Choosing Bootstrap Method for the Estimation of the Uncertainty of Traffic Noise Measurements

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Abstract—The environmental acoustic noise is considered as a big risk for today's population health. Consequently, the regulations in many countries commit themselves to control the exposition of people, imposing limits to the noise level. In the comparison between the measured value and the threshold, the uncertainty of the measured value has to be taken into account. In this paper, a procedure for the evaluation of the uncertainty of traffic noise measurements due to the variability of the measurand is proposed. A deep analysis of five bootstrap (normal, basic percentile, t-Student, bias corrected percentile, bias corrected and accelerated percentile) methods is performed to obtain accurate confidence intervals for the indicator $L_{eq,A}$ without necessity to make normal theory assumptions. From the comparison with the classical method (according to ISO GUM), the novel approach reveals to be more effective for estimating both the expected value and the uncertainty of the short term equivalent sound pressure level when a large data set is not available.

Keywords— traffic noise measurements, measurement uncertainty, statistical analysis, bootstrap method

I. INTRODUCTION

The World Health Organisation identified the noise as the most significant risk to the health of today's population in relation to the number of people exposed [1]. Many studies show that high sound pressure levels can damage human health in different ways (hearing, communication capabilities, behaviour) and so today there is a large interest in activities aimed at controlling noise. In all these cases, the main reference is the A-weighting equivalent level of environmental noise:

$$L_{eq,A} = 10\,\mathrm{Log}\,\frac{1}{T}\int_T \left[\frac{p_A(t)}{p_{rif}}\right]^2 \mathrm{d}t$$

which indicates the level of a continuous stationary noise having the same acoustic energy content of the floating noise under measurement compared to the average sensitivity curve in terms of frequency of the human auditory system. The comparison between this value and the limits stated in the legislation is a problem, because the difference between a measurable value and a threshold does not involve a straightforward comparison of values, given that such measurement results have to be expressed [2] in a manner consistent with the principle of the ISO GUM [3]. In other words, the uncertainty of the measured value has to be taken into account [4]-[8], also in order to obtain an assessment of the risk associated with the decision taken. The task of establishing the decision-making rules to test the compliance of a product to specifications, taking into account the uncertainty of the measurement, relies on [2], [9] and [10]. Both show that the higher the uncertainty of measurement, the smaller the degree of compliance, but neither shows how to assess the level of confidence in the outcome of the comparison, therefore resulting in heightened levels of risk that a wrong decision is being made [2]. Sometimes the uncertainty of acoustic noise measurement may be greater than the reference threshold stated by the standard for that source. For this reason, nowadays the determination of the uncertainty in the measurement of environmental acoustic noise is a very sensitive subject. Laboratories wishing to be accredited according to the ISO 17025 standard must be able to evaluate the uncertainties in measurement. However, reference standards and guidelines for the estimation of uncertainty in environmental acoustics are not always rigorous. Information on uncertainty is increasingly requested by customers, especially by residents in the case of noise monitoring around roads, railways and airports.

For the determination of the uncertainty in the measurement of acoustic pressure levels, several sources of uncertainty can be considered. C. Rosin in [11] with Ishikawa diagrams allocates the uncertainties of the various parameters into five families according to their sources:

- Method of measurement: calibration, A-weighted filter, tolerance
- Operator: procedures, skill
- Machine: linearity, drift and resolution of equipment, frequency response and bandwidth
- Environment: temperature, humidity, electromagnetic field sensitivity, pressure
- Measurand: the sound

Applying the ISO GUM, the uncertainty estimation of a sound level meter class 1 involves consideration of the technical specifications of the instrumentation and of the technical standards on electro-acoustics. For example, about a generic stationary outdoor source, the deviation from the nominal value ($u_{instrum}$), which takes into account inherent uncertainties contributions (i.e. temperature, humidity, pressure, linearity, A weighting curve, microphone isotropy), was estimated about 0.49 dB [12]. Further uncorrelated components are represented by the standard uncertainty associated to the distance between source and receiver (u_{dist}), the standard uncertainty associated to the distance of microphone from reflective walls (u_{refl}), the standard uncertainty associated to the height of the microphone above the ground (u_{height}).

However, to provide an adequate estimation of total uncertainty associated with the measurement of the equivalent level of environmental noise, the intrinsic variability of the measurand ($u_{measurand}$) cannot be ignored. In [13] the authors focused on the variability of the acoustic phenomenon, as further component to be added to the above-mentioned instrumental, methodological and operating uncertainties.

In order to choose the most effective way to evaluate the uncertainty component $u_{measurand}$ in environmental acoustic noise measurements, the present work focuses on the comparison of five non classical models for constructing confidence intervals based on the bootstrap resampling method [14]. The bootstrap is a computationally intensive statistical technique that allows one to make inferences from data without making strong distributional assumptions about the statistic that is calculated and/or the data [15].

The paper is organized as follows: in Section II, an overview of the main proposals for the uncertainty estimation of traffic noise is reported together with a brief description of the bootstrap techniques. Section III describes the application of the novel approach to measurement data from an acquisition campaign, once a suitable outlier detection algorithm ([16],[17]) has been performed. Finally, in Section IV the main remarkable results are summarized.

II. UNCERTAINTY ESTIMATION: STATE OF ART

For the uncertainty estimation of environmental acoustic noise measurements, it is usually assumed that results are associated to well-known distributions.

Some authors, such as M. Paviotti and S. Kephalopoulos in [18], performed studies about noise measurement uncertainty, focusing on road traffic data and estimating long-term environmental noise indicator (urban road with 25000 vehicles pass-bys on average per day). The main assumption of the analysis is the Gaussian model for the originating population.

T. Wszolek and M. Klaczynski in [19] further explored the theme examining real statistical distributions of 24-h traffic noise levels registered in 55 reference points. From measurements and analysis the authors, using the Lilliefors test and the Kolmogorov-Smirnov test, showed that for the significance level $\alpha=0.05$ none of the acoustic data series obtained from measuring reference points, neither for day-time nor night-time, exhibits characteristics of the normal distribution. Additionally they verified that the measured distributions are not related to any statistical distribution known in the literature. Based on the completed variance analysis, nevertheless, the observed distributions tend to Gaussian distributions both for night-time and day-time data with the increasing traffic intensity observed in a given measurement section. In other words, even if the partial results used for estimation of the uncertainty value have been attributed statistical distributions that are not normal, then the distribution of the resultant variable still tends to a normal distribution.

Such hypothesis raises some doubts or even criticism [20]-[22]. A different approach, proposed in [23], consists in analyzing the uncertainty of the long-term noise indicators L_{DEN} and L_N with the bootstrap method, which does not have limitations in terms of form and properties of considered statistics. These indicators are related to noise annoyance values of the long-term average sound level A in the day-evening-night periods (L_{DEN}) and night periods (L_N) in dB.

$$L_{DEN} = 10\log\left[\frac{1}{365}\sum_{i=1}^{365} 10^{0,1*L_{DEN,i}}\right]$$

$$L_N = 10\log\left[\frac{1}{365}\sum_{i=1}^{365} 10^{0,1*L_{N,i}}\right]$$

with:

$$L_{DEN,i} = 10\log\left(\frac{1}{24}\left(12 * 10^{0,1*L_{D,i}} + 4 * 10^{0,1*(L_{E,i+5})} + 8 * 10^{0,1*(L_{N,i}+10)}\right)\right)$$

$$L_{N,i} = 10\log\left[\frac{1}{K}\sum_{i=1}^{K} 10^{0,1*(L_{Aeq,T})_i}\right]$$

where K is the sample size, $(L_{Aeq,T})_i$ is the equivalent sound level for the i^{th} sample, in [dB], $L_{D,i}$ is the day sound A level, determined from the day–time noise exposure i.e. from 6:00 a.m. to 6:00 p.m., in [dB], $L_{E,i}$ is the evening sound A level, determined from the noise exposures from 6:00 p.m. to 10:00 p.m., in [dB], and $L_{N,i}$ is the night sound A level, determined for the night periods i.e. from 10:00 p.m. to 6:00 a.m., in [dB].

In particular, W. Batko and B. Stepien in [24] proposed the bootstrap method because it is very difficult to know the probability distribution of the long-term indicators of the noise in the environment. It has a wide applicability and can be very useful in many cases in order to get estimations, which can be difficult to be obtained with other methods. The main idea is that a number of new data sets, which are referred to as bootstrap samples, can be generated from the initial data set by sampling with replacement. With this resampling scheme, these distributions can be seen as approximations to the true distributions of the estimators, and then a good estimate can be obtained of the distribution of a statistics of interest, such as bias, standard deviation and so on. Given a one-dimensional random variable X and a sample (x_1, x_2, ..., x_n) which is a realization of $X = \{X_i, i = 1,2,\dots,n\}$, the hypothesis of this method is that the probability density function F of X is unknown. Termed $R(x, F)$ a certain statistics determined on the sample space, the basic bootstrap procedure has these standard steps:

1. designing the probability distribution by means of the following function

$$\mathrm{P}\ (X_B\ = x_K) = \frac{1}{n}$$

for $k = 1,2,\dots,n$

called the bootstrap distribution from sample and denoted by $\hat{F}$, where n is the sample size;

2. sampling, independently, according to the distribution of values $(x_1^*, x_2^* \dots . x_n^*)$, which are treated as the realisation of variable $x^* = (X_1^*, X_2^* \dots . X_n^*)$ and it is called the bootstrap sample;

3. distribution of statistics R is approximated by means of the bootstrap distribution: $\vartheta^* = R\left(x^*; \hat{F}\right)$.

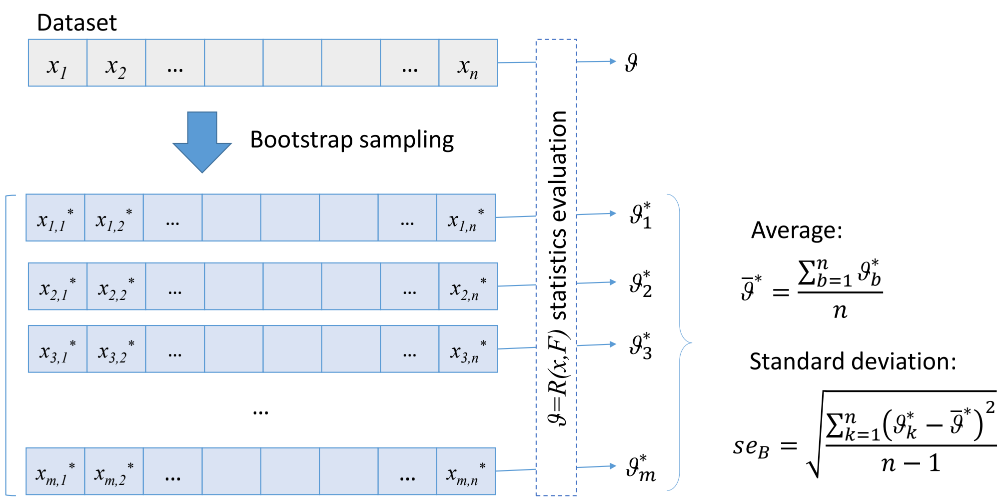


Fig. 1: Algorithm of bootstrap method

The distribution of variable ϑ^* is approximated by means the Monte Carlo method. The histogram of statistics ϑ^* is determined as a function of the bootstrap samples repeated m times. The algorithm of the estimation s_{eB} of the expected value and the uncertainty of the long-term noise indicators performed by the bootstrap method is shown in Fig. 1. The values of the long-term noise indicators determined by the bootstrap method are more accurate because uncertainties estimated by the bootstrap method are smaller than the ones obtained by the classical estimation method. In [24] the simulation experiment of the determination of the expected value and the uncertainty was carried out by means of two methods, classical and bootstrap. The obtained results show that: i) the bootstrap estimates have smaller errors (the difference between the measured value and the estimated one was smaller); ii) the uncertainty determined by the bootstrap method has a smaller value. For all these reasons, the bootstrap method for the estimation of the uncertainty as well as the expected value of the noise indicators is a promising calculation tool.

The most important application field of bootstrap method is the construction of confidence intervals [25]. In literature there are various bootstrap methods for the determination of these intervals, the main ones are:

- normal bootstrap method (NORM), which approximates confidence interval with bootstrapped bias and standard error [15];

- basic percentile bootstrap method (PER), which proceeds in a similar way to the normal bootstrap one, using percentiles of the bootstrap distribution. In this case the confidence interval is invariant for monotonic trasformations and is range-preserving, that is its endpoints always fall within the range of variation of the parameter of interest. Confidence interval endpoints for the desired confidence level *1-α* are $\left[\vartheta - \frac{\vartheta_{1-\frac{\alpha}{2}} \cdot se}{\sqrt{n}}, \vartheta - \frac{\vartheta_{\alpha/2} \cdot se}{\sqrt{n}}\right]$ [15], [26], [27];
- bias corrected percentile bootstrap method (CPER), which has a weaker assumption and allows the mean of the transformed estimate to differ from the population mean. CPER interval endpoints depend on one number z_0 called the bias correction, that is calculated from the bootstrap sampling distribution [15], [26], [27];

- bias corrected and accelerated percentile bootstrap method (BCA), which is based on knowledge of percentiles of the bootstrap distribution. BCA interval endpoints depend on two numbers: the acceleration and the bias correction [27];

- bootstrap-t method (STUD), which is a more general version of the bootstrap, [28]-[30], uses a Student's t statistic. In Fig. 2, a flow chart of the bootstrap-t method is shown. The basic principle of this extension of the bootstrap method is the construction of the confidence intervals by using t-student statistics. For the each bootstrap i-th resample, a second level bootstrap gives h new samples and the statistics $t_i^* = (\vartheta_i^* - \vartheta)/se_{i*}$ is calculated by bootstrapping the resample, where ϑ_i^* is the estimate from the i-th bootstrap resample and se_{i*} is an estimate for standard deviation of the same resample. The percentiles $t_{1-\alpha/2}^*$ and $t_{\alpha/2}^*$ are used in order to approximate the confidence interval for the desired confidence level $1\text{-}\alpha$, equal to $[\vartheta - t_{1-\alpha/2}^* \cdot se,\ \vartheta - t_{\alpha/2}^* \cdot se]$, where ϑ is the estimation on the original dataset and se is the standard deviation of the original dataset.

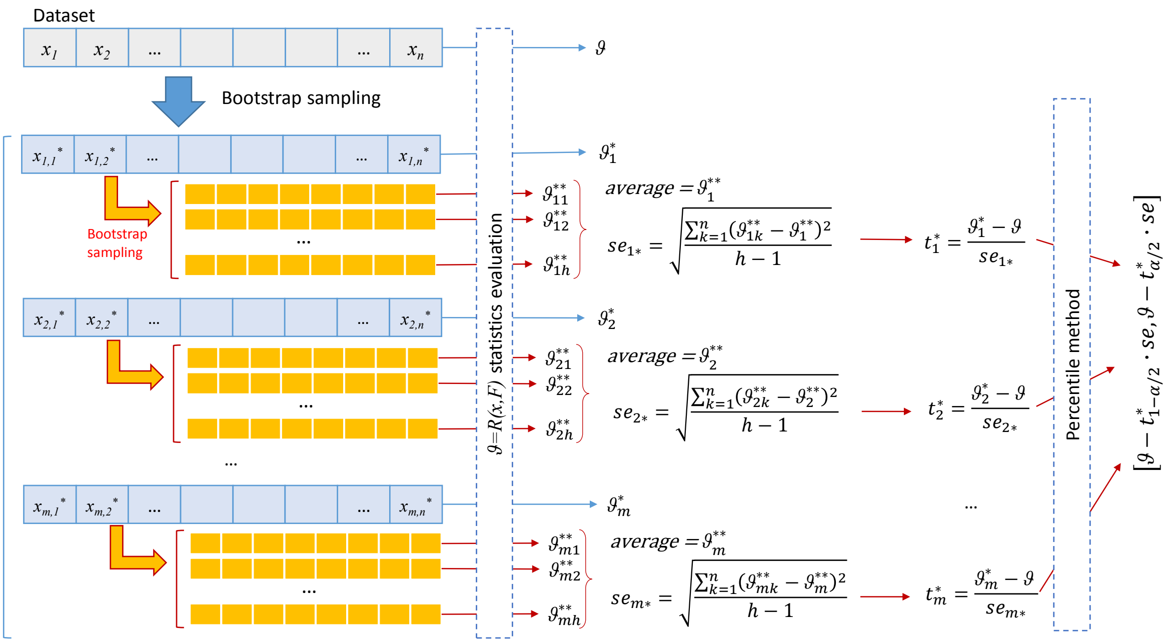


Fig. 2: The bootstrap-t method

III. COMPARISON OF BOOTSTRAP METHODS

The standard uncertainty has to be evaluated as a standard deviation of the probability density function (PDF) attributed to the measurand [3]. In the practice, such a PDF is determined on a finite number of observation, collected in a sample. Besides, in the area of traffic noise measurements it could be very useful to estimate the uncertainty to be attributed to long time periods by acquiring and processing only time windows of reduced duration. The authors, to choose the most effective method in order to evaluate the uncertainty interval, have performed a comparison of five bootstrap techniques reported in Section II: NORM, PER, CPER, BCA, STUD. The analysis was carried out on measurements of traffic noise: the Sound Level Meter (Larson Davis 831, Class 1 Environmental Noise & Bulding Acoustics Analyzer) was placed in proximity of motorway (at a distance of 1 m from the road and a height of 4 m from the ground). Data collection was performed during one day (wind lower than 5 m/second and witouth rain) by considering the time interval from 9 a.m. to 12 p.m as observation time window. About the instrument setting, the time constant and the measurement time have been considered equal to 1 second and 15 minutes respectively. More in details, a set of 16 acquisitions has been collected, with each acquisition block including 900 equivalent noise levels $L_{eq,Ai}$. In this Section, at first, the impossibility that a definite PDF could be attribute to traffic noise data is documented with tests on real data.

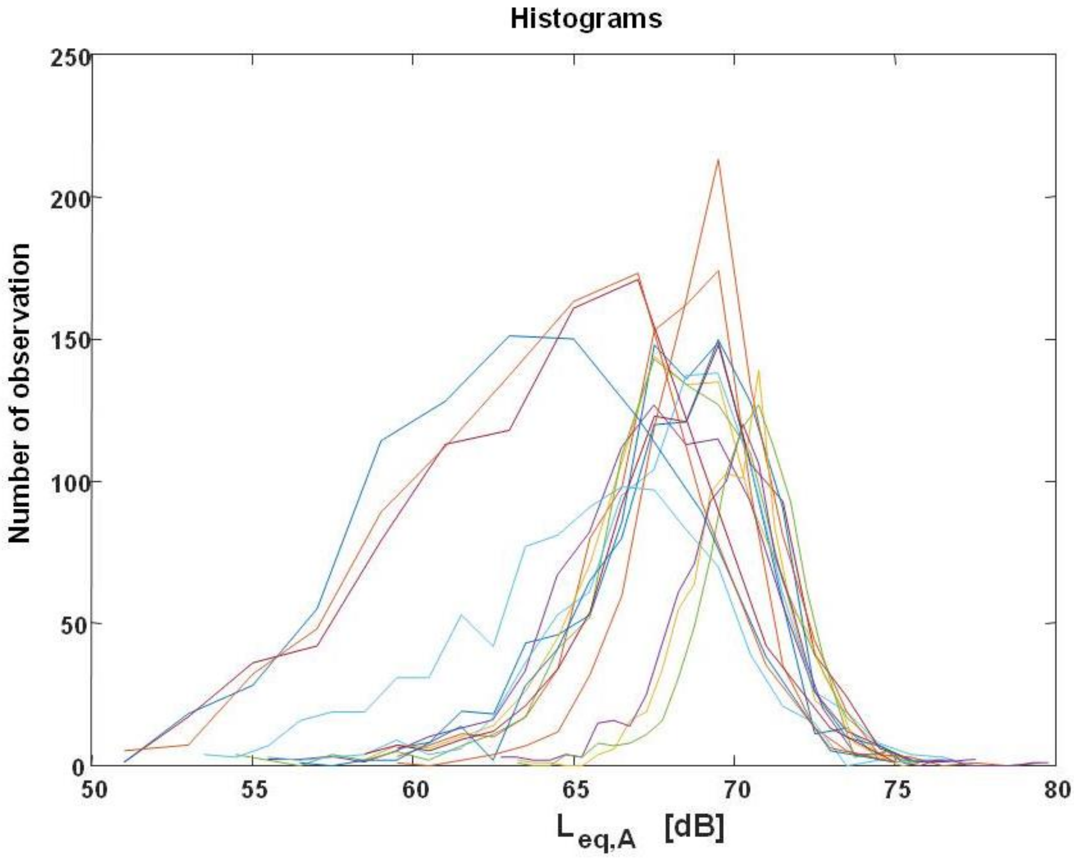
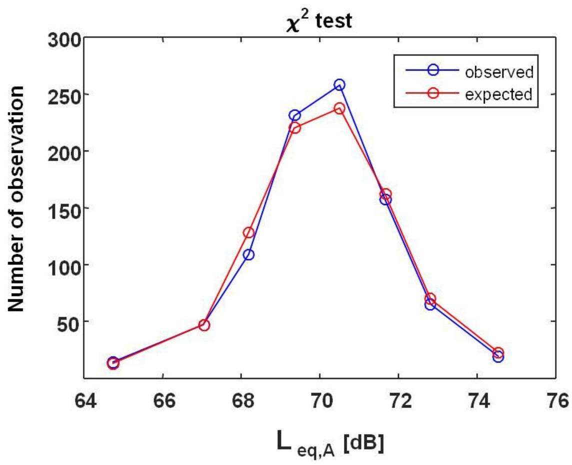
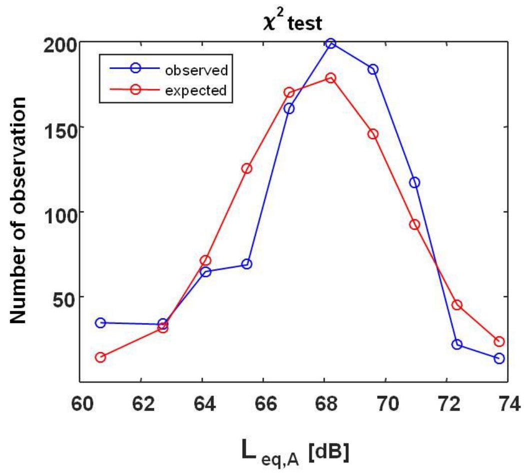


Fig. 3: Comparison between histograms of the 16 Leq acquisitions

In order to characterize statistically the distribution of data, the Matlab environment was used. For each sequence, the histogram of the equivalent noise levels is determined in a first step (Fig. 3) and then the χ^2 tests (Fig. 4) are applied on these noise level sequences, in order to verify if a Gaussian model can be hypothesized for the originating population For the χ^2 tests, the significance level α has been changed between 0 and 1, being α the probability of rejecting the null hypothesis when it is true. As a result, for all reasonable values of α, all sequences except the 10th turned out not to be Gaussian (Figs. 4a, 4b).



a)



b)

Fig. 4: Comparison between observed distribution and expected Gaussian distribution in sequences no. 10 (a) and no. 1 (b).

For each test, the level of statistical significance γ was also determined. Chosen an expected statistical distribution (Gaussian in the present case), the γ value is the probability of observing the given sample or one which is in worse agreement with the expected distribution, given that the null hypothesis is true. Then, the smaller the γ value, the worse the agreement with the expected distribution. From Fig. 5, it is easy to see that only the 10th sequence of data can be considered Gaussian, since the γ value is close to 1: Extremely low values of γ strongly indicate that those datasets are not Gaussian, since when the γ value is less than a predetermined significance level (e.g. 0.05), it means that the null hypotheses (H_0: "dataset is Gaussian") is rejected.

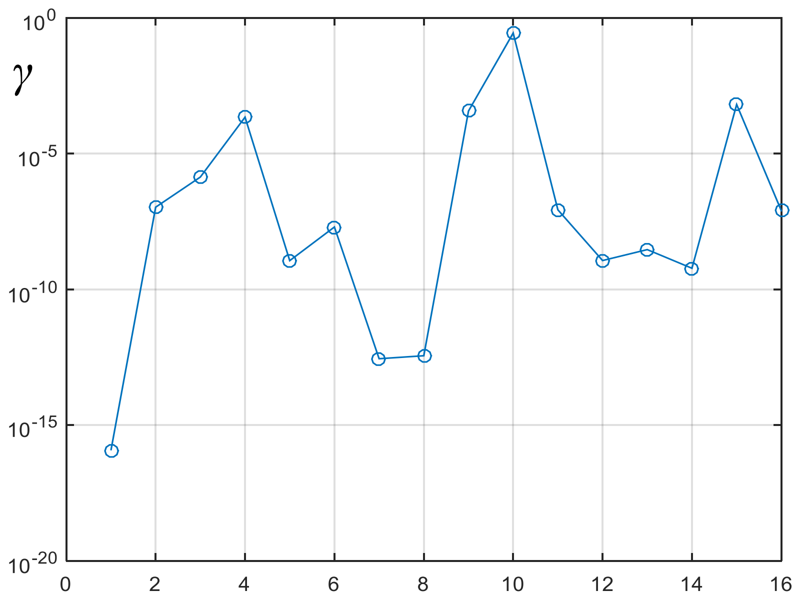
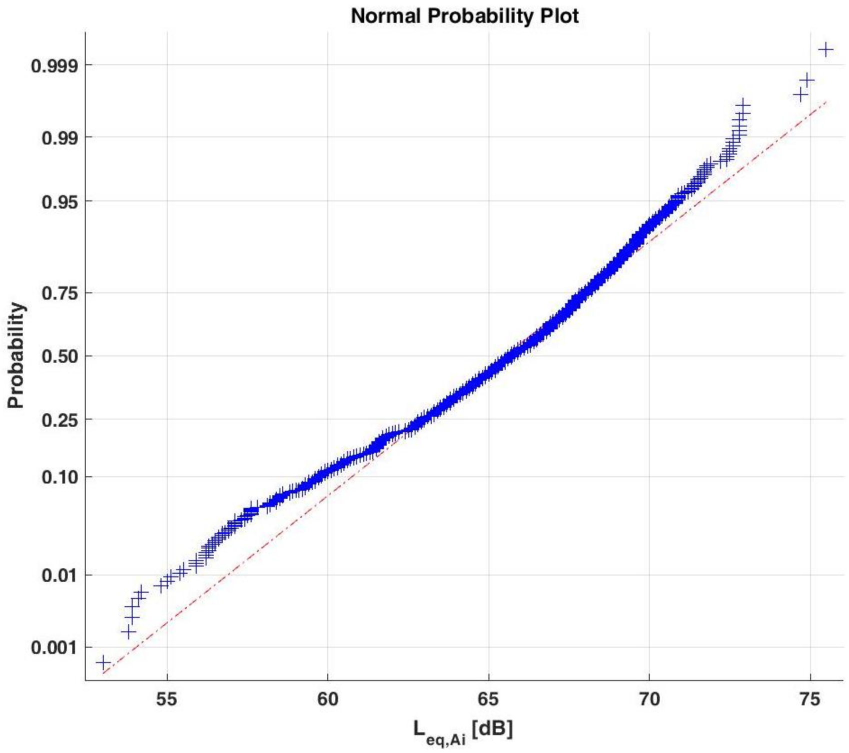
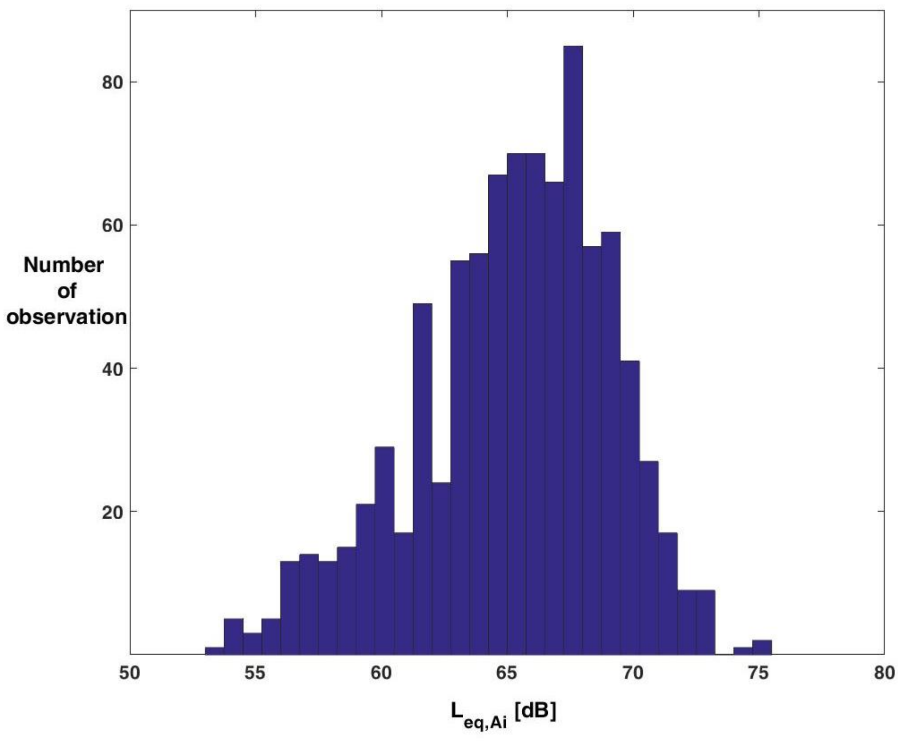


Fig. 5: Levels of statistical significance γ of all 16 acquisitions

In the following, a sequence of non gaussian data has been considered to carry out the comparison of the bootstrap methods previously introduced in terms of the 95% confidence intervals for $L_{eq,A}$. In details, in Figs. 6a, 6b the normal plot and histogram of the corresponding $L_{eq,Ai}$ are reported for the 13th sequence.



a)



b)

Fig. 6: Measured $L_{eq,Ai}$ for 13th sequence: a) Normal Probability Plot; b) Histogram of occurrences

One thousand (1000) random samples of size n = 60 have been extracted from the considered data; the sample size has been chosen with the intention to study the influence on the estimated $L_{eq,A}$ of the population (13th sequence) from a reduced time window.

The reconstruction of the probability density function of the noise indicator has been performed separately on the basis of each sample, by considering three different values for the number of bootstrap replications (m = 250, m = 500, m = 1000). In other words, 1000 bootstrap distributions have been obtained for each value of m. From these bootstrap distributions 95% confidence intervals have been calculated for each of the 5 presented bootstrap methods. Then, focus has been devoted to the widths of estimated confidence intervals (the difference between the upper and the lower confidence limits).

Thus, 1000-element distributions of 95% confidence interval widths of noise indicator $L_{eq,A}$ have been obtained for each combination of bootstrap model and number of replication. Fig. 7 shows all the obtained distributions in terms of the corresponding histograms (30 class bins have been considered), whereas the statistical parameters (minimum, maximum, mean, median, and standard deviation) are reported in Table I.

Table I. Statistical parameters of 95% confidence intervals widths for different bootstrap methods

	Parameter	Unit	NORM	PER	STUD	CPER	BCA
m = 250	**min**	[dB]	1.20	1.18	1.04	1.06	1.07
	mean		1.78	1.78	1.90	1.77	1.79
	median		1.75	1.75	1.84	1.74	1.74
	max		2.90	2.87	3.74	2.67	3.22
	std		0.26	0.27	0.39	0.26	0.30
m = 500	**min**	[dB]	1.12	1.22	1.01	1.17	1.15
	mean		1.79	1.79	1.93	1.78	1.80
	median		1.75	1.74	1.84	1.74	1.75
	max		2.80	2.87	4.45	2.87	2.92
	std		0.26	0.27	0.38	0.26	0.29
m = 1000	**min**	[dB]	1.11	1.11	1.17	1.17	1.11
	mean		1.79	1.79	1.91	1.79	1.80
	median		1.75	1.75	1.84	1.75	1.75
	max		2.84	2.84	3.69	2.69	2.81
	std		0.26	0.26	0.35	0.26	0.29

It can be noted that the examined distributions resulting from the application of all the bootstrap methods have statistical parameters very close to each other. Only a slightly greater standard deviation is observed for the distribution computed with STUD method. Thus, a further statistical analysis has been carried out in order to exclude the revealed differences are statistically significant.

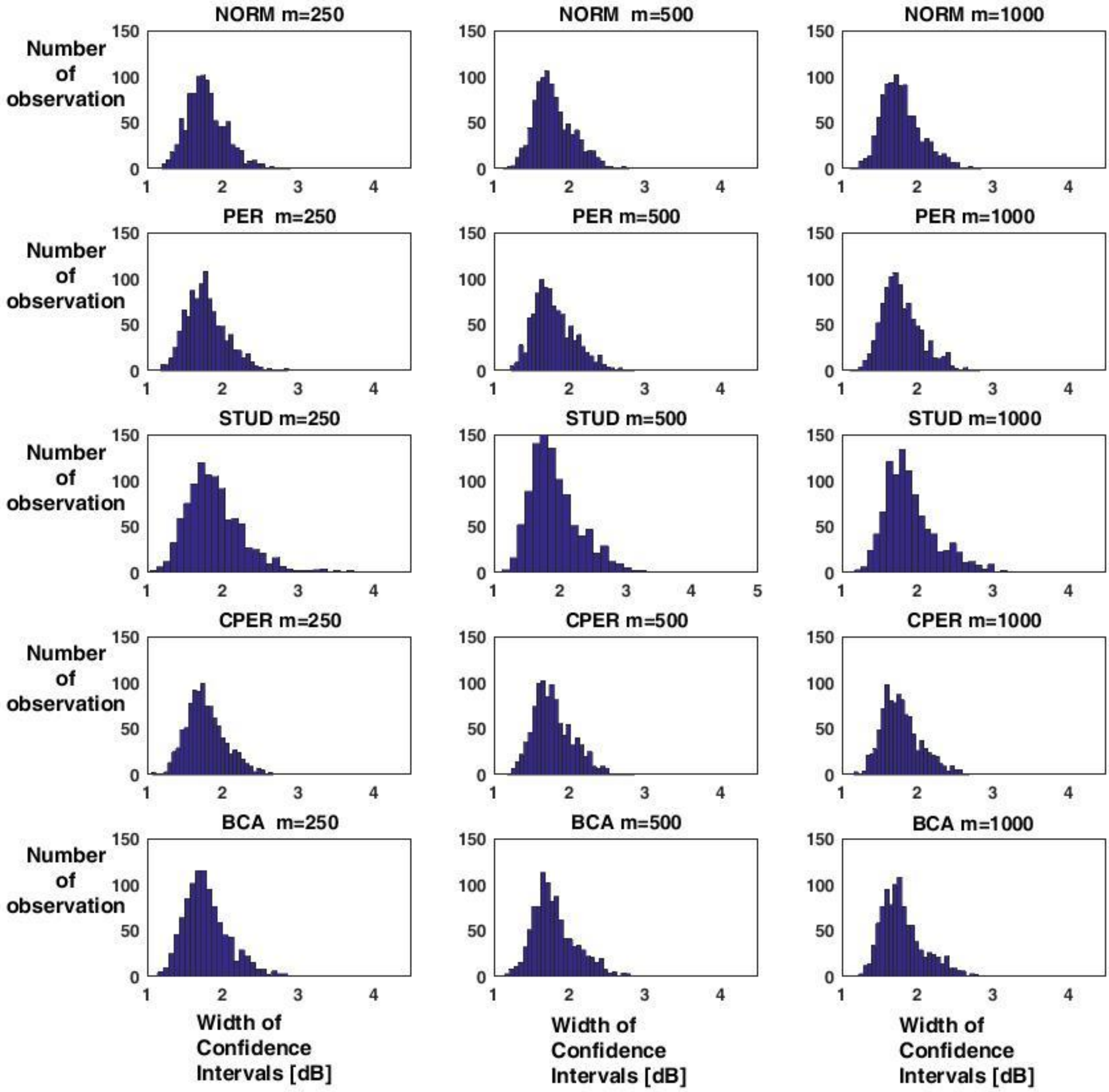


Fig. 7: Histograms of 95% Confidence Interval widths (based on 1000 repetitions)

Firstly, the Kruskal-Wallis non-parametric test has been performed at confidence level α= 0:05; the obtained probability value (p= $5.8 \cdot 10^{-10}$) indicates that there are statistically significant differences between the bootstrap techniques. Then, the Tukey-Kramer non-parametric test has been performed at confidence level α= 0:05; the corresponding result are reported in terms of the graphical representation in Fig. 8. The graphs show the average value of rank together with the confidence level for each of the models. Any two compared group averages are statistically different when their intervals are disjoint. Overlapping intervals indicate that there are no statistically significant differences between the compared group averages.

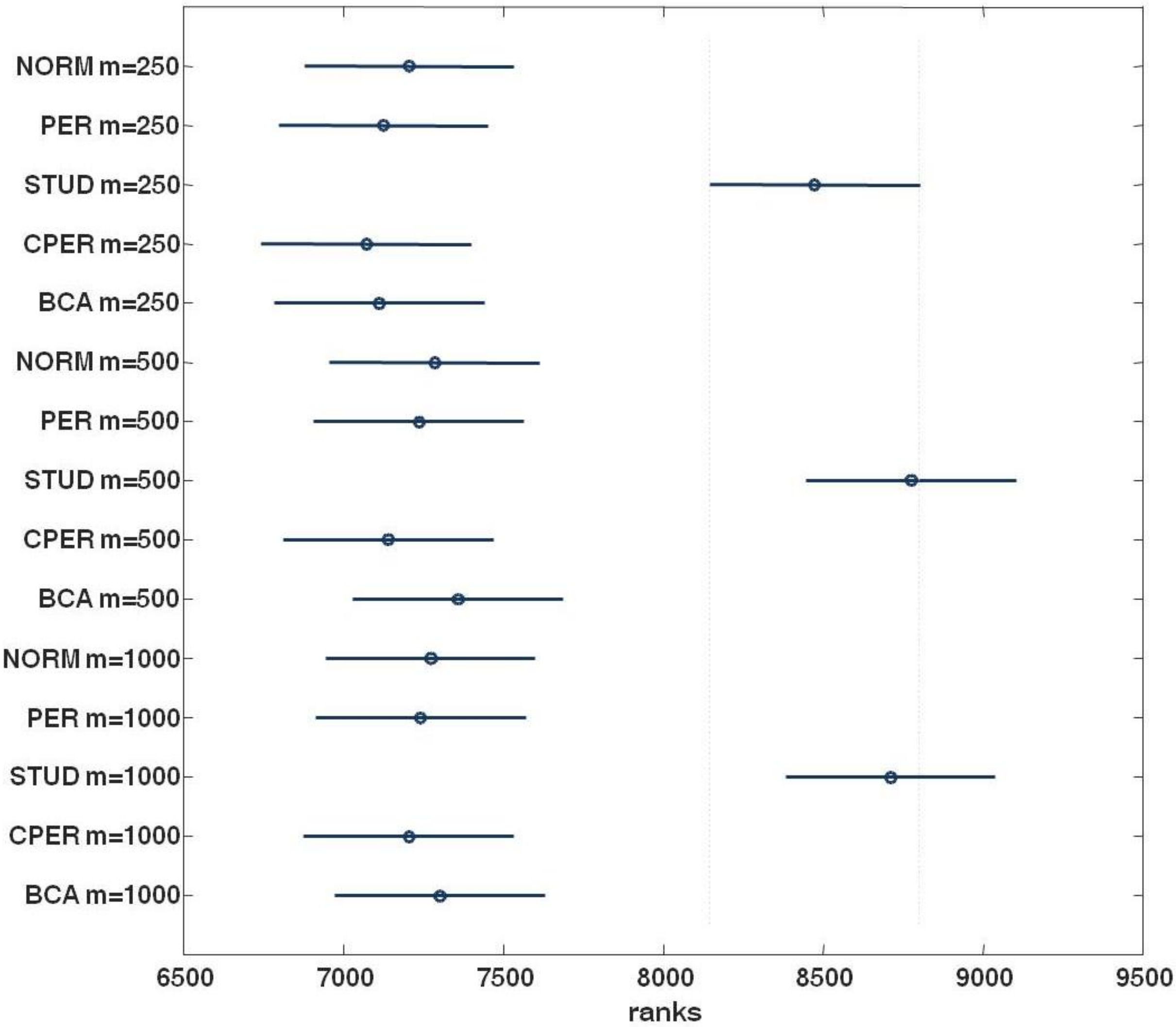


Fig. 8: Multiple comparison performed through the Tukey-Kramer Test

Two main results can be noted: *i)* for each bootstrap method, there are not statistically significant differences in the distributions resulting from different number *m* of replications; *ii)* the confidence interval widths determined with the application of the STUD algorithm are significantly statistically different from widths determined by means of other bootstrap methods, whereas there are no statistically significant differences between 95% condence interval widths obtained by means of NORM, PER, CPER, and BCA techniques. As an example, the probability values obtained from the test for *m*= 250 are summarized in Table II.

Table II. p-values of the Tukey-Kramer Test

NORM	PER	STUD	CPER	BCA	
-	1.00	4.34E-07	1.00	1.00	**NORM**
	-	4.29E-07	1.00	1.00	**PER**
		-	4.29E-07	4.29E-07	**STUD**
			-	1.00	**CPER**
				-	**BCA**

As a further result, for the determination of CI about the $L_{eq,A}$, the authors recommend the CPER algorithm, whose application exhibits the greater repeatability (the smaller standard deviation observed for the computed width distribution, see Table I).

As a last step of analysis, the results of the proposed approach have been compared with the estimation of the expected value for the short term noise indicator and of the corresponding uncertainty by classic method (according to the ISO GUM).

In more details, the expected value of the equivalent sound pressure level (referring to the observation time = 15 minutes) - in the classical approach - is determined by Eq. (1):

$$\bar{L}_{eq,A} = 10 \log\left(\frac{1}{n}\sum_{1=1}^{n} 10^{0,1\, L_{eq,Ai}}\right)$$

where n is the size of the considered sample.

The corresponding uncertainty is determined by Eq. (2):

$$u(\bar{L}_{eq,A}) = \sqrt{\frac{\sum_{i=1}^{n}\left(L_{eq,Ai} - \bar{L}_{eq,A}\right)^2}{n(n-1)}}$$

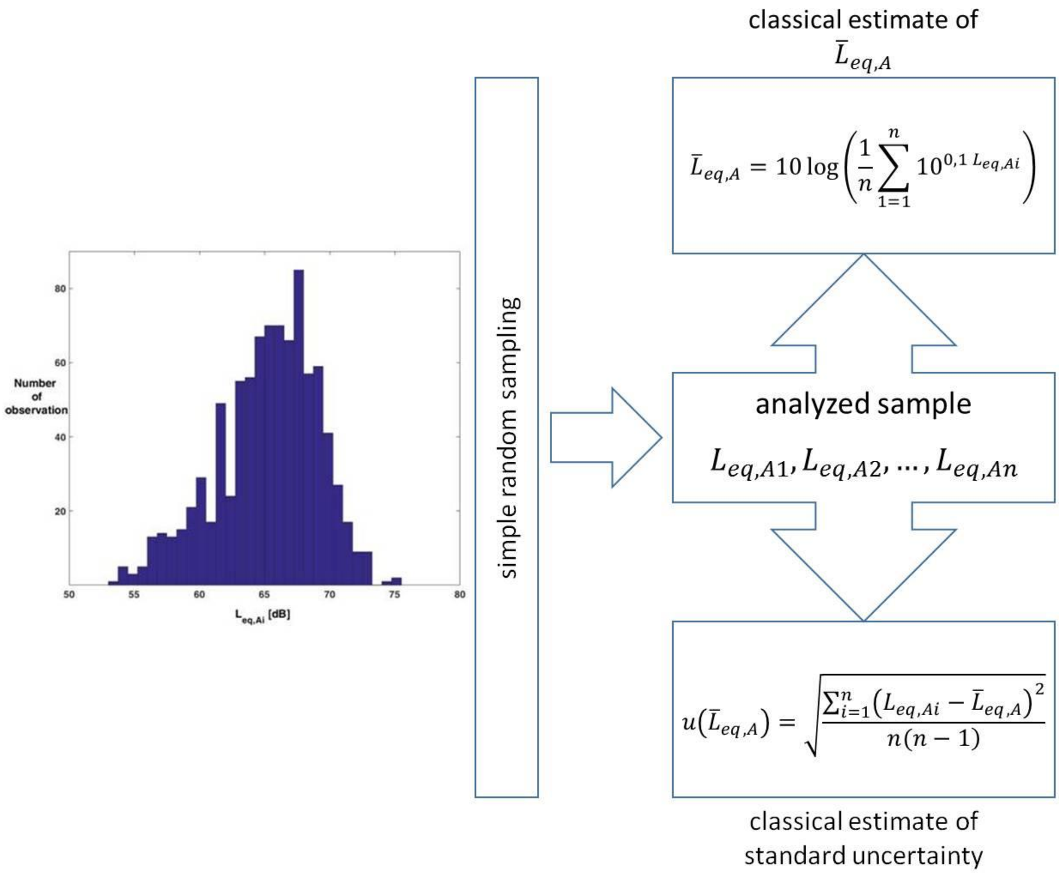


Fig. 9: Estimation of expected value and type uncertainty according to the classical method

The calculation scheme according to the classical approach is depicted in Fig. 9.

According to the bootstrap approach, the expected value of the equivalent sound pressure level is determined as the midpoint of the CI resulting from the application of the CPER algorithm. The corresponding uncertainty is calculated as standard deviation of the expected values with respect to k iterations of the CPER algorithm, according to the scheme in Fig. 10.

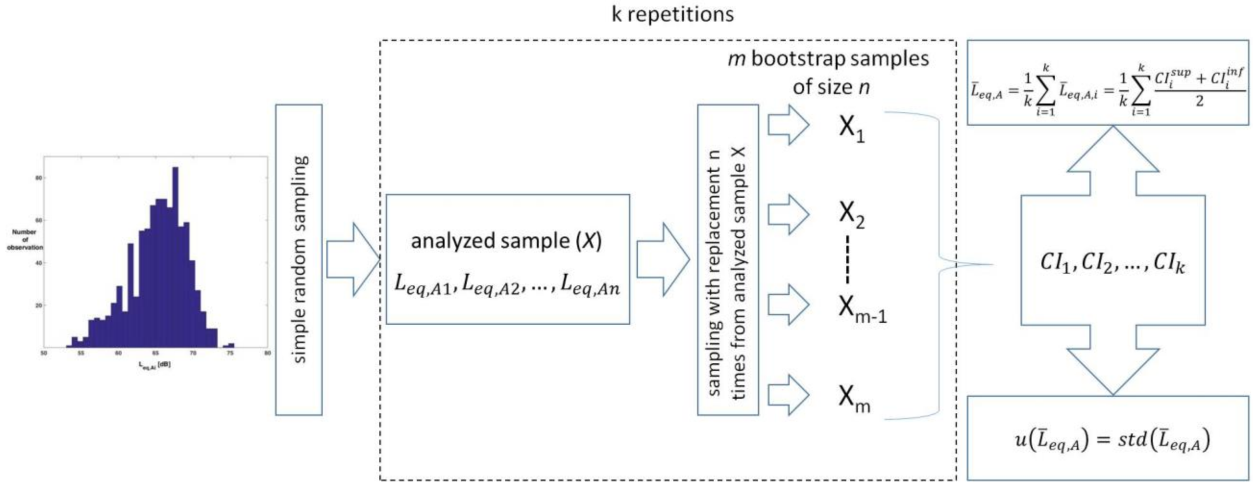


Fig. 10: Estimation of expected value and type uncertainty according to the bootstrap approach (CPER algorithm)

Table III summarizes the comparison results when the classical and bootstrap approaches have been applied to the samples (with different size) randomly extracted by 13th sequence. About the classical method, for each sample size, both the expected value and uncertainty are achieved as averaged values on $k = 1000$ repetitions (in order to reduce the influence of the random sampling from the population). The same number of iterations has been considered to estimate the uncertainty of the equivalent sound pressure level according to the bootstrap approach. The references are determined by the classical method applied to the whole population (900 samples), resulting in 66.74 dB and 0.14 dB for the expected value and the uncertainty respectively.

As you may note, both the classical and proposed methods lead to a very good estimation of the expected value for the short term noise indicator when a reduced number of samples are considered. Moreover, the reduced sample size introduces an overestimation of the true uncertainty, that is more evident for the classical method. Consequently, the bootstrap approach seems to be a very promising technique for the prediction of the environmental noise indicator from a reduced set of measurement data.

Table III Comparison of classical and bootstrap method for the estimation of the equivalent sound pressure level

	Classical Method			Bootstrap Method		
Sample Size (n)	**Expected Value [dB]**	**Uncertainty [dB]**	**Δu [dB]**	**Expected Value [dB]**	**Uncertainty [dB]**	**Δu [dB]**
50	66.72	0.58	0.44	66.70	0.50	0.35
60	66.72	0.54	0.40	66.70	0.48	0.34
100	66.72	0.42	0.28	66.74	0.35	0.21
120	66.73	0.38	0.24	66.74	0.32	0.18
150	66.74	0.34	0.20	66.74	0.27	0.13

IV. CONCLUDING REMARKS

An original proposal has been introduced for the evaluation of the uncertainty of traffic noise measurement. It aims to provide a Confidence Interval (CI) for the indicator $L_{eq,A}$ through two main steps: first, the outliers are removed from the measured data with a method based on K-neighbors distance; then, the estimation of the uncertainty interval is performed with a bootstrap procedure based on resampling method. The main parameters of the bootstrapping (i.e. the number of bootstrap replications *m* and the kind of algorithm for CI computation, NORM, PER, STUD, CPER and BCA) have been compared with respect to experimental data acquired during the measurement of traffic noise on a motorway with fast moving vehicles. More in details, a statistical analysis has been carried out about the widths of 95% confidence intervals and the comparison has been performed through the non-parametric Tukey-Kramer test at significance level $\alpha = 0.05$.

As results of the comparison, the minimum number of bootstrap replications (m=250) suggested in literature as rule of thumb has revealed enough to estimate an accurate CI for the noise indicator: the adoption of greater values of *m* does not provide statistically significant differences but it introduces a greater computational burden. About the different bootstrap algorithms, the CI widths obtained using the STUD model are statistically significantly different from other methods and the corresponding distributions are characterized by the greater standard deviation. The application of NORM, PER, CPER and BCA methods leads to similar behaviour (statistically significant differences have been not observed). In particular the results from the CPER algorithm exhibit the greater repeatability for the CI about the $L_{eq,A}$: the smaller standard deviation observed for the computed width distribution. Moreover, a comparison between the proposed approach and the classical method has been performed to estimate the uncertainty of the short term noise indicator. Again, the bootstrap approach has been revealed very promising in the prediction of the expected value and uncertianty of the traffic noise when a reduced dataset is considered. Thanks to its streamlined structure, the procedure illustrated can be easily integrated into applied noise measurement instrumentation, in order to provide real-time information on the uncertainty of the measurand.

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