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Rumor spreading models with random denials

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Abstract

The concept of denial is introduced on rumor spreading processes. The denials occur with a certain rate and they reset to start the initial situation. A population of N individuals is subdivided into ignorants, spreaders and stiflers; at the initial time there is only one spreader and the rest of the population is ignorant. The denials are introduced in the classic DK model and in its generalization, in which a spreader can transmit the rumor at most to k ignorants. The steady state densities are analyzed for these models. Finally, a numerical analysis is performed to study the rule of the involved parameters and to compare the proposed models.

Keywords: Rumor spreading, DK-model, Denials.

1. Introduction

Recently, great attention has been payed to models for the spread of rumor. Indeed, the rumor spreading plays an important role in various contexts of the real life, for instance, it can shape the public opinion in a country, greatly impacts financial markets and causes panic in a society. To analyze the spreading and cessation of the rumor, models are often formulated as social contagion processes (cf., for example, [1], [6], [7]). The first classical rumor spreading model, the DK-m, was proposed by Daley and Kendal in 1960s (cf. [3], [4]). Subsequently, Maki and Thomson developed an other classical model (MT-m) (cf. [9]). In both models people are divided into three groups, the ones who know and transmit the rumor (spreaders), the ones who do not know the rumor (ignorants) and the ones who know the rumor but do not transmit it (stiflers), the rumor spreads through pair-wise contacts between spreaders and the other people. In the DK- m, spreader-ignorant contact will convert the ignorant to spreader; spreader-spreader contact will convert both spreaders to stiflers and spreader-stifler contact will stifle the spreader. In the MT-m when a spreader contacts another spreader, only the initiating one becomes a stifler. A large amount of works have studied the dynamics and limit behaviors of these systems and their variants (cf. [8], [10], [11], [12]).

Recently, increasing attention has been payed on spreading processes because they can be connected to modern technology, business marketing and sociology (cf. [5]). Examples of such processes are virus propagation in social and computer networks, the diffusion of innovations, the occurrence of information cascades in social and economic systems, information diffusion in a society through the word of mouth mechanism ([13] and references therein, [14], [15], [16]).

In this scenario we insert the concept of denial. The denials occur at random instants of time, they reset the system to the initial condition (i.e. only one person is able to spread the rumor and all others are ignorant) and then the process starts following the previous rules. Generally, during the spreading of a rumor one can consider the effect of an external entity that denies the rumor so the process is reseted to the initial state, i.e. there is only one spreader, the initial one, that renews the spreading process. For example, if we consider the rumor as a worm, the denial represents the effect of an anti-virus that restores the initial condition in which the hacker reinforces the virus (or he designs a new virus). In business marketing, the rumor is the advertisement of a product, the denial can be an information that discredits the product (in this case the society improves the product or defends oneself from the accuses), or the launch of a new concurrent product. In both the cases after the denial the rumor restarts with a new advertisement. In a political campaign, we can explain the rumor as the promoting of a candidate, the denial can be the consequence of a scandal, the re-starting is the refusal of the scandal.

We introduce the denials in two models: the classic DK-m, and in a variant (model B) in which each spreader can transmit the rumor at most k times before becoming a stifler. Obviously, if k tends to infinity the model B can be connected to the DK-m. The work is organized into 3 sections. In Section 2 we analyze the DK-m with denials and we study the stationary density; for this model we show that at most the half of the population can be informed about the rumor. In Section 3 we consider the model B subject to denials and we discuss the stationary density. In this case the percentage of ignorants depends on k, in addition the rate at which the denials occur. Moreover, when k grows the model behaves like the DK-model with denials and a good match is found already for $k = 6$.

2. Model A: DK-model with denials

We consider a population consisting of N individuals which, with respect to the rumor, are subdivided into three classes: ignorants, spreaders and stiflers. As in [2] and [9], we assume that the rumor spreads by directed contact of the spreaders with others in the population and $\lambda > 0$ represents the rate of contacts between two individuals. The contacts between the spreaders and the rest of the population are governed by the following set of rules:

- when a spreader contacts an ignorant, the ignorant becomes a spreader;
- when a spreader contacts another spreader both become stiflers;
- when a spreader contacts a stifler, the spreader becomes stifler.

Moreover, we assume that a denial can occur at a rate $\xi > 0$. A denial transforms all spreaders and stiflers into ignorants except one spreader that remains the same. We assume that the contacts occur independently from the denials. We denote by $X(t)$, $Y(t)$ and $Z(t)$ the density of population that are ignorants, spreaders and stiflers at time t, respectively. It results:

$$X(t) + Y(t) + Z(t) = 1 \tag{2.1}$$

and at the initial time there is only one spreader and the rest of population is ignorant, i.e.

$$X(0) = 1 - \frac{1}{N}, \quad Y(0) = \frac{1}{N}, \quad Z(0) = 0. \tag{2.2}$$

Therefore, if t_i for $i = 1, 2, \ldots$ denote the times of renewal in which the denials occur then it results $X(t_i) = 1 - 1/N$, $Y((t_i) = 1/N$ and $Z((t_i) = 0$. A denial re-establishes the initial condition (2.2). For $\Delta t > 0$, the rumor spreading mechanism is described by the following equations:

$$\begin{aligned} X(t+\Delta t) &= X(t) - \lambda\,\Delta t\, X(t)\,Y(t) + \xi\,\Delta t\left[1 - X(t) - \frac{1}{N}\right] \\ Y(t+\Delta t) &= Y(t) + \lambda\,\Delta t\,Y(t)\left[X(t) - Y(t) + \frac{1}{N} - Z(t)\right] - \xi\,\Delta t\left[Y(t) - \frac{1}{N}\right] \\ Z(t+\Delta t) &= Z(t) + \lambda\,\Delta t\,Y(t)\left[Y(t) - \frac{1}{N} + Z(t)\right] - \xi\,\Delta t\,Z(t), \end{aligned} \tag{2.3}$$

obtained by decomposing the last step. Assuming that N is sufficiently large, we can approximate $Y(t)\left[Y(t) - \frac{1}{N}\right] \simeq Y^2(t)$; so that, for $\Delta t \to 0$, Equations (2.3) become:

$$\begin{aligned} \frac{dX(t)}{dt} &= -\lambda\,X(t)\,Y(t) + \xi\left[1 - \frac{1}{N}\right] - \xi\,X(t) \\ \frac{dY(t)}{dt} &= Y(t)\left\{\lambda\,[X(t) - Y(t) - Z(t)] + \frac{\lambda}{N} - \xi\right\} + \frac{\xi}{N} \\ \frac{dZ(t)}{dt} &= \lambda\,Y(t)\,[Y(t) + Z(t)] - \xi\,Z(t). \end{aligned} \tag{2.4}$$

In Figures 1 the densities of the three subpopulations are plotted for $N = 100$, $\lambda = 1$ and various choices of ξ. The full, dashed and dot-dashed curves represent the percentage of the ignorants, spreaders and stiflers respectively. As we formally show in the next subsection, we observe from Figure 1 that the system (2.4) admits a steady state behavior, reached in less time for increasing values of ξ, and the steady state X is always greater than 0.5 for $\xi > 0$, whereas $X = 0.2$ for $\xi = 0$ according to the classical DK-model.

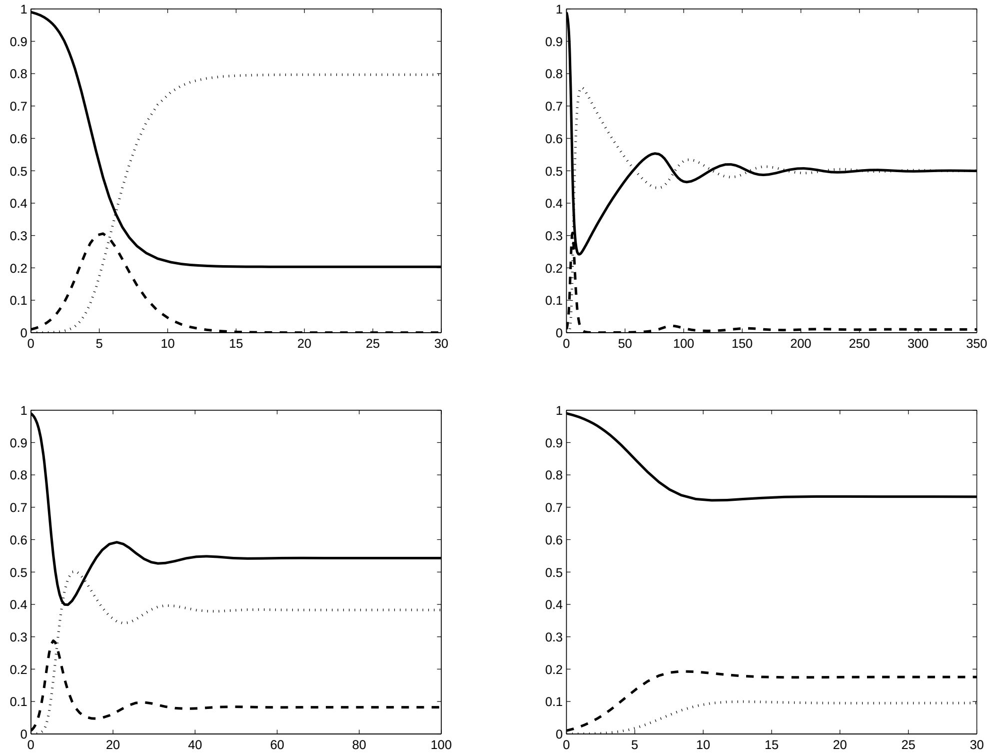

Figure 1: For the DK-model with denials, X (full curve), Y (dashed curve) and Z (dot-dashed) for $N = 100$, $\lambda = 1$ and $\xi = 0,\ 0.01,\ 0.1,\ 0.5$ from left to the right.

2.1. The steady state analysis

Setting

$$X = \lim_{t\to\infty} X(t), \quad Y = \lim_{t\to\infty} Y(t), \quad Z = \lim_{t\to\infty} Z(t),$$

from (2.4), recalling (2.1), one obtains

$$X\,Y - \rho\Big(1 - \frac{1}{N} - X\Big) = 0, \qquad Y\Big(-\rho + 2\,X - 1\Big) + \frac{\rho}{N} = 0 \tag{2.5}$$

with $\rho = \xi/\lambda > 0$. By solving (2.5) we determine the stationary density of the ignorants and spreaders, the stationary density of the stiflers can be obtained recalling (2.1).

Theorem 1. *For the DK-model with denials, the steady-state density of the ignorants is*

$$X = \frac{N\rho + 3N - 1 - \sqrt{N^2(1-\rho)^2 + 1 + 6N\rho + 2N}}{4N}. \tag{2.6}$$

Moreover, one has

$$\lim_{N\to\infty} X = \begin{cases} \dfrac{\rho+1}{2} & \text{if } \rho < 1 \\ 1 & \text{if } \rho \geq 1. \end{cases} \tag{2.7}$$

Proof. By comparing the equations of (2.5) one can have two different expressions of Y from which it follows:

$$2NX^2 - (N\rho + 3N - 1)X + (N-1)(\rho+1) = 0. \tag{2.8}$$

Let $X_{1,2}$ be the roots of (2.8) with $X_1 > X_2$; they are real and one has

$$X_{1,2} = \frac{N\rho + 3N - 1 \pm \sqrt{N^2(1-\rho)^2 + 1 + 6N\rho + 2N}}{4N}. \tag{2.9}$$

Note that $X_2 > 0$ being $N\rho + 3N - 1 - \sqrt{N^2(1-\rho)^2 + 1 + 6N\rho + 2N} > N\rho + 3N - 1 - \sqrt{(N\rho + 3N - 1)^2} = 0$ Moreover, since $\sqrt{(N\rho + 3N - 1)^2 - 8N(N-1)(\rho + 1)} > -[1 + N(1-\rho)]$ it follows $X_2 < 1$.
Similarly, $X_1 > 1$ is satisfied because $\sqrt{(N\rho + 3N - 1)^2 - 8N(N-1)(\rho + 1)} > 1 + N(1-\rho)$. Therefore, only X_2 is a consistent solution for the considered problem, so we conclude that the density of the ignorants in the population tends to X_2 in the steady-state regime. Finally, Eq. (2.7) follows taking the limit of (2.6) for $N \to \infty$.

Proposition 1. *The density of the ignorants $X(\rho)$ is positive, increasing and its maximum is*

$$\lim_{\rho\to\infty} X(\rho) = \frac{N-1}{N} \xrightarrow[N\to\infty]{} 1.$$

Proof. From (2.6) one has $\frac{dX(\rho)}{d\rho} > 0$, so $X(\rho)$ is an increasing function. Furthermore, $X(\rho) = 0$ only when $\rho = -1$, that is impossible because $\rho = \xi/\lambda > 0$. Since $X(\rho)$ is monotone, we conclude that its supremum corresponds to $\rho = \infty$.

In Table 1 the asymptotic percentage of ignorants is showed for $N = 50, 100, 1000$ and for varying ρ. In the last row the values of X are listed for $N \to \infty$ (via (2.7)). Note that X tends to 1 when ρ increases and the speed of this growth increases with N.

ρ	0.00	0.01	0.02	0.04	0.06	0.08	0.10	0.30	0.50	0.70	0.90	1.00
$N = 50$	0.20	0.49	0.49	0.50	0.51	0.52	0.53	0.63	0.72	0.80	0.87	0.89
$N = 100$	0.20	0.49	0.50	0.51	0.52	0.53	0.54	0.64	0.73	0.82	0.90	0.92
$N = 1000$	0.20	0.50	0.51	0.52	0.53	0.54	0.55	0.65	0.74	0.84	0.94	0.97
$N \to \infty$	0.20	0.50	0.51	0.52	0.53	0.54	0.55	0.65	0.75	0.85	0.95	1

Table 1: For the model A, the asymptotic percentage of ignorants for different choices of N and for varying ρ.

From (2.6) it follows that

Remark 1. *For $\nu > 1/2$, the density $X(\rho) = \nu$ if and only if*

$$\rho = \frac{N(2\nu^2 - 3\nu + 1) + \nu - 1}{N(\nu - 1) + 1} \xrightarrow[N\to\infty]{} 2\nu - 1.$$

From Remark 1, if $\nu = 0.8$ then $\rho = 0.6$ and the 20% of the population knows the rumor. In Table 2 we list the values of ρ corresponding to the desired percentage of the ignorants $X > 0.5$ for $N = 1000$.

X	0.51	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.99
ρ	0.02	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.98

Table 2: Values of ρ corresponding to the percentage of the ignorants $X > 0.5$ for the DK-m with denials with $N = 1000$.

3. Model B: An alternative model

As before, we consider a population consisting of N individuals, we assume that the rumor spreads by directed contact of the spreaders with others in the population and each individual mets another one with rate $\lambda > 0$. In this model we suppose that the population is divided into $k + 2$ groups for $k = 1, 2, \ldots$: ignorants, spreaders that have spreaded the rumor i times for $i = 1, 2, \ldots, k$ and stiflers. We call the i-th spreader the spreader that has told the rumor i times. The 0-th spreader is the one who has not yet told the rumor. Hence, the 0-th spreader is the initial spreader that starts the diffusion process or the ignorant who has just become a spreader. In this model we suppose that a spreader can spread the rumor only k times when he mets an ignorant; so, when the $(k-1)$-th spreader mets an ignorant he spreads the rumor and then he becomes a stiflers. The contacts are governed by the following set of rules:

- for $i = 0, 1, \ldots, k-2$, when the i-th spreader contacts an ignorant, the ignorant becomes a 0-th spreader and the i-th spreader becomes an $(i+1)$-th spreader;
- for $i = k-1$, when the $(k-1)$-th spreader contacts an ignorant, the ignorant becomes a 0-th spreader and the $(k-1)$-th spreader becomes a stiflers;
- when a spreader of any class contacts another spreader of any group, both become stiflers;
- when a spreader of any class contacts a stifler, the spreader becomes stifler.

Moreover, as before, we assume that denials occur with $\xi > 0$. We denote by $X(t)$, $Y_i(t)$ $(i = 0, 1, \ldots, k-1)$ and $Z(t)$ the densities of population that are ignorants, i-th spreaders $i = 0, 1, \ldots, k-1$ and stiflers at time t, respectively and with $Y(t) = \sum_{i=0}^{k-1} Y_i(t)$ the density of spreaders at time t. The conditions (2.1) and (2.2) hold. Assuming that N is sufficiently large so we can approximate $Y_i(t)\Big[Y_i(t) - \frac{1}{N}\Big] \simeq Y_i^2(t)$ for $i = 0, 1, \ldots, k-1$, the rumor spreading mechanism is described by the following equations:

$$\begin{aligned}
\frac{dX(t)}{dt} &= -\lambda X(t)Y(t) + \xi\Big[1 - \frac{1}{N}\Big] - \xi X(t)\\
\frac{dY_0(t)}{dt} &= \lambda X(t)Y(t) - \lambda Y_0(t) - \xi\Big[Y_0(t) - \frac{1}{N}\Big]\\
\frac{dY_i(t)}{dt} &= \lambda X(t)Y_{i-1}(t) - \lambda Y_i(t) - \xi Y_i(t), \qquad (i = 1, 2, \ldots, k-1)\\
\frac{dZ(t)}{dt} &= \lambda Y(t)\Big[1 - X(t)\Big] + \lambda X(t)Y_{k-1}(t) - \xi Z(t).
\end{aligned} \tag{3.1}$$

In Figures 2 and 3 the densities of the three subpopulations are shown for $N = 1000$, $\lambda = 1$, and various choices of ξ. In particular, in Figure 2 we have set $k = 2$, whereas $k = 4$ in Figure 3. The full, dashed and dot-dashed curves represent the percentages of the ignorants, spreaders and stiflers, respectively. Moreover, for any choices of k the percentage of ignorants increases with ξ; on the other hand, for fixed ξ we note an increasing of the rumor spreading when k grows.

3.1. The steady state analysis

Setting

$$X = \lim_{t\to\infty} X(t), \quad Y_i = \lim_{t\to\infty} Y(t) \quad (i = 0, 1, \ldots, k-1), \quad Y = \sum_{i=0}^{k} Y_i, \quad Z = \lim_{t\to\infty} Z(t),$$

and $\rho = \xi/\lambda$, from (3.1), assuming N sufficiently large so that $1 - 1/N \cong 1$ and recalling (2.1), one obtains

$$-XY + \rho\Big(1 - X\Big) = 0, \qquad XY - Y_0 - \rho Y_0 = 0, \qquad XY_{i-1} - Y_i - \rho Y_i = 0, \qquad (i = 0, 1, \ldots, k-1). \tag{3.2}$$

Proposition 2. *For the Model B, if $\rho > 0$ the steady-state density of the ignorants satisfies the following equation:*

$$(X-1)(X-A) = 0, \tag{3.3}$$

if $k = 1$; whereas, for $k \geq 2$ one has

$$X^{k+1} + (A-1)\sum_{j=0}^{k-2} A^j X^{k-j} - A^{k-1}(A+1)X + A^k = 0, \tag{3.4}$$

with $A = \rho + 1$.

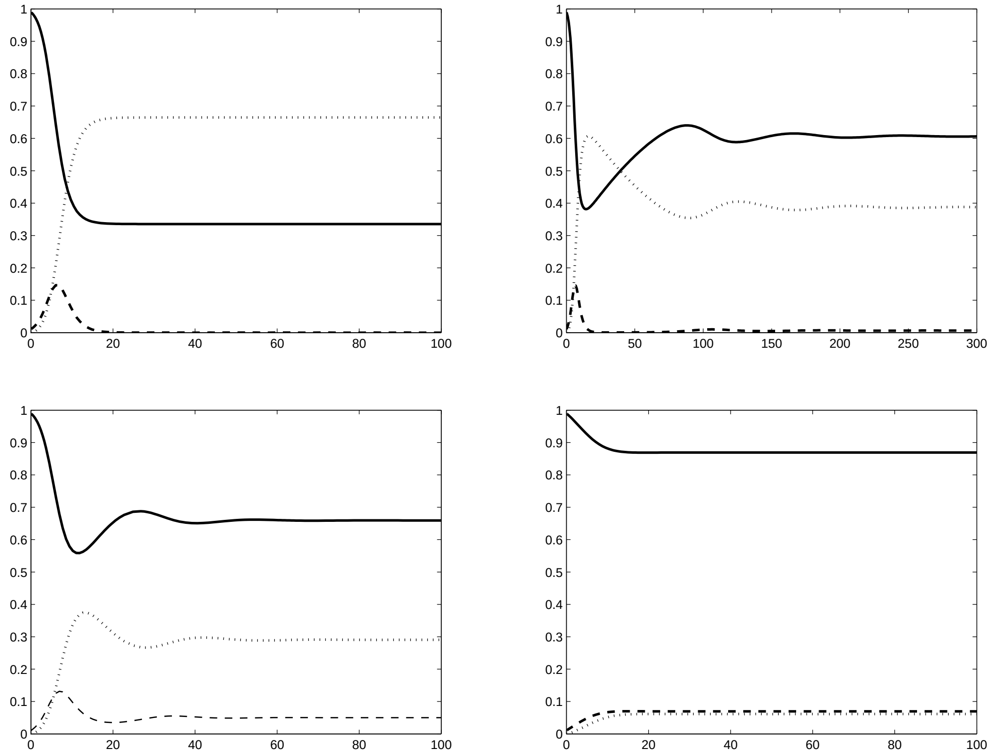

Figure 2: For the model B, X (full curve), Y (dashed curve) and Z (dot-dashed curve) for $k = 2$, $\lambda = 1$ and $\xi = 0,\ 0.01,\ 0.1,\ 0.5$ from left to right.

Proof. By adding the first two equations of (3.2) we get $Y_0 = \rho(1-X)/A$. On the other hand, by solving iteratively the last equation of (3.2) and by using the previous expression of Y_0 one has $Y_i = X^i\rho(1-X)/A^{i+1}$ $(i = 1, 2, \ldots, k-1)$. So

$$Y = \frac{\rho}{A}(1-X)\sum_{i=0}^{k-1}\left(\frac{X}{A}\right)^i = \rho\frac{1-X}{A-X}\left[1-\left(\frac{X}{A}\right)^k\right]. \tag{3.5}$$

By substituting (3.5) in the first equation of (3.2) it results

$$-X(1-X)\sum_{i=0}^{k-1}X^{k-1-i}A^i + A^k\left(1-X-\frac{1}{N}\right) = 0. \tag{3.6}$$

After some computation, (3.6) becomes (3.3) for $k = 1$ and (3.4) otherwise.

The solutions of (3.3) are simple to obtain, they are 1 and A. Since we are interested into solution less than 1, if $1 \simeq 1-1/N$, we conclude that $X = 1$ is the acceptable solution. Then if $k = 1$ and N is sufficiently large, the rumor does not spread asymptotically so the system remains in the initial condition.
Now, we focus on (3.4). Note that Eq. (3.4) can be written as $\tilde{f}_k(x) = (x-1)f_k(x) = 0$, where

$$f_k(x) = x^k + Ax^{k-1} + A^2x^{k-2} + \ldots + A^{k-1}x - A^k. \tag{3.7}$$

Hence, a solution of (3.4) is $X = 1$, the remaining solutions coincide with the zeros of $f_k(x)$. These zeros aren't computable explicitly, but we can determine the range in which the solution of our interest is. Indeed

Proposition 3. *The equation $f_k(x) = 0$ has a unique real solution in the interval* $(0,\ A)$.

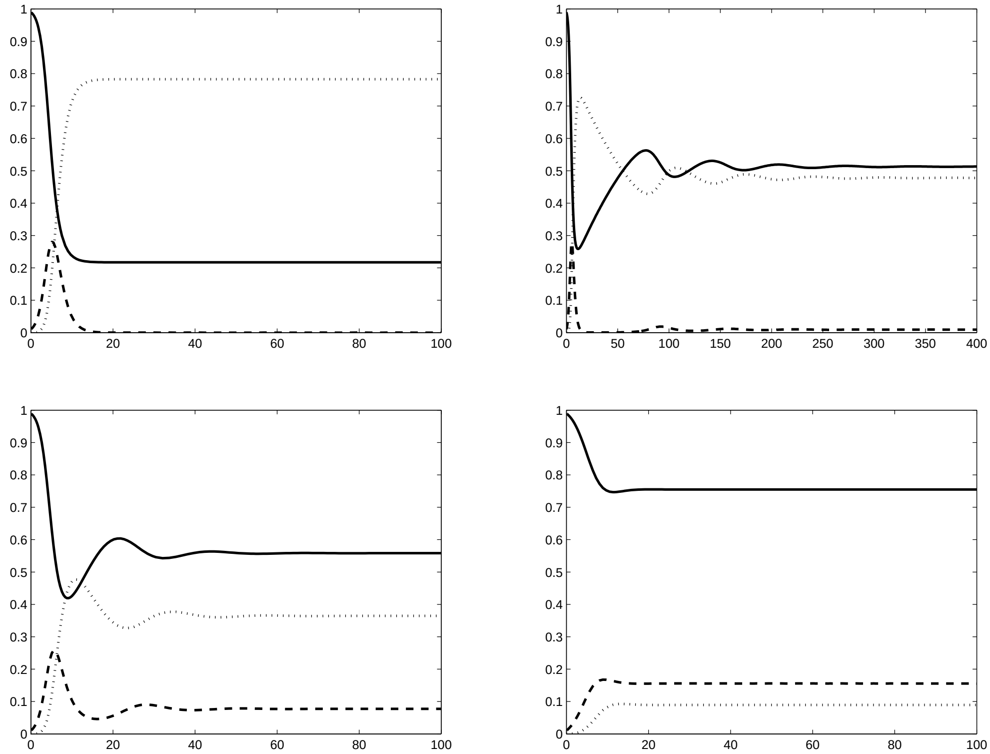

Figure 3: As in Figure 2 with $k = 4$.

Proof. Noting that for $k \geq 2$, $f_k(0) = -A^k < 0$ and $f_k(A) = (k-1)A^k > 0$, we obtain that $f_k(x) = 0$ has at least one real root in $(0, A)$. Moreover, there is a unique real solution in $(0, A)$ because $f_k(x)$ is a continuous increasing function being $f_k'(x) > 0$.

Let x_k be the unique real solution of $f_k(x) = 0$ in the interval $(0, A)$. We write $x_k = d_k A$, with $0 < d_k < 1$.

Proposition 4. *$\{d_k\}_{k\geq 2}$ is a decreasing succession. Moreover, $d_2 = (\sqrt{5}-1)/2$ and $d_\infty = \lim_{k\to\infty} d_k = 0.5$.*

Proof. By recalling that $A > 1$, for $x < A$ one has:

$$f_k(x) - f_{k+1}(x) > \sum_{i=0}^{k-1} x^{k-1}A^i - \sum_{i=1}^{k} x^{k+1-i}A^i - x^{k+1} = (1-A)x\frac{x^k - A^k}{x - A} - x^{k+1} > \frac{x^{k+1}}{x-A}(1-x) > \frac{x^{k+1}}{x-A}(1-A) > 0.$$

Hence $\{x_k\}_{k\geq 2}$ is a decreasing succession and consequently $\{d_k\}_{k\geq 2}$ is decreasing too. Note that $f_2(x) = 0$ if and only if $x^2 + Ax - A^2 = 0$, so $d_2 = (\sqrt{5}-1)/2$. Moreover, because $\lim_{k\to\infty} f_k(d_k A) = 0$ if and only if $\sum_{j=1}^{\infty} d_\infty^j - 1 = 0$, we have $d_\infty = 0.5$.

Proposition 5. *For $k \geq 2$, if $\rho < 1/d_2 - 1$ the polynomial $f_k(x)$ has always a unique real zero less than* 1.

Proof. Let $k \geq 2$, in order that $f_k(x) = 0$ has always a unique real root less than 1, we need that $A = 1 + \rho < \frac{1}{d_k}$, i.e. $0 < \rho < \frac{1}{d_k} - 1$.The thesis follows recalling Proposition 4.

When $f_k(x)$ has a zero less than 1, the solution of $f_k(x) = 0$ represents the percentage of ignorants X. Otherwise, when all the solutions of $f_k(x) = 0$ are greater than 1, the rumor does not spread.

In Table 3, for $\rho = 0.5$ and some choices of k, we show the values of d_k, of $1/d_k - 1$ and the final proportion of ignorants X in the population. In the second row the values of d_k are listed, such values are decreasing and they converge to 0.5, as showed in Proposition 4. The third row shows the values $1/d_k - 1$, note that, since $\rho = 0.5 < 1/d_k - 1$, it follows that $X < 1$ as listed in the fourth row. Moreover, X decreases as k increases.

In Table 4 we list the proportion of ignorants for the the model B for various choices of ρ and k. Note that the values of X decrease as k increases because the rumor has more chance to spread. Moreover, as ρ increases X, grows to 1 and, for $\rho \geq 1$ the rumor does not spread at all. Fixed $\rho > 0$ the values for $k = 100$ coincide with the corresponding values of Model A listed in Table 1. In particular, one has that for $k \geq 6$ the percentage of the ignorants reaches about the same value obtained for $k \to \infty$ confirming the theory of "six degrees of separation". According to this theory, everyone and everything is six, or fewer, steps away from any other person in the world, so that a chain of "a friend of a friend" statements can be made to connect any two people in a maximum of six steps. Therefore, to spread as much as possible the rumor it is advisable to choice $k \geq 6$ and $\rho \leq 0.1$. In these cases at most 50% of the population knows the rumor.

k	2	3	4	5	6	7	8	9	10	...	100
d_k	0.6180	0.5436	0.5187	0.5086	0.5041	0.5020	0.5009	0.5004	0.5002	...	0.5
$1/d_k - 1$	0.6180	0.8392	0.9275	0.9659	0.9835	0.9919	0.9960	0.9980	0.9990	...	1
X	0.9270	0.8155	0.7781	0.7629	0.7562	0.7530	0.7514	0.7507	0.7503	...	0.75

Table 3: For $\rho = 0.5$ and some choices of k, the values of d_k, $1/d_k - 1$ and X are listed.

ρ	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 7$	...	$k = 10$	...	$k = 100$
0.00	0.618034	0.543689	0.518790	0.508660	0.504138	0.502017	...	0.500245	...	0.5
0.01	0.624214	0.549126	0.523978	0.513747	0.509180	0.507037	...	0.505248	...	0.505
0.02	0.630395	0.554563	0.529166	0.518834	0.514221	0.512057	...	0.510250	...	0.51
0.04	0.642755	0.565437	0.539542	0.529007	0.524304	0.522098	...	0.520255	...	0.52
0.06	0.655116	0.576310	0.549917	0.539180	0.534387	0.532138	...	0.530260	...	0.53
0.08	0.667477	0.587184	0.560293	0.549353	0.544469	0.542178	...	0.540265	...	0.54
0.10	0.679837	0.598058	0.570669	0.559526	0.554552	0.552219	...	0.550270	...	0.55
0.30	0.803444	0.706796	0.674427	0.661259	0.655380	0.652622	...	0.650319	...	0.65
0.50	0.927051	0.815534	0.778185	0.762991	0.756207	0.753026	...	0.750368	...	0.75
0.70	1	0.924271	0.881943	0.864723	0.857035	0.853429	...	0.850417	...	0.85
0.90	1	1	0.985701	0.966455	0.957863	0.953832	...	0.950466	...	0.95
1.00	1	1	1	1	1	1	...	1	...	1

Table 4: For some choices of ρ and k, the values of X are listed.

4. Conclusions

In this paper we introduce the effect of denials on two rumor spreading processes, based on epidemiological models. The denials re-set to start the initial situation in which there is only one spreader and the rest of the population is ignorant. We consider the well-known DK model with denials, and an alternative model (model B), in which denails occur and each spreader can transmitt the rumor at most k times. For both models we study the steady state densities and we focus on the asymptotic percentage of ignorants to identify the density of the population that knows the rumor. A scrutinized numerical analysis is performed to study the effect of denials on varying parameters and to compare the proposed models. We note that in both cases the asymptotic percentage of ignorants increases when the rate of the denials ξ grows respect to the rate of the contacts λ; in particular, if the size of the population is large and $\xi \geq \lambda$, the rumor does not spread at all. For the model B the density of individuals that knows the rumor increase with k, since the rumor has more chance to spread. Moreover, the model B behaves like the DK-m with denials when k increases, in particular a good match is found already for $k = 6$, confirming "six degrees of separation" theory. Finally, in both models we obtain that at most the half of the population can be informed about the rumor.

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