

A generalized mean-field theory for the t - J model: the single-pole COM solution

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Abstract. We have reported the most general mean-field approximation for the t - J model obtained in the framework of the Composite Operator Method within the simplest one-pole approximation. The nearest-neighbor charge-charge and spin-spin correlators have been computed in a projection (one-loop) approximation with the result of reducing the self-consistent parameters just to two: the chemical potential and the kinetic energy. The properties of the system have been studied as functions of the filling and the temperature and compared with well established numerical results in order to assess the reliability of the obtained approximated solution. This latter is very well suited to study low-intermediate fillings and small dopings in case of frustration.

1 Introduction

The study of strongly correlated electronic systems (SCES) [1–3] is very relevant to current literature as it is one of the most intriguing open challenges in theoretical condensed matter physics [4,5], and also because strongly correlated materials exhibit an extremely rich variety of unconventional experimental features with very promising technological applications [6–10]. Even though deceptively simple, model Hamiltonians such as the Hubbard model [11–13] and its strong-coupling derivative, the t - J model [14–16], seem capable to host the emergence of extremely interesting and still puzzling phenomenologies such as the Mott-Hubbard metal-insulator transition [17], exotic charge, spin and/or orbital ordering [18–21], non-Fermi-liquid normal phases characterized and dominated by short-range spin and/or charge correlations [21–23], as well as high-temperature superconductivity [22–28]. According to this, the study of the properties of these models is still very active [29–44] and follows the current trends in the field: spatial inhomogeneity (defects, interfaces, heterostructures, etc.) [8–10,45–51] and time dependence [52–60].

Solving so much correlated models is an incredibly difficult task and there is always need for new methods and, in particular, for non-perturbative ones. In this

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short manuscript, we use an operatorial approach [1] and first apply the Composite Operator Method (COM) [22,61–64], in its simplest version, to the t - J model. Operatorial approaches (the Hubbard approximations [11–13], the Projection Operator Method [65,66], the works of Mori [67], Rowe [68], Roth [69], and the Spectral Density Approach [70]) study SCES searching for the new quasi-particles emerging in the system because of the strong correlations in terms of operators describing the new elementary excitations and their unconventional properties. We will establish the lowest-order approximation in the COM framework, just a one-pole approximation, and analyze the properties of what is the most general mean-field solution. We will also compare the results with well established numerical ones to assess the reliability of the solution and investigate the routes towards its improvement. In particular, we will analyze the behavior of the self-consistent parameters of the solution, which amount to the chemical potential and the correlator defining the kinetic energy in the Hamiltonian. The nearest-neighbor charge–charge and spin–spin correlators will be computed in a projection (one-loop) approximation and their properties analyzed in detail as functions of filling and temperature.

2 Model and method

2.1 Model and notation

The Hamiltonian of the two-dimensional t - J model – 3-site terms excluded – reads as

$$\mathcal{H} = -4t \sum_{\mathbf{i}} \xi^\dagger(\mathbf{i}) \cdot \xi^\alpha(\mathbf{i}) + \frac{1}{2}J \sum_{\mathbf{i}} [\nu_k(\mathbf{i}) \nu_k^\alpha(\mathbf{i}) - \nu(\mathbf{i}) \nu^\alpha(\mathbf{i})] - \mu \sum_{\mathbf{i}} \nu(\mathbf{i}) \quad (1)$$

where t is the nearest-neighbor hopping integral (energy unit hereafter), J is the exchange integral and μ is the chemical potential.

The fermionic field $\xi^\dagger(\mathbf{i}) = (\xi_\uparrow^\dagger(\mathbf{i}), \xi_\downarrow^\dagger(\mathbf{i}))$ is written in spinorial notation (\cdot stands for the inner (scalar) product in spin space) and Heisenberg picture ($i = (\mathbf{i}, t_i)$), $\mathbf{i} = \mathbf{R}_i$ is a vector of the two-dimensional Bravais lattice. $\xi_\sigma(\mathbf{i}) = \left(1 - c_\sigma^\dagger(\mathbf{i}) c_\sigma(\mathbf{i})\right) c_\sigma(\mathbf{i})$ is the projected Hubbard operator taking care of the single occupation of site \mathbf{i} with an electron of spin σ , that is of the transitions $n = 0 \leftrightarrow n = 1$ (σ). It is worth noting that $\xi(\mathbf{i})$ allows to simplify the notation with respect to the canonical operator $c(\mathbf{i})$: $\xi(\mathbf{i})$ absorbs in its definition the single-occupation-per-site projector, which is an essential and indispensable characteristic of the t - J model, and allows to use conventional operatorial methods without need to impose the single-occupation-per-site projection as an external constraint.

$\nu_\sigma(\mathbf{i}) = \xi_\sigma^\dagger(\mathbf{i}) \xi_\sigma(\mathbf{i})$ is the particle density operator for spin σ at site \mathbf{i} and $\nu(\mathbf{i}) = \sum_\sigma \nu_\sigma(\mathbf{i}) = \xi^\dagger(\mathbf{i}) \cdot \xi(\mathbf{i})$ is the total particle density operator at site \mathbf{i} . $\nu_k(\mathbf{i}) = \xi^\dagger(\mathbf{i}) \cdot \sigma_k \cdot \xi(\mathbf{i})$ is the spin density operator at site \mathbf{i} , being σ_k the Pauli matrices; the ordinary components of the spin operator are in the following relationship: $S_k = \frac{1}{2} \nu_k$. \hbar has been fixed to 1. We also have $\sigma_\mu = (\mathbf{1}, \boldsymbol{\sigma})$ [$\sigma^\mu = (-\mathbf{1}, \boldsymbol{\sigma})$] and, correspondingly, $\nu_\mu(\mathbf{i}) = \xi^\dagger(\mathbf{i}) \cdot \sigma_\mu \cdot \xi(\mathbf{i})$ is the charge ($\mu = 0$) and spin ($\mu = 1, 2, 3$) density operator at site \mathbf{i} .

Finally, $\alpha_{\mathbf{ij}} = \frac{1}{4} \delta_{\mathbf{ij}}$ is the nearest-neighbor projector and, hereafter, for any operator $\psi(\mathbf{i})$, we use the notation $\psi^\kappa(\mathbf{i}, t) = \sum_{\mathbf{j}} \kappa_{\mathbf{ij}} \psi(\mathbf{j}, t)$ where $\kappa_{\mathbf{ij}}$ can be any function of the two sites \mathbf{i} and \mathbf{j} and, in particular, a projector. Accordingly, for instance, the

kinetic term \mathcal{K} of the Hamiltonian has been rewritten as

$$\begin{aligned} \mathcal{K} &= -t \sum_{\langle \mathbf{i}, \mathbf{j} \rangle} \xi^\dagger(\mathbf{i}, t) \cdot \xi(\mathbf{j}, t) = -t \sum_{\mathbf{i}, \mathbf{j}} \delta_{\langle \mathbf{i}, \mathbf{j} \rangle} \xi^\dagger(\mathbf{i}, t) \cdot \xi(\mathbf{j}, t) \\ &= -4t \sum_{\mathbf{i}, \mathbf{j}} \alpha_{\mathbf{ij}} \xi^\dagger(\mathbf{i}, t) \cdot \xi(\mathbf{j}, t) = -4t \sum_{\mathbf{i}} \xi^\dagger(\mathbf{i}) \cdot \xi^\alpha(\mathbf{i}). \end{aligned} \quad (2)$$

Also the nearest-neighbor spin-spin and charge-charge interaction terms of the Hamiltonian have been rewritten using the same very compact notation.

Actually, we will rewrite the Hamiltonian (1) in the following form

$$\mathcal{H} = \sum_{\mathbf{i}} \xi^\dagger(\mathbf{i}) \cdot \left[-4t \xi^\alpha(\mathbf{i}) + J \left(\tilde{\xi}_0(\mathbf{i}) + \tilde{\xi}_s(\mathbf{i}) \right) - \left(\mu + \frac{1}{2} J \right) \xi(\mathbf{i}) \right] \quad (3)$$

where $\tilde{\xi}_0(\mathbf{i}) = \frac{1}{2}(1 - \nu^\alpha(\mathbf{i})) \xi(\mathbf{i})$ and $\tilde{\xi}_s(\mathbf{i}) = \frac{1}{2} \nu_k^\alpha(\mathbf{i}) \sigma_k \cdot \xi(\mathbf{i})$ allow to recast the Hamiltonian in terms of electronic transitions dressed by nearest-neighbor charge and spin fluctuations.

2.2 Method

2.2.1 Current

The current $J(\mathbf{i})$ of the field $\xi(\mathbf{i})$ reads as

$$\begin{aligned} J(\mathbf{i}) &= i \frac{\partial}{\partial t} \xi(\mathbf{i}) = [\xi(\mathbf{i}), \mathcal{H}] \\ &= -2t (\xi^\alpha(\mathbf{i}) + 2\xi_0(\mathbf{i}) + 2\xi_s(\mathbf{i})) + 2J \left(\tilde{\xi}_0(\mathbf{i}) + \tilde{\xi}_s(\mathbf{i}) \right) - (\mu + J) \xi(\mathbf{i}) \end{aligned} \quad (4)$$

where $\xi_0(\mathbf{i}) = \frac{1}{2}(1 - \nu(\mathbf{i})) \xi^\alpha(\mathbf{i})$ and $\xi_s(\mathbf{i}) = \frac{1}{2} \nu_k(\mathbf{i}) \sigma_k \cdot \xi^\alpha(\mathbf{i})$ are just the companions of the higher-order fields defined in the Hamiltonian. It is worth noting that the projective nature of the model Hamiltonian, and of the fields appearing within, leads to the emergence of new fields in the current and to a hierarchy of equations of motion even only by the kinetic energy term, which is usually considered harmless. The t - J model is a strongly interacting model not only for the exchange term explicitly present in the Hamiltonian, but at least equally for its projective nature affecting all its components.

2.2.2 Normalization

The field $\xi(\mathbf{i})$ has a peculiar commutation relation that reminds us of its projective nature and of the obvious and remarkable difference with the original electronic field $c(\mathbf{i})$ [$\{c(\mathbf{i}), c^\dagger(\mathbf{j})\} = \delta_{\mathbf{ij}}$]:

$$\{\xi(\mathbf{i}), \xi^\dagger(\mathbf{j})\} = \delta_{\mathbf{ij}} \left(\mathbf{1} + \frac{1}{2} \sigma^\mu \nu_\mu(\mathbf{i}) \right) \quad (5)$$

that gives the following *normalization* in the homogenous paramagnetic phase

$$I_{\mathbf{ij}} = \langle \{\xi(\mathbf{i}), \xi^\dagger(\mathbf{j})\} \rangle = \delta_{\mathbf{ij}} \left(1 - \frac{1}{2} \nu \right) \quad (6)$$

where $\nu = \langle \nu(\mathbf{i}) \rangle$ is the actual filling of the system (number of particle per site) and $\langle \dots \rangle$ is the quantum mechanical average in the grand-canonical ensemble.

2.2.3 Projection

Once one decides to retain only a single pole in the Green's function, and obviously the one describing the dynamics of the electrons in the most direct way, the COM acts as a generalized mean-field theory with the advantage of using a clear and controlled projection procedure:

$$J(i) = i \frac{\partial}{\partial t} \xi(i) \cong \sum_{\mathbf{j}} \varepsilon_{\mathbf{ij}} \xi(j). \quad (7)$$

Such a generalized mean-field approximation falls back to the standard mean-field (although the most general one) if one projects the current of the operator $c(i)$ on $c(i)$ itself. In the t - J model, this is just impossible by construction as the operator $c(i)$ has lost any significance because of the ubiquitous single-occupation-per-site projector that leads to the emergence of $\xi(i)$. In this respect, the t - J model is so strongly correlated by construction that the standard mean-field approximation cannot even be computed and the one presented here is the simplest possible within the same framework: projection of the dynamics (i.e. the current) of one operator on itself.

Computing the average of the commutation of both sides of equation (7) with the chosen basic field and applying the Fourier transform, we obtain

$$\langle \{J(\mathbf{i}), \xi^\dagger(\mathbf{l})\} \rangle = \sum_{\mathbf{j}} \varepsilon_{\mathbf{ij}} \langle \{ \xi(\mathbf{j}), \xi^\dagger(\mathbf{l}) \} \rangle \quad (8)$$

$$\varepsilon(\mathbf{k}) = m(\mathbf{k}) I^{-1}(\mathbf{k}) \quad (9)$$

where $m(\mathbf{k}) = \mathcal{F} \langle \{J(\mathbf{i}), \xi^\dagger(\mathbf{j})\} \rangle$ and $I(\mathbf{k}) = \mathcal{F}[I_{\mathbf{ij}}] = \mathcal{F} \langle \{ \xi(\mathbf{i}), \xi^\dagger(\mathbf{j}) \} \rangle = 1 - \frac{1}{2}\nu$.

If a Hamiltonian $\bar{\mathcal{H}}$ can be exactly diagonalized in terms of quasi-particles represented by field operators $\psi(i)$ such that $J_\psi(i) = i \frac{\partial}{\partial t} \psi(i) = [\psi(i), \bar{\mathcal{H}}] = \sum_{\mathbf{j}} \bar{\varepsilon}_{\mathbf{ij}} \psi(j)$, we define $\psi(i)$ as an *eigenoperator* of the Hamiltonian $\bar{\mathcal{H}}$ and $\bar{\varepsilon}_{\mathbf{ij}}$ as its *eigenenergy*. Accordingly, $\varepsilon(\mathbf{k}) = \mathcal{F}[\varepsilon_{\mathbf{ij}}]$ is the approximate *eigenenergy* of the approximate *eigenoperator* $\xi(i)$ of the Hamiltonian \mathcal{H} . Such a procedure is equivalent to project the current $J(i)$ onto the basic field $\xi(i)$ and therefore compute the most general mean-field theory possible:

$$J(\mathbf{i}) \cong P_\xi [J(\mathbf{i})] \quad (10)$$

where

$$P_\psi [\phi(\mathbf{i})] = \sum_{\mathbf{j}} \mathcal{F}^{-1} \left\{ \frac{\mathcal{F} \langle \{ \phi(\mathbf{i}), \psi^\dagger(\mathbf{j}) \} \rangle}{\mathcal{F} \langle \{ \psi(\mathbf{i}), \psi^\dagger(\mathbf{j}) \} \rangle} \right\} \psi(\mathbf{j}) \quad (11)$$

is the projection functional for field operators. In particular, we have the following expressions for the thermal averages of the commutators of the fields appearing in the current

$$\langle \{ \xi_0(\mathbf{i}), \xi^\dagger(\mathbf{j}) \} \rangle = -\frac{1}{2} \delta_{\mathbf{ij}} C^\alpha + \frac{1}{4} \alpha_{\mathbf{ij}} (2 - 3\nu + \chi^\alpha) \quad (12)$$

$$\langle \{ \xi_s(\mathbf{i}), \xi^\dagger(\mathbf{j}) \} \rangle = \frac{3}{2} \delta_{\mathbf{ij}} C^\alpha + \frac{3}{4} \alpha_{\mathbf{ij}} \chi_s^\alpha \quad (13)$$

$$\langle \{ \tilde{\xi}_0(\mathbf{i}), \xi^\dagger(\mathbf{j}) \} \rangle = \frac{1}{4} \delta_{\mathbf{ij}} (2 - 3\nu + \chi_c^\alpha) - \frac{1}{2} \alpha_{\mathbf{ij}} C^\alpha \quad (14)$$

$$\langle \{ \tilde{\xi}_s(\mathbf{i}), \xi^\dagger(\mathbf{j}) \} \rangle = \frac{3}{4} \delta_{\mathbf{ij}} \chi_s^\alpha + \frac{3}{2} \alpha_{\mathbf{ij}} C^\alpha. \quad (15)$$

Through this procedure, we obtain the following expression for $m(\mathbf{k})$

$$m(\mathbf{k}) = -\mu I(\mathbf{k}) - 4t [C^\alpha + \alpha(\mathbf{k})(1 - \nu + p)] + 2J \left[p - \frac{\nu}{2} + \alpha(\mathbf{k}) C^\alpha \right] \quad (16)$$

where $C^\alpha = \langle \xi^\alpha(\mathbf{i}) \xi^\dagger(\mathbf{i}) \rangle$ is the main contribution to the kinetic energy $K = 8tC^\alpha$ and $p = \frac{1}{4}(\chi_c^\alpha + 3\chi_s^\alpha)$ is a linear combination of the charge, $\chi_c^\alpha = \langle \nu(\mathbf{i}) \nu^\alpha(\mathbf{i}) \rangle$, and spin, $\chi_s^\alpha = \frac{1}{3} \langle \nu_k(\mathbf{i}) \nu_k^\alpha(\mathbf{i}) \rangle$, nearest-neighbor correlation functions.

2.2.4 Green's function and elementary excitations

The approximated expression of the current (7) allows to obtain a closed expression for the relevant thermal retarded Green's function $G(i, j) = \mathcal{R} \langle \{ \xi(i), \xi^\dagger(j) \} \rangle$ and, in particular, for its Fourier transform $G(\mathbf{k}, \omega) = \mathcal{F}[G(i, j)]$

$$i \frac{\partial}{\partial t_i} G(i, j) = i \delta(t_i - t_j) I_{\mathbf{ij}} + \sum_{\mathbf{l}} \varepsilon_{\mathbf{il}} G(\mathbf{l}, t_i; j) \quad (17)$$

$$G(\mathbf{k}, \omega) = \frac{I(\mathbf{k})}{\omega - \varepsilon(\mathbf{k}) + i\delta}. \quad (18)$$

Such an expression for $G(\mathbf{k}, \omega)$ immediately clarifies the relevance of the *normalization* $I(\mathbf{k})$ and of the *eigenenergy* $\varepsilon(\mathbf{k})$ through the obvious correspondence with the Landau theory: they are the (spectral) weight and the dispersion, respectively, of the only relevant quasi-particle (elementary excitation) in this approximation. Actually, these two quantities completely characterize such quasi-particle and, correspondingly, all properties of the system.

2.2.5 Self-consistency

In order to compute the properties of the system through the Green's function, it is now necessary to fix the values of the correlators appearing in the *eigenenergy* $\varepsilon(\mathbf{k})$: C^α , χ_c^α , and χ_s^α .

The correlator C^α can be easily obtained self-consistently through the relation dictated by the fluctuation-dissipation theorem between the Green's function just introduced and the correlators of the fields appearing in this latter

$$\begin{aligned} C(i, j) &= \langle \xi(i) \xi^\dagger(j) \rangle = \mathcal{F}^{-1} \{ [1 - f_{\mathbb{F}}(\omega)] \Im [G(\mathbf{k}, \omega)] \} \\ &= \mathcal{F}^{-1} \{ 2\pi [1 - f_{\mathbb{F}}(\omega)] I(\mathbf{k}) \delta(\omega - \varepsilon(\mathbf{k})) \} \end{aligned} \quad (19)$$

that leads to

$$C^\alpha = \langle \xi^\alpha(\mathbf{i}) \xi^\dagger(\mathbf{i}) \rangle = \frac{2\pi}{N} \sum_{\mathbf{k}} [1 - f_F(\varepsilon(\mathbf{k}))] \alpha(\mathbf{k}) I(\mathbf{k}) \quad (20)$$

where $f_F(\omega) = \frac{1}{e^{\frac{\omega}{T}} + 1}$ is the Fermi function and T is the temperature in units of t (k_B has been fixed to 1).

The correlators χ_c^α and χ_s^α cannot be obtained directly from the Green's function as they are higher-order correlators. A very straightforward, but still very reasonable, approximation, it is to compute them through the same projection technique used to approximate the current and obtain the generalized mean-field approximation. We can rewrite the two correlators as

$$\chi_c^\alpha = \langle \nu(\mathbf{i}) \nu^\alpha(\mathbf{i}) \rangle = \nu - 2 \langle \xi^\dagger(\mathbf{i}) \cdot \tilde{\xi}_0(\mathbf{i}) \rangle \quad (21)$$

$$\chi_s^\alpha = \frac{1}{3} \langle \nu_k(\mathbf{i}) \nu_k^\alpha(\mathbf{i}) \rangle = \frac{2}{3} \langle \xi^\dagger(\mathbf{i}) \cdot \tilde{\xi}_s(\mathbf{i}) \rangle. \quad (22)$$

Then, we can approximate the two fields appearing in such expressions as

$$\tilde{\xi}_0(\mathbf{i}) \cong P_\xi [\tilde{\xi}_0(\mathbf{i})] = \frac{2 - 3\nu + \chi_c^\alpha}{4(1 - \frac{\nu}{2})} \xi(\mathbf{i}) - \frac{C^\alpha}{2(1 - \frac{\nu}{2})} \xi^\alpha(\mathbf{i}) \quad (23)$$

$$\tilde{\xi}_s(\mathbf{i}) \cong P_\xi [\tilde{\xi}_s(\mathbf{i})] = \frac{3\chi_s^\alpha}{4(1 - \frac{\nu}{2})} \xi(\mathbf{i}) + \frac{3C^\alpha}{2(1 - \frac{\nu}{2})} \xi^\alpha(\mathbf{i}). \quad (24)$$

Finally, substituting the previous expressions in those of χ_c^α and χ_s^α , we obtain - after solving the resulting equations -

$$\chi_c^\alpha = \nu^2 - 2(C^\alpha)^2 \quad (25)$$

$$\chi_s^\alpha = -\frac{2}{1 - \nu} (C^\alpha)^2 \quad (26)$$

that leads to the following straightforward expression for the parameter p

$$p = \frac{\nu^2}{4} - \frac{1}{2} \frac{4 - \nu}{1 - \nu} (C^\alpha)^2 \quad (27)$$

and allows to determine the *eigenenergy* $\varepsilon(\mathbf{k})$ as a function only of the self-consistent parameter C^α .

Actually, we need also to determine the value of the chemical potential μ in the *eigenenergy* $\varepsilon(\mathbf{k})$ in order to study the system for a chosen value of ν , once the Hamiltonian parameters, t and J , and temperature T have been fixed. This requires to implement a second self-consistent equation and solve the two of them simultaneously. The needed second equation can be obtained directly from the same-site ξ correlation function

$$\frac{2\pi}{N} \sum_{\mathbf{k}} [1 - f_F(\varepsilon(\mathbf{k}))] I(\mathbf{k}) = \langle \xi(\mathbf{i}) \xi^\dagger(\mathbf{i}) \rangle = 1 - \nu \quad (28)$$

whose value is obtained on the right-hand side from a direct evaluation, exploiting the local algebra closed by the operator ξ , and on the left-hand side from the fluctuation-dissipation theorem that links its expression to the elements of the Green's function and therefore to the *eigenenergy* $\varepsilon(\mathbf{k})$ and the chemical potential μ therein.

Starting from educated guesses for the values of C^α and μ that allow to compute an educated guess for the *eigenenergy* $\varepsilon(\mathbf{k})$, we can find the solution of the model through the set of two self-consistent equations

$$\begin{cases} C^\alpha = \frac{2\pi}{N} \sum_{\mathbf{k}} [1 - f_F(\varepsilon(\mathbf{k}))] \alpha(\mathbf{k}) I(\mathbf{k}) \\ \nu = 1 - \frac{2\pi}{N} \sum_{\mathbf{k}} [1 - f_F(\varepsilon(\mathbf{k}))] I(\mathbf{k}). \end{cases} \quad (29)$$

3 Results

In Figure 1 (left panel), we report C^α as a function of ν for $J = 0.1$ and $T \in [0, 1.2]$ in steps of 0.1. The black circles are Exact Diagonalization (ED) data for a 4×4 cluster at $T = 0$ from reference [71]. The overall behavior is the one expected taking into account that the kinetic energy is just $K = 8tC^\alpha$: the electrons have a finite mobility only when the average number of particles per site is finite ($\nu > 0$) and the system is not completely filled ($\nu < 1$), according to the projective constraint that allows only one particle per site independently of its spin. Counterintuitively, on increasing the temperature, the mobility reduces. Actually, such a reduction can be understood in terms of virtual exchange processes, possible only when there is an antiferromagnetic spin alignment between nearest-neighbor sites, becoming less and less favored on increasing the temperature because of the complementary reduction of the nearest-neighbor spin-spin correlation function χ_s^α [see Fig. 1 (right panel)]. At higher and higher temperatures ferromagnetic alignments of nearest-neighbor spins are more and more probable as higher-energy excited states become accessible.

The comparison with the ED results is partly satisfactory and partly enlightening regarding the actual characteristics of the approximate solution found. On one hand, for low-intermediate fillings ($\nu \lesssim \frac{2}{3}$), we have a very good agreement although the finiteness of the numerical cluster requires to use a larger value of the temperature to balance the finite level spacing. On the other hand, for high fillings (low dopings) it is evident a more and more marked discrepancy between the analytical and the numerical results. It can be understood looking again at the nearest-neighbor spin-spin correlation function χ_s^α [see Fig. 1 (right panel)] that shows a minimum in that region of doping and vanishes at $\nu = 1$. Actually, our approximate solution is strictly paramagnetic and this contrasts with the actual solution one expects at $\nu = 1$ that is strictly antiferromagnetic (just antiferromagnetic Heisenberg). However, in case frustration is present in the system (just the next-nearest-neighbor hopping term t' will be sufficient), this is exactly what we expect: a not-ordered (paramagnetic) solution at all dopings. Anyway, we better take this into account in the interpretation of all other presented quantities.

The chemical potential μ is reported in Figure 2 (left panel) as a function of ν for $J = 0.1$ and $T \in [0, 1.2]$ in steps of 0.1. It is worth noting the presence of an almost perfect isosbestic point at $\nu \approx 0.725$. At the lower temperatures, the compressibility of the system is definitely decreasing more and more on approaching $\nu = 1$ and is very low and constant in the low-doping region. This is reflected also in the behavior of the charge fluctuations as it is possible to observe in Figure 2 (right panel) where we report the nearest-neighbor charge-charge correlation function χ_c^α diminished of its non-fluctuating component ν^2 . It is again evident the effect of the projective constraint that does not allow any charge fluctuations at $\nu = 1$ and leaves the system in condition to sustain them efficiently only in proximity of *half-filling* ($\nu = 0.5$). There, at the

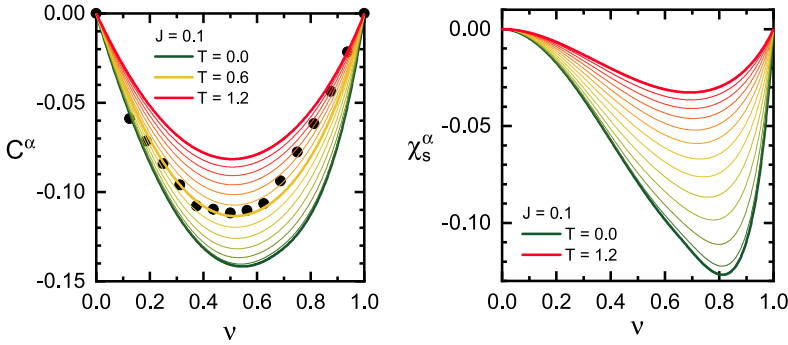


Fig. 1. (Left) C^α and (right) χ_s^α as functions of ν for $J = 0.1$ and $T \in [0, 1.2]$ in steps of 0.1. The black circles in the left panel are ED data for a 4×4 cluster at $T = 0$ from reference [71].

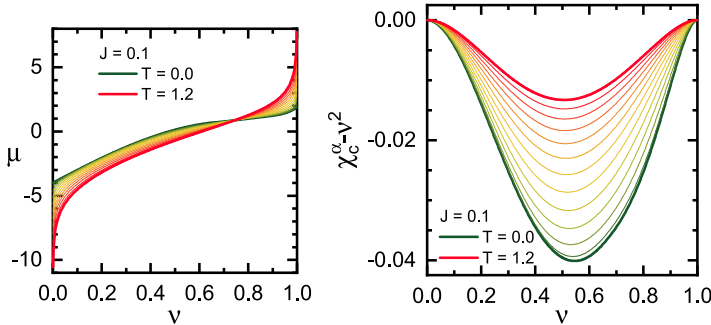


Fig. 2. (Left) μ and (right) $\chi_c^\alpha - \nu^2$ as functions of ν for the same parameters of Figure 1.

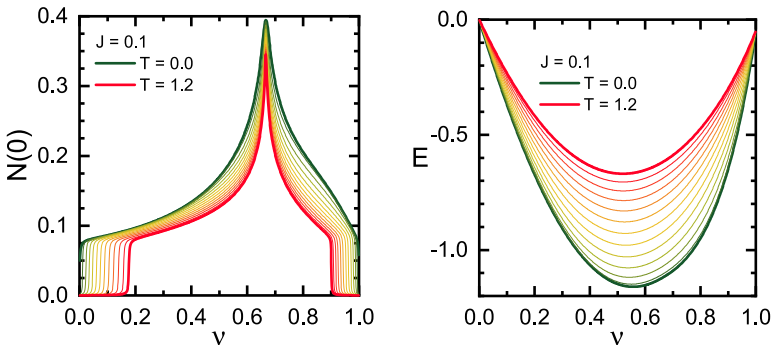


Fig. 3. (Left) $N(0)$ and (right) E as functions of ν for the same parameters of Figure 1.

lower temperatures, we can register the signature of an incipient checkerboard charge order that is again strongly frustrated by our choice to study a homogeneous solution.

Finally, in Figure 3, we report the density of states at the chemical potential $N(0)$ (left panel) and the internal energy per site E (right panel) as functions of ν for $J = 0.1$ and $T \in [0, 1.2]$ in steps of 0.1. The behavior of $N(0)$ clarifies the reason why $\nu_c \approx \frac{2}{3}$ is somehow special for C^α : at ν_c , independently of temperature, we have the presence of a van Hove singularity in the density of states. For fillings lower than ν_c , the system presents a dispersion that is reasonably well described by our paramagnetic one; above ν_c , strong antiferromagnetic correlations will induce a

flattening of the dispersion, leading to the reduction of the available states, that we do not observe in our still paramagnetic dispersion. We have a similar behavior on increasing the temperature (also the numerical data meet our analytical values of C^α for larger and larger values of the temperature on reducing the doping) that is instead dictated by the reduction of spin–spin fluctuations accompanying a reduction of the (virtual) mobility.

4 Conclusions and remarks

In conclusion, we have reported an approximate solution of the t - J model and characterized its properties. The solution showed to be very reliable in the low-intermediate region of filling. At low doping, being forced to be homogeneous and paramagnetic, it does not feature the strong antiferromagnetic correlations of numerical solutions, but it better resembles the actual situation present in real systems where frustration destroys long-range order at very low values of doping. The solution can be further improved by both enlarging the basis of fields (truncating the hierarchy of equations of motion at a higher level and moving away from the simplest generalized mean-field theory) and computing the spin and charge dynamics in the same framework to obtain a richer and more accurate behavior for these fundamental ingredients of the approximation. Such options are actually under current investigation.

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