow based on α-Cuts arithmetics," in Proc. 39th North Amer. Power Symp., Las Cruces, NM, Oct. 2007, pp. 683691.

[28] M. Cortes-Carmona, R. Palma-Behnke, G. Jimenez-Estevez, "Fuzzy Arithmetic for the DC Load Flow," *IEEE Transactions on Power Systems*, vol.25, no.1, pp.206,214, Feb. 2010.

[29] B.A. Gomes, and J.T. Saraiva, "Demand and generation cost uncertainty modeling in power system optimization studies," *Electric Power System Research*, vol. 79, no. 6, pp. 1000-1009, 2009.

[30] D. Saez, F. Avila, D. Olivares, C. Canizares, and L. Marin, "Fuzzy Prediction Interval Models for Forecasting Renewable Resources and Loads in Microgrids," *IEEE Trans. on Smart Grid*, vol. 6, no. 2, pp.548-556, 2015.

[31] L.A. Zadeh, "Fuzzy sets as a basis for a theory of possibility," *Fuzzy Sets and Systems 1*, vol. 100, pp. 9-34, 1999.

[32] A. Piegat, "On practical problems with the explanation of the difference between possibility and probability," *Control and Cybernetics*, vol. 34, no.2, pp.505-524, 2005.

[33] C.H. Yeh, and H. Deng, "A practical approach to fuzzy utilities comparison in fuzzy multicriteria analysis," *International Journal of Approximate Reasoning*, vol. 35, no. 2, pp. 179-194, 2004.

[34] V. Calderaro, V. Galdi, M. Cortes-Carmona, and R. Palma-Behnke, "Fuzzy load-shedding strategy in distribution systems," in *Proc. ISDA Int. Conf.*, 2011, pp.319,324.

[35] X. Wu, X. Wang, and Z. Bie, "Optimal generation scheduling of a microgrid," in *Proc. IEEE ISGT Europe Int. Conf.*, 2012.

[36] A. Migdalas, P.M. Pardalos, and P. Varbrand, "Multilevel optimization: algorithms and applications," Dordrecht, The Netherlands: Kluwer Academic Publisher.

[37] P. Kundur, "Power System Stability and Control," EPRI, McGrow Hill, 1994.

[38] M.E. Baran, and F.F. Wu, "Optimal capacitor placement on radial distribution systems," *IEEE Trans. on Power Delivery*, vol. 4 n. 1, pp. 725734, 1989.

Vito Calderaro (M10) received the M.Sc. degree in electronic engineering and the Ph.D. degree in information engineering from the University of Salerno, Italy, in 2001 and 2006, respectively. He is a Research Fellow with the Department of Industrial Engineering. He served as reviewer for international conferences and journals. His current research focuses on integration of distributed generation systems on electrical distribution networks, soft computing methodologies in power system applications, and protection and diagnosis of complex systems.

Vincenzo Galdi (M98) received the M.Sc. degree in Electronic Engineering from the University of Salerno, Italy, in 1994, and the Ph.D. degree in Electrical Engineering from the University of Napoli, Italy, in 1999. Since 2003, he has been an Associate Professor at the University of Salerno. His current research interests include application of ICT to power systems, to power management and energy efficiency in transportation systems, smartgrids.

Giuseppe Graber (S14) received the M.Sc. degree in Electronic Engineering from the University of Salerno Italy, in 2011. Since 2013 he has been pursuing the Ph.D. degree in Information Engineering at University of Salerno. His main research interests focus on energy storage systems in electrified transportation, impact of electric vehicles on distribution networks, and soft computing methodologies in power system applications.

Antonio Piccolo (M06) received the M.Sc. degree in electrical engineering from the University of Napoli, Italy. Since 1990, he has been a Full Professor at University of Salerno, Italy. He served as reviewer and session chairman for many international conferences. He has authored more than 130 papers mainly in international journals and conferences in the fields of power systems. His main fields of interest are the application of ICT to electric energy infrastructures and transportation systems.