CHAPTER 7: SCRIPTING COLLABORATION FOR COMPETENCE-BASED MATHEMATICS LEARNING: A CASE STUDY ON ARGUMENTATION

Giovannina Albano[1], Umberto Dello Iacono[2]
University of Salerno, Italy
galbano@unisa.it, udelloiacono@unisa.it

Full address for correspondence:
c/o DIEM - Università degli Studi di Salerno
Via Giovanni Paolo II, 132
84084 - Fisciano (SA)
Italy

E-mail address: galbano@unisa.it

Abstract: This work concerns the use of scripting collaboration in order to implement an innovative approach to competence-based mathematics learning. In this chapter, we show how Digital Interactive Storytelling in Mathematics (DIST-M) can be used to engage students in shared argumentative experiences and how the technology supports the students' reprocessing and appropriation within their knowing. The design is based on a network of theories and students are engaged in activities within a storytelling experience. The activities use both experiential and discursive approaches to mathematics learning, integrating individual and social tasks, defined by external scripts. We merge free tools and define new applications, which allow to integrate and manipulate dynamic graphs as well as to construct open sentences starting from available blocks of words. We also discuss the outcomes of a case study.

Keywords: Mathematics education, Collaboration script, Digital Storytelling, Argumentation, Linguistic cohesion.

1. INTRODUCTION

This paper concerns the definition of a design methodology, Digital Interactive Storytelling in Mathematics (DIST-M), for competence-based mathematics learning in e-learning environment. It is based on the assumption that such an environment can be arranged in a way that a good exploitation of platform tools and a well-structured collaboration among peers can act as an "expert" and scaffold students in achieving their learning goal (Albano, Dello Iacono, & Mariotti, 2017; Albano, Dello Iacono, & Fiorentino, 2016). The underpinning theoretical approach is framed in the socio-constructivist view of learning, where students construct their own knowledge and are actively engaged in social interactions (Vygotsky, 1978). The DIST-M consists of collaboration scripts, aimed at regulating and structuring roles and interaction in a collaborative setting (King, 2007). As suggested by the name, the described methodology is implemented in a storytelling framework, where the students are characters in a story and they interact facing problems, whose solution is needed to advance. The design on one hand can motivate learners and, on another hand, can have benefits of the integration between narrative and logical-scientific thought (Zan, 2011).

In this chapter, we apply the DIST-M in a case study concerning the argumentative competence in mathematics and analyze whether it can promote the production of written arguments according to a register shared in the mathematical scientific community. In fact, as shown by PISA results, a critical challenge for 15 years old students is expressing arguments and conclusions in written form (Turner & Adams, 2012). In the frame of discursive approach to mathematics learning, seen as initiation to a particular mathematical discourse (Sfard, 2001), Ferrari (2004) shows that mathematical language and written literate registers of ordinary language share many features. Thus, he concludes that being familiar with written communications is a prerequisite to promote advanced mathematical thinking. To this aim, there is a need of a shift from the request of just solving a problem to the request of verbal and or written explanations.

In the following, first, we share the theoretical framework, then we describe the design of DIST-M. Finally, in the case study we discuss its outcomes from a qualitative point of view, taking into account the collaborative and argumentative features of the design. The analysis will highlight the arguments produced by the students and assumes a linguistic perspective, focusing on the organization of the verbal texts, as cohesive texts, which means words and sentences perceived as a whole entity.

2. THEORETICAL FRAMEWORK

This work approaches mathematics learning, taking into account a network of theories: discursive approach (Sfard, 2001), computer supported collaboration script (King, 2007), especially with respect to argumentative competence (Weinberger *et al*., 2007).

Sfard's (2001) notes that, communication in mathematics cannot be considered simply an aid to thinking, but, primarly because mathematics learning can be defined as initiation to a particular (mathematical) discourse, it should be thought of as thinking. When students construct arguments, they elaborate and explain to themselves the concepts that they are justifying (Baker, 2003). Such explanations to themselves help the students to integrate new information within cognitive structures previously existing (Chi *et al*., 1989). When students share their explanations in a work group, they are expected to produce arguments, in form of communicable texts (Boero, 1999), that are socially acceptable, and, in particular, "mathematically acceptable" (Mariotti, 2006). In the frame of the discursive approach to mathematics learning (Sfard, 2001), the quality of such texts is strictly linked to the quality of thinking, and thus of argumentation, so that the use of literate registers should be considered an indicator of quality, and at the same time an educational objective (Ferrari, 2004).

In spite of the benefits of collaborative work, it is well known that collaboration is not spontaneous and successful without being well structured (Laurillard, 2013). To this aim, educational scripts can be externally imposed, regulating roles and actions that the students are expected to assume and to carry out in order learning occurs successfully in collaborative learning (King, 2007). Their use has been implemented in computer-based environments (Weinberger *et al*., 2009), where external scripts can establish the roles of the participants and the sequence of the tasks to be performed (King, 2007), regulating the interaction and collaboration in order to foster suitable cognitive processes. Two levels can be distinguished in a script: one macro and one other micro. The former concerns how to group and to assign the roles, whilst the latter defines a suitable sequence of events and of specific tasks to be carried out for the effectiveness of the cooperation. The aim of the educational scripts is that they will be interiorized along the time through the social practice (Vygotsky, 1978), in order to bring the students to be more autonomous learners.

Literature also shows the failure of spontaneous collaboration in order to produce arguments (Kuhn, Shaw, & Felton, 1997). Andriessen, Baker, & Suthers (2003) notes that computer based collaboration scripts can help, since they allow the students to communicate among them by means of text-based interfaces and to write and read texts and messages. This modality let them to be faster than in face-to-face setting in order to read and revise their own speech and the peers' ones (Pea, 1994). Moreover, it also encourages weak students to participate in debates, supporting the recovery of previous gaps.

Weinberger *et al*. (2007) distinguishes three types of components into a computer based collaboration script on argumentation: epistemic, argumentative, social. The epistemic components aim to structure the collaborative activities, focusing on the discussion's content and on the steps needed to carry out the task. They can support students in finding suitable solving strategies. The argumentative components aim to support the construction of arguments admissible with respect to a fixed formal system. The social components define the interactions among the students in order to promote the construction of knowledge. They take care of the engagement of the students in collaborative activities that cannot occur spontaneously, as for instance encouraging students to reply critically to their peers' contributions.

3. THE DIST-M METHODOLOGY

The methodology defined in Distance Interactive Storytelling (DIST) is competence oriented, framed in an e-learning environment. DIST makes use of digital storytelling, which becomes interactive due to the use of applications allowing the student to manipulate objects (graphics, multimedia, etc.) and due to feedbacks given by the e-learning platform. When a DIST concerns a mathematical competence, it will be called DIST-M, short for Digital Interactive Storytelling in Mathematics.

The DIST is organized as collection of Frames. Each Frame is sequence of scripts and each script consists in one or more tasks, where a task is a learning activity. The first Frame, called Introduction, aims to let the student become familiar with the digital environment, the storytelling and the content pre-requisites. All the other Frames, labelled as "Frame of Level," aim to mediate various levels of the specific competence at stake. Thus, starting from a Frame of Level 1, focused on a basic level of competency, Frames of greater level mediate the same competence at higher levels.

The tasks can be individual, collaborative or mixed. In the individual tasks, the student is expected to work and to delivery her products individually, not communicating with peers. In the second, collaboration among peers is guided by the task design and it is realized by means of constraints in the use of tools such as chat, forum, wiki, etc. In the mixed tasks, the student can take advantage of communication with peers (usually by chat) but she is required to delivery her work individually. The underpinning idea is that scripts should be designed as sequence of collaborative and individual tasks so that learning is first socialized and then interiorized, consistent with a Vygotskian view (Vygotsky, 1978).

4. THE DESIGN OF A DIST-M CONCERNING ARGUMENTATION

In the following, we describe a DIST-M on argumentative competency. The DIST-M aims at providing the students with a methodology of construction and communication of arguments in mathematics. In the literature, some authors studied the effectiveness of collaborative scripts in fostering mathematical argumentation skills (Kollar *et al*., 2014, Vogel *et al*., 2015) focusing on the construction of a conjecture and on its proving process by means of explanation, arguments, counterarguments and synthesis. In our case, we focus on the elaboration of a communicable text (Boero, 1999) as an answer to a question, explaining the correctness of the answer based on mathematical arguments and expressed in a literate register (Ferrari, 2004) and according to shared socio-mathematical norms which makes the statement acceptable in the reference scientific community (Mariotti, 2006). The design of the scripts within the DIST-M is based on the idea of transferring the mediation role of the teacher to the peers and to the device. Thus, it foresees various types of interactions, both with the device and the peers, and it enables students to produce personal arguments, to compare with the peers' ones and to elaborate their own final argumentative text in a literate register. In this view, the epistemic components of the script do not explicitly give solving strategies, but they aim to recover previous knowledge and skills gaps (see section 4.2) and to activate processes of individual and collaborative reflection, whose side-effects should be self-regulation and self-correction. The argumentative components of the DIST-M aim to let the student explain her reasoning to her peers and then to convert it in a literate register. The social components of the DIST-M have been designed in order to promote the interaction among peers, for instance in order to agree upon a common answer and statement, to ask and give help to peers in troubles, and so on.

The actual implementation of the DIST-M includes the Frame Introduction, consisting of one script, and the Frame of Level 1, consisting of three scripts, named Chapter 1, Chapter 2 and Chapter 3. It has been implemented in Moodle, a common e-learning platform (https://moodle.org). Among its tools, we have used: Chat (for informal and speed communication); Answer and Question Forum (as students cannot view other students' posts before posting their own comment); Wiki (as a LogBook, where to write information or add images useful for the continuation of the activities); Lesson Module (to allow personalized learning path according to the student's needs along the DIST-M); Task Module (to share the students' products within a group). Further, we have integrated Moodle pages with comic strips, realized by Tondoo (www.toondoo.com), to implement the storytelling, and with new interactive manipulative objects, realized by GeoGebra (www.geogebra.org) as shown in the next section.

4.1. The technological innovations

Three new resources, consisting of interactive applications created by GeoGebra, have been defined: Tutorial, Interactive Graphical Question and Interactive Semi-Open Question. Once created, they have been loaded to the Community for the users of GeoGebra (https://www.geogebra.org/materials) and they have been made visible by the URL address and the HTML code has been incorporated in the short answer question page (a single word or short phrase answer should be provided) of the Moodle Lesson. In the following chapters, we will detail each resource.

The Tutorial enables the student to inteact with a designed object (graphs, text boxes, tables, etc.) in order to find a configuration of the object as answer to a given question. The application, at the end of the student's manipulation, gives back a code, according to the correctness or not of the given answer. The student is expected to insert, in a suitable text box of the short answer of the Lesson page, such code, which steers the student to a subsequent personalized path.

The Interactive Graphical Question (IGQ) differs from Tutorial, since the code given back does not correspond only to the two options (correct or incorrect answer). Given that in many cases, various decisions and configurations of the graphical objects can be correct, the IGQ allows for a variety of parameters, so each configuration can be correct (all parameters are admissible), semi-correct (some parameters are admissible), wrong (no parameter is admissible). Thus, the application has been designed in order to be able to generate dynamically a code that reveals the manipulation of the student with respect the two previous features. Then the code allows a very fine subsequent personalization.

The Interactive Semi-Open Question (ISQ) allows to construct the answer to a given question by assembling some available words-blocks by means of dragging. The expected answer should be constituted as a main sentence linked to a secondary one, which concerns the arguments to support what stated in the main sentence. The correctness of the answer depends on the two sentences, thus we can have correct, semi-correct and wrong answers. The code given back is generated dynamically taking into account which words-blocks have been used and how they have been assembled, allowing to know exactly which sentence the student created and thus to foresee fine personalized paths. From the technological point of view, the ISQ allows the envirorонment to overcome, at least partially, the problem of automatic assessment of open-ended questions. Actually, it can be very near to a real open-ended question if the words-blocks are suitably chosen to allow the student to construct sentences alike in language and thought to the ones she actually uses in a similar situation. From the educational point of view, the careful selection of the words-blocks to be made available can foster the argumentative competence. In fact, in our case, they can make evident the general structure of an argumentation in a literate register, highlighting the causal conjunction between the main sentence and the subordinate one, independently on the order of the two sentences (Albano, Dello Iacono, & Mariotti, 2017).

4.2. The Frame Introduction

The Frame Introduction aims to steer the student within the storytelling and the digital environment. It consists of a single script that introduces the student in the story "Discovery Programme". As an example, on Discovery Programme noted "Life on the planet Terra is at risk: a huge impact with a meteorite is foreseen in 2150, which can cause the extinction of life on the planet. All the scientists in the world are working hard in order to find a solution as soon as possible. A new planet in the solar system has been discovered and the NASA has launched the space probe COLOMBO to collect data from the new planet, which need to be analyzed in order to test if life is possible on it." In this case, the student plays the role of a scientist of the NASA, as member of an équipe (team) supervised by Professor Garcia (guide and voice of the storytelling) (Figure 7.1).

Figure 7.1: The équipe in Discovery Programme

Along the storytelling, the student is going to face questions concerning statistics and statistical graphics, needed for the équipe work. Thus, the mathematical content at stake concerns representation and management of graphics of descriptive statistics, which constituted the contents of the Tutorials (which are not the focus in this paper).

4.3. The Frame of Level 1

In the following, we go into details of the script Chapter 1 within the Frame of Level 1. The design, shown in Figure 7.2 is based on both experiential and discursive approach. So, on one hand, the student can manipulate interactive objects in order to formulate and test hypotheses and, on the other hand, she is expected to debate with herself and with the peers.

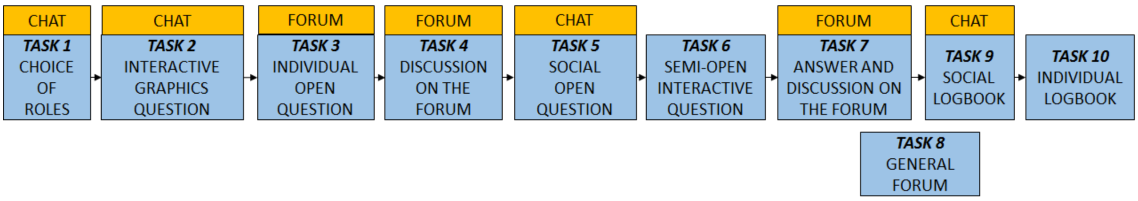

Figure 7.2: Design of Script Chapter 1

At beginning (task 1) each student is expected to choose a role to play in her group, by negotiating it with the colleagues. The roles foreseen are the following:

- the Captain, the leader of the group who takes care of engaging all the mates in the discussions and in the decision processes (social literacy);
- the Scientific Official, who is in charge of collecting and summarizing all the mates' answers concerning mathematical questions to be solved during the mission (mathematical literacy);
- the Technological Official, who supports the mates who are in troubles in using the platform (digital literacy);
- the Communication Official, who reports and summarize the conversations of the mates when a shared communication/answer is required (communicational literacy).

Then, the group is required to work with an ISQ, where suitable manipulation of a graphical object gives the answer to a posed question (task 2 part 1). According to the code, corresponding to the final object's configuration chosen by the student (see section 5.2), a personalized reflective question is delivered (task 2 part 2). It aims to steer the student towards self-regulation processes by means of being aware of what done and why: further possible correct configurations of the object, if she was successful; reasons of her choice, in case of semi-correct configuration; and on what has brought her to generate a wrong configuration, in case of failure.

Task 3 requires the student to answer to an individual open-ended question, aimed to shift from the previous experience to a general case. It should bring out the elaboration of arguments to justify the given answer. Each student has to post her answer in a Question and Answer Forum, in order to avoid the influence of peers' arguments and to force the participation.

When all the students posted their answers, in order to elaborate a shared answer (task 4), a discussion is started in the same Forum. The use of the forum guarantees that everyone completes the previous task, otherwise they cannot access to the peers' answers, and differing from the chat where communication is immediate and not so formal, in the forum there is an implicit request of a shift towards a more literate register. Once agreed upon the answer, the students deliver it by the Moodle Task module, in collaborative setting so that each member takes her responsibility to deliver the shared answer (task 5).

Later (task 6), the student individually converts it into a more literate text, assembling suitable words-blocks (see ISQ, section 4.1). The words-blocks have been constructed in order to highlight the causal structure of the sentences, that is the causal conjunctions (i.e. *since*, *because*, etc.) constitute single blocks, allowing to link two sentences (main and conditional ones) constructed by more other blocks. The ISQ recognizes the correctness of the construction independently on the order of the sentences.

The task 7 requires the student to post the answer constructed in the Question and Answer Forum together with explanation of her reasoning and to discuss all the answers in the thread. If she was successful in the previous task, then she is acknowledged of this (she has the title of Champion) and she is asked to help her classmates. A further general Forum is activated where all the Champions (eventually, also a teacher who can be essential when there is no champion) are available for anybody at risk (task 8).

At the end of the activity, the students are required to edit a Social LogBook (task 9) and an Individual LogBook (task 10). The first one is composed using a Moodle collaborative wiki, aiming to collect and store all the cognitive information useful for the mission. The second one is referred to a metacognitive reflection of the student on the activity, on the difficulties encountered and how she overcome them.

The various tasks of the script refer to components that can be social, epistemic and argumentative. All the tasks that envisage debate, discussion, comparison among the students can be ascribed to social components (tasks 1, 4, 5, 7, 8, 9). As the aim of the script is to foster argumentation, most of the tasks encourage the students to reflect in order to explain, to clarify, to produce arguments, so they can be considered argumentative components. Finally, as the arguments the students should produce concern problem solving activities, solving strategies, recovery of errors, many tasks can be seen as epistemic components (tasks 2, 3).

5. THE CASE STUDY

5.1. The mathematical problem

The mathematical problem of the case study concerns the invariance of the angle of a circular sector with respect to the length of the radius of the circle. The tasks of the script aim to put the student's attention on the relationship between the size of the angle and the radius of the circle. In task 2, a circle with unit radius is presented, where a circular sector corresponding to a 72 ° angle represents the percentage of red stone on the new planet (20%) and the student must enlarge the chart manipulating the radius and / or the size of the angle (by means of an IGQ), unchanging the percentage.

After manipulating the circle graph, a scientist of another group appears asking the student a question: “Can you explain how you chose the angle?”, in case of semi-incorrect or incorrect configuration, or “I would like to take as radius X and to leave the angle at 72°. Does anything change?”, in the case of a correct configuration with a radius different than X. The student faces the same question several times in order to generalize the experience and the results achieved: “How does the size of the angle of colored sector changing the radius? Explain your answer”, first individually (task 3), then collaboratively (task 5) and finally by means the ISQ (task 6).

5.2. Methodology

The Frame Introduction and Chapter 1 of the DIST-M have been tested in a pilot involving eleven 10th Grade students from Grammar High School “Virgilio” in Southern Italy. The students were randomly split into 4 groups, 3 of which consisting in 3 members and 1 consisting in 2 ones. In groups of 3 members, one of the students played 2 roles (task 1), whilst in the group of 2 members, each student played 2 roles. Each student worked on her PC, for twenty hours, logged into the platform by username (such as S1, S2 etc.) and password, and communicated within a group only by means of the tools of the platform.

5.3. Analysis Tools

Our DIST-M aims to support the production of written verbal arguments by the students. Some theoretical models of analysis of the arguments do not refer to language, but we assume that a written argument is, firstly, a written text and the production of a correct text is closely intertwined with an acceptable explanation. This is why we chose to use a linguistic approach to analyze the data, using tools such as textual cohesion that consider the text a single entity rather than a collection of words and disorganized sentences (Halliday and Hasan, 1976). Cohesion is different from coherence, although they are very interrelated. The coherence identifies the connection among the various sentences and, thus, allows to give them continuity of sense, that depends on who writes and who interprets the text, starting from her encyclopedic knowledge. Therefore, coherence is closely linked to the interlocutors rather than to the language itself. Instead, the cohesion concerns the grammatical way in which the sentences are related to each other. Then it refers to the linguistic tools needed to achieve coherence and helps to highlight it, although a text can be perceived as coherent without cohesion markers (Thompson, 1996). Lexical or grammatical repetitions and conjunctions are markers which can be used to realize the cohesion. A lexical repetition consists in repetition of words. It is a very powerful form of cohesion and it is often used to emphasize or strengthen a concept. As grammatical repetition we consider only the reference, which is . used to indicate whether a term is repeated somewhere earlier in the text (it has already been said) or if it has not yet appeared in the text (it is new). The conjunction is a cohesion marker which connects any two parts of speech. It can be external, when it reflects a state of fact, internal, when it refers exclusively to the organization of the text.

6. ANALYSIS AND DISCUSSION OF THE OUTCOMES

In this section, we analyze the work of each group of students in the study and share the results and outcomes as case studies. In particular, we analyze the students' transcripts with respect to the impact of the social components, (collaboration in task 4 and task 5), and the argumentative component (the ISQ in task 6) on the production of argumentative texts. For this reason, we look at the transcripts related first to the task 2 - part 2 and task 3 (individual phase before collaboration and ISQ) and then to task 7 (individual phase after collaboration and ISQ). Moreover, we examine transcripts related to Social Open Question (task 5), Social Logbook (task 9) and Individual Logbook (task 10) to go more in depth.

As was mentioned, a qualitative approach was used to to deepen our understanding of the quality of argumentation. More specifically, for each transcript, we looked at the existence of arguments and the quality (cohesion) of the text. We define the following qualitative model (Table 7.1) which allows us to classify the argumentative texts with respect to the links among various pieces realized by means of cohesion markers (Thompson, 1996):

Table 7.1: Qualitative model

Level 0	Text with no argument. An example is S9's answer to task 3: "the angle always remains the same by changing the radius"
Level 1	Text without markers or with generic markers (such as 'and') or with even less generic markers but that do not expose the links. For instance, S8's answer in task 2: "It does not change anything because the angle always remains the same, along with the percentage" (there is a non-generic marker, "because", but the links are not explicit)
Level 2	Some links are also appropriately explained but there are some pieces that are not related: typically, one of the relationships is well highlighted, while the others are in the shadows (that is, it significantly requires the collaboration of the reader). See S8's answer in task 3: "The colored part increases as the circumference and radius increase, but the angle is always the same, i.e. 72° and the percentage is 20%" (there is the explanation of 72° as percentage of 20 %, but the speech is not clear and a significant collaboration from the reader is required)
Level 3	All the relevant links are explained through markers. See S8's answer in task 7: "The angle does not vary because it is directly proportional to the circumference. If the radius increases, with it also the circumference, but this does not affect the angle amplitude because grades remain unchanged and also the percentage. The increase of the radius makes the graph clearer when added more data, but the colored part, that is, the red rocks, will always remain 20% and 72°" (non-generic markers, explicit links).

We assume level 2 as sufficient level of argumentation in the sense that it is perceived that the student has in mind the argument but she is able to communicate only partially. Table 7.2 summarizes the outcomes according to the defined model and level for each student, labelled S#:

Table 7.2: Levels of argument for each student in task 2, 3 and 7

Task\Group	**1**	**2**	**3**	**4**
2	S1: 1 S2: 2 S3: 0	S4: 0 S5: 0 S6: 1	S7: 0 S8: 1 S9: 0	S10: 1 S11: 1
3	S1: 0 S2: 2 S3: 1	S4: 0 S5: 1 S6: 2	S7: 0 S8: 2 S9: 0	S10: 2 S11: 2
7	S1: 2 S2: 3 S3: 2	S4: 1 S5: 2 S6: 2	S7: 2 S8: 3 S9: 2	S10: 3 S11: 3

As shown in the above data, we can observe a slight change from the task 2 to the task 3. Recall that task 2 part 2 requires the student to reflect on what happens in different cases of her manipulation and task 3 asks for a generalization of what found previously. Only few students, at task 3, are able to do that with partially explicit arguments (mark 2), some others are not able to give any argument (level 0) and nobody is able to write an argumentative text (level 3).

In the following only one group has been discussed in detail, but similar results were observed in the others.

We now share some excerpts from the group 1 (Table 7.3). All the students in this group succeeded in IGQ (task 2 part 1), and were then asked if something changes in the circle graph choosing a radius different from the one taken into account by the student during the manipulation (task 2 part 1).

Table 7.3: Excerpts from the group 1 in task 2 and task 3

Task	S#	Excerpt	Analysis
2	S1	Nothing changes because the percentages and the degrees are always the same.	Only one external conjunction (“because”) not exposing the links.

	S2 S3	Nothing changes because, incrementing the radius, only the circumference grows and thus the degrees are always the same. The radius increases but the size does not vary.	Two external conjunctions (“because”, “thus”), some pieces are not related. No argument.
3	S1 S2 S3	The angle does not change varying the radius. The colored part increases according the increment of the circumference but the angle is constant (72°) as well as the percentage (20%) Nothing changes, since the degrees, increasing or decreasing, the circumference remains constant.	No argument. Two external conjunctions (“but”, “as well as”), more explicit arguments with respect to the one given previously, but some pieces are not related. Only one external conjunction (“since”) not exposing the links.

Table 7.2 also shows a meaningful improvement from task 3 to task 7. Let us see some excerpts from the group 1 (Table 7.4). Every student constructs arguments at a sufficient level (level 2), in particular the student S2 improved beyond (from level 2, already reached at task 3, at level 3)

Table 7.4: Excerpts from the group 1 in task 7

Task	S#	Excerpt	Analysis
7	S1 S2	The angle does not vary because it is directly proportional to the circumference, since increasing the circumference the percentage of the angle does not vary, because the degrees are always 360°. The angle is directly proportional to the circumference thus it does not vary. Increasing the radius, there is an increase of the size of the circumference but not of the size of the angle within. The degrees of the	The student goes beyond the sentence constructed by the words-blocks (first part), adding explanations that refer to somehow implicit arguments. Two external conjunctions in the second part (“since”, “because”) and two lexical repetitions (“angle”, “circumference”). Some links are also appropriately explained but there are some pieces not related. The student goes beyond the sentence constructed by the words-blocks (first part), adding many explanations that refers to explicit arguments. Two external conjunctions (“but”, “then”), some lexical repetitions (“angle”, “circumference”) and

		circumference are always 360° and the percentage is always 100. Then any change we make to the size of the circumference, the angle will remain always constant and fixed.	a substitution (“but not of” instead of “but there is not an increase of”). All the relevant links are explained.
	S3	The angle does not vary because it is directly proportional to the area of the circle, because increasing the circumference the angle remains constant. The size of the angle does not change since the radius does not move but it only increases.	The student goes beyond the sentence constructed by the words-blocks (first part), adding explanations that refer to somehow implicit arguments. Two external conjunctions (“since”, “because”) and a lexical repetition (“angle”). Some links are also appropriately explained but there are some pieces not related.

The tasks 3 and 7 require the student to answer the same question, before and after social tasks (tasks 4 and 5) and ISQ (task 6).

During the discussion before task 5, the students chose to deliver the answer given by S2 (table 2 task 3). They said: “Anyway all the answers are equal”. Actually, this is not true (Table 7.2), but they probably refer to the shared belief that the angle does not change. At the same time, the choice of S2’s sentence seems to make evident that all the members are somehow implicitly aware that the S2 sentence is the more complete, because it gives an argument too, even if not an explicit argument. We also note that S2 is not the communication official, thus she is not in charge of reporting the shared answer, so this means that the group really choices her sentence.

Next, we examine a chat in task 6:

S3: the explanation?

S1: the reasoning

S3: what do we write?

S2: let me think about

I guess that the radius is directly proportional to…

I DON’T KNOW :’((((((((

It is worthwhile to note that this (argumentative component) reveals the fact that an argument is expected and brought the students to think about. It seems not casual that S2 is the one more engaged in the reasoning, as she was the one who already reasoned (implicitly). We note that she seems to be in crisis because she is not able to say in words her thinking but this conflict highlights that something is wrong from the mathematical point of view and she becomes aware of this. In fact, S2 wrote in her logbook:

I had some difficulties because the size of the circumference could be whatever. And I knew that the answer I constructed with the words-blocks was wrong, but anyway I justified the reasoning after (see Table 7.4).

As conclusion, we share an excerpt from the logbook of the group 1:

We started the chapter 1 analyzing the give aerogram. After a sequence of procedures justified by an argument, we achieved a common theory, that is: the angle does not vary because it is directly proportional to the area of the circle. The angle is directly proportional to the circumference and thus it does not change. Increasing the radius, there is a growth of the circumference, but not of the size of the angle within. The degrees of the circumference are always 360 and the total percentage is always 100. Thus, any change we make to the size of the circumference, the angle will be always constant and fixed.

The above excerpt highlights a metacognitive process realized by the group working together. Looking back to the experience, they become aware of a shift from procedures to the need of a justification by arguments and finally to the achievement of a common theory (that is what they report, in the above excerpt, from the second line on).

Finally, comparing S2's excerpt (Table 7.4) and the above logbook excerpt, we can note that S2 acts as an expert in the group and her interactions with the group's mates allow an improvement of each member with respect to the argumentative competence. So, she mediates the underpinning educational objective, as desired, fostered by the script's design, requesting comparisons within the group, shared answer, conversion by words-blocks.

As the script is designed to scaffold argumentation, it was successful, even if the group was not able to give the correct answer from the mathematical point of view. Looking at the transcripts, students achieved an intuitive understanding of the mathematics but they lack some contents (i.e. direct proportionality) needed to convert their understanding in texts.

7. CONCLUSIONS

In this paper, we have introduced a general methodology to support competence-based learning. This method, DIST-M, represents an adaptation of the DIST methods, specifically created to support mathematics learning in an e-learning environment. The specific activity discussed focused on developing argumentative competence, using DIST-M methodology within a Moodle platform. The implementation involved using computer-supported collaboration scripts aimed to foster the students' shift from investigating and reasoning to communicate verbally what found, by constructing sentences with evidence of arguments in a style typical of scientific communication. The implementation utilized the Moodle features and supported collaboration among peers as well as individual work. Collaborative tasks utilize the chat or forum features, depending on what linguistic register is to be supported, and the particular setting of the forum to be used (Questions and Answer) avoid the students undue influence on each other. Analogously, the group features in Moodle-hosted tasks compels the students to take part in the activity. Individual tasks also benefit from the technology: the student can manipulate dynamical objects, graphical or textual, with automatic tracking, thus personalized recovery paths are delivered.

The analysis of the students' transcripts in chat and forum, in a linguistic perspective, have shown an improvement on the level of arguments given by each student. Moreover, they show how the script supports reasoning on mathematical concepts.

Looking at the tasks' flow, we conjecture that the improvement has been mainly fostered by two key points:

- the social tasks, that require to negotiate a delivery shared by all the members of the group, seem to come to light the need of producing arguments to support the answer; this request was present from the beginning, but many students did not give arguments when delivered individually;
- the argumentative task, that supports the treatment from the sentence in a colloquial register chosen by the group to sentence in a literate register, foster not only to refine the argument, but mainly to deepen the students' mathematical understanding as shown by their further cohesive sentences produced to explain their reasoning.

The above outcomes encourage further investigation of the design and effectiveness of DIST-M for promoting the ability of converting reasoning in constructing arguments expressed by cohesive texts.

8. REFERENCES

Albano, G., Dello Iacono, U., & Mariotti, M. A. (2017). A computer-based collaboration script to mediate verbal argumentation in mathematics. To appears in *Proceeding of Cerme10*.

Albano, G., Dello Iacono, U., & Fiorentino, G. (2016). An online Vygotskian learning activity model in mathematics. *Journal of e-Learning and Knowledge Society*, *12*(3).

Andriessen, J., Baker, M., & Suthers, D. (2003). Argumentation, computer support, and the educational context of confronting cognitions. In J. Andriessen, M. Baker, & D. Suthers (Eds.), *Arguing to learn: Confronting cognitions in computer-supported collaborative learning environments* (pp. 1–25). Dordrecht: Kluwer.

Baker, M. (2003). Computer-mediated argumentative interactions for the co-elaboration of scientific notations. In J. Andriessen, M. Baker, & D. Suthers (Eds.), *Arguing to learn: Confronting cognitions in computer-supported collaborative learning environments* (pp. 47–78). Dordrecht, Netherlands: Kluwer.

Boero, P. (1999). Argumentation and mathematical proof: A complex, productive, unavoidable relationship in mathematics and mathematics education. International newsletter on the teaching and learning of mathematical proof. (July/August 1999).

Chi, M. T. H., Bassok, M., Lewis, M. W., Reimann, P., & Glaser, R. (1989). Self-explanations: How students study and use examples in learning to solve problems. *Cogn. Sci*. 13: 145-182.

Ferrari, P.L. (2004). Mathematical Language and Advanced Mathematics Learning. In M. Johnsen Høines & F.A. Berit (Eds.), *Proc. of the 28th Conference of the International Group for the Psychology of Mathematics Education: PME 28* (2, pp. 383-390). Bergen (N): Univ. College.

Halliday, M. A., & Hasan, R. (1976). *Cohesion in English*. Longman, London.

King, A. (2007). Scripting collaborative learning processes: A cognitive perspective. In: F. Fischer, I. Kollar, H. Mandl, & J. Haake (eds.), *Scripting computer-supported collaborative learning: Cognitive, computational and educational perspectives* (pp. 13-37). New York: Springer.

Kollar, I., Ufer, S., Reichersdorfer, E., Vogel, F., Fischer, F., & Reiss, K. (2014). Effects of collaboration scripts and heuristic worked examples on the acquisition of mathematical argumentation skills of teacher students with different levels of prior achievement. *Learning and Instruction*,32, 22–36.

Kuhn, D., Shaw, V., & Felton, M. (1997). Effects of dyadic interaction on argumentative reasoning. *Cognition and Instruction*, 15(3), 287-315.

Laurillard, D. (2013). *Teaching as a design science: Building pedagogical patterns for learning and technology*. Routledge.

Mariotti, M. A. (2006). Proof and proving in mathematics education. In A. Gutiérrez and P. Boero (Eds.), *Handbook of research on the psychology of mathematics education: Past, present and future*, pp. 173–204. Rotterdam: Sense.

Pea, R. D. (1994). Seeing what we build together: Distributed multimedia learning environments for transformative communications. Special Issue: Computer support for collaborative learning. *Journal of the Learning Sciences*, 3(3), 285-299.

Sfard, A. (2001). Learning mathematics as developing a discourse. In R. Speiser, C. A. Maher, & C. N. Walter (Eds.), *Proceedings of the Twenty-Third Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 23-43). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.

Thompson, G. (1996). *Introducing functional grammar*. London: Routledge.

Turner, R., & Adams, R.J. (2012). Some drivers of test item difficulty in mathematics: an analysis of the competency rubric. Paper presented at AERA Annual Meeting, April 2012, Vancouver, Canada.

Vogel, F., Kollar, I., Ufer, S., Reichersdorfer, E., Reiss, K., & Fischer, F. (2015). Fostering argumentation skills in mathematics with adaptable collaboration scripts: only viable for good self-regulators? In O. Lindwall, P. Häkkinen, T. Koschmann, P. Tchounikine, & S. Ludvigsen (Eds.), *Exploring the material conditions of learning. The Computer Supported Collaborative Learning Conference (CSCL) 2015 - Volume II* (pp. 576-580). International Society of the Learning Sciences: University of Gothenburg.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Weinberger, A., Stegmann, K., Fischer, F., & Mandl, H. (2007). Scripting argumentative knowledge construction in computer-supported learning environments. In F. Fischer, H. Mandl, J. Haake, & I. Kollar (Eds.), *Scripting computer-supported communication of knowledge-cognitive, computational and educational perspectives* (pp. 191–211). New York: Springer.

Weinberger, A., Kollar, I., Dimitriadis, Y., Mäkitalo-Siegl, K., & Fischer, F. (2009). Computer-supported collaboration scripts: Perspectives from educational psychology and computer science. In N. Balachef, S. Ludvigsen, T. De Jong, A. Lazonder, & S. Barnes (Eds.), *Technology-enhanced learning: Principles and products* (pp. 155–174). Dordrecht: Springer

Zan, R. (2011). The crucial role of narrative thought in understanding story problems. In K. Kislenko (Ed.): *Current State of Research on Mathematical Beliefs XVI* (pp. 287-305). Tallinn Estonia: Tallinn University.